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NON-LINEAR ROBOT CONTROL: METHOD OF EXACT LINEARIZATION AND DECOUPLING BY STATE FEEDBACK AND ALTERNATIVE CONTROL DESIGN METHODS

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The dynamics of robots has to be considered as non-linear because of the couplings between the robot axes and the effects of the Coulomb friction within the joints. For the first type of non-linearities a state feedback control is often designed by the method of exact linearization and non-linear decoupling. But there are disadvantages of such control design: (i) exact knowledge about the system model is needed to avoid all the related sensitivity problems; (ii) discontinuous non-linearities, such as e.g. the Coulomb friction cannot be analysed by this method. Therefore two other design approaches are proposed: the method of "reverse linearization" and the method of estimation and compensation of non-linearities. They result in a robust independent joint control of robots.

1. Introduction

Today's industrial robots are almost exclusively equipped with independent joint controllers for the position control, although the robot dynamics is highly non-linear e.g. due to the position-dependent inertia moments and the coupling effects by the Coriolis moments. Therefore, for the improvement of the control performance different control concepts to decouple and compensate the non-linear dynamics have been developed in the robotics research for years (Spong *et al.*, 1992). Such concepts can be divided into two groups. On the one hand, multivariable controllers based on the multibody models of robot dynamics are designed, e.g. using the method of exact linearization by state feedback. On the other hand, the structure of independent joint control is kept and the non-linear effects are compensated by feedforwarding the "computed torques" obtained from desired trajectories or by the feedback of joint torques which are measured at the driven sides of each robot axis ("joint torque control").

When we compare the two model-based methods, the method of exact linearization is methodically more precise than the computed torque method, although it is more complicated (due to the on-line state feedback) than the latter one, in which the required feedforwarding torques can be computed off-line. In both cases, however, complete knowledge of models is assumed. Parameter inaccuracies and incompletely

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known friction effects lead herewith to the need of an additional robust control design. These problems disappear with the method of joint torque control. But to this end additional measurement devices are required.

Avoiding the disadvantages of the mentioned control design methods — complete knowledge of a system model and sensitivity problems or additional measurements — the method of non-linearity estimation and compensation can alternatively be applied (Müller, 1993; 1994a). This control method, proposed in this contribution, works with the usual measurements and the necessary information for the compensation is obtained by observers. It will be shown that this approach leads to the design of robust position controllers for robots. In addition to the robustness, its decentralized structure offers further advantages, where the concept of independent joint control can still be used. This method has produced good results in practical applications.

2. Model of Robot Dynamics

The dynamics of a robot with elastic joints and its motor drive dynamics can be modelled according to (Spong and Vidyasagar, 1989) as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{K}(\mathbf{q}-\mathbf{p}) = \mathbf{0} \tag{1}$$

$$\mathbf{J}\ddot{\mathbf{p}} + \mathbf{D}\dot{\mathbf{p}} - \mathbf{K}(\mathbf{q}-\mathbf{p}) = \mathbf{m} \tag{2}$$

$$\mathbf{T}\dot{\mathbf{m}} + \mathbf{m} = \mathbf{G}\mathbf{u} \tag{3}$$

where $\mathbf{q}$ is the vector of joint coordinates, $\mathbf{M}(\mathbf{q})$ is the positive definite mass matrix, $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$ represents the Coriolis and the centripetal as well as the gravitational forces. The vector $\mathbf{p}$ defines the motor angles relative to the gear ratios, whereas $\mathbf{J}$, $\mathbf{D}$ and $\mathbf{K}$ are diagonal matrices which stand for the effective moments of inertia of the motors and the dampings and stiffnesses between the motors and robot arms, respectively. The vector $\mathbf{m}$ represents the drive torques of the motors. The matrices $\mathbf{T}$ and $\mathbf{G}$ are diagonal and contain the time constants and the gains of the motors. Finally, $\mathbf{u}$ is the vector of input voltages of the electrical motors.

If effects of friction are also considered (as it will be done below), then they are included within the vector $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$.

3. Exact Linearization and Non-linear Decoupling by State Feedback

3.1. Control Design Method

The method of exact linearization and non-linear decoupling by state feedback is well-established (Isidori, 1989; Nijmeijer and van der Schaft, 1990). Therefore, the method itself will not be discussed here. But its application to the robot control problem (1)–(3) can be presented explicitly using the special structure of the mechanical system where each robot axis is controlled by a separate actuator. By assuming the joint coordinates as the output variables we can introduce new variables $\mathbf{z}_i$, $i = 1,\ldots,5$, as follows:

$$\mathbf{z}_1 = \mathbf{y} = \mathbf{q} \tag{4a}$$

$$\mathbf{z}_2 = \dot{\mathbf{y}} = \dot{\mathbf{q}} \tag{4b}$$

$$\mathbf{z}_3 = \ddot{\mathbf{y}} = -\mathbf{M}^{-1}(\mathbf{q})\Big[\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{K}(\mathbf{q}-\mathbf{p})\Big] \tag{4c}$$

$$\mathbf{z}_4 = \mathbf{y}^{(3)} = \frac{\mathrm{d}}{\mathrm{d}t}\ddot{\mathbf{y}} = \mathbf{a}_4(\mathbf{q},\dot{\mathbf{q}},\mathbf{p}) + \mathbf{M}^{-1}(\mathbf{q})\mathbf{K}\dot{\mathbf{p}} \tag{4d}$$

$$\mathbf{z}_5 = \mathbf{y}^{(4)} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{y}^{(3)} = \mathbf{a}_5(\mathbf{q},\dot{\mathbf{q}},\mathbf{p},\dot{\mathbf{p}}) + \mathbf{M}^{-1}(\mathbf{q})\mathbf{K}\mathbf{J}^{-1}\mathbf{m}[1mm] \tag{4e}$$

The control input vector additionally appears in the equation

$$\dot{\mathbf{z}}_5 = \mathbf{y}^{(5)} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{y}^{(4)} = \mathbf{a}_6(\mathbf{q},\dot{\mathbf{q}},\mathbf{p},\dot{\mathbf{p}},\mathbf{m}) + \mathbf{M}^{-1}(\mathbf{q})\mathbf{K}\mathbf{J}^{-1}\mathbf{T}^{-1}\mathbf{G}\mathbf{u} \tag{5}$$

The vector functions $\mathbf{a}_4$, $\mathbf{a}_5$, $\mathbf{a}_6$ are obtained by differentiation of the preceding equations having in mind eqns. (1)–(3). Choosing the control inputs as

$$\mathbf{u} = \mathbf{G}^{-1}\mathbf{T}\mathbf{J}\mathbf{K}^{-1}\mathbf{M}(\mathbf{q})(\mathbf{v} - \mathbf{a}_6) \tag{6}$$

a decoupled linear system

$$\mathbf{q}^{(5)} = \mathbf{v} \tag{7}$$

is obtained. Here $\mathbf{v}$ is the new input vector. If according to the path planning a desired trajectory $\mathbf{r}(t) = \mathbf{q}_d(t)$ has to be realized, then it can be reached by the control

$$\mathbf{v} = \mathbf{q}_d^{(5)} + \mathbf{k}_4(\mathbf{q}_d^{(4)} - \mathbf{q}^{(4)}) + \mathbf{k}_3(\mathbf{q}_d^{(3)} - \mathbf{q}^{(3)}) + \mathbf{k}_2(\ddot{\mathbf{q}}_d - \ddot{\mathbf{q}}) + \mathbf{k}_1(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{k}_0(\mathbf{q}_d - \mathbf{q}) \tag{8}$$

in which $\mathbf{k}_i$, $i = 0,\ldots,4$, are diagonal matrices with suitably chosen positive diagonal elements which determine the stable dynamics of each joint controller.

3.2. Problems

As fruitful and beautiful the method of exact linearization and non-linear decoupling by state feedback is from a theoretical point of view, as many problems appear in the case of its practical application. Some of them are briefly summarized

- Depending on the chosen output variables the so-called "zero dynamics" may appear which have to be asymptotically stable (Isidori, 1989; Nijmeijer and van der Schaft, 1990). Here, using $\mathbf{y} = \mathbf{q}$, this problem does not appear. But if we use more realistic output variables $\mathbf{y} = \mathbf{p}$, then there is a problem of zero dynamics which is not asymptotically but only marginally stable (in the sense of Lyapunov).
- The computational effort for the calculation of the functions $\mathbf{a}_4$, $\mathbf{a}_5$, $\mathbf{a}_6$ is generally great. Therefore symbolic computer programs have been developed using computer algebra, cf. (Birk and Zeitz, 1991).

- Because the derivatives of the non-linear functions are needed, the functions are assumed to be sufficiently smooth. But in many applications this condition is not satisfied, e.g. in the case of the Coulomb friction, backlash, impacts, saturations. Here, we have essentially the problem of the Coulomb friction.
- The knowledge of an exact mathematical model is required. Therefore, the sensitivity problem appears with respect to unmodelled effects or uncertain parameters. To overcome this problem special design methods are applied to guarantee robustness introducing an outer control loop, cf. (Spong and Vidyasagar, 1989).
- Due to the complete decoupling the amount of energy of the control may be high. For decoupling also "good" couplings of the system are counteracted unnecessarily.
- Generally, the realization of the control (6) is very cumbersome or even impossible. Usually, all the state variables are fed back and have to be known. It is unrealistic to assume that all the state variables are measured; this would be too expensive. The application of state observers is a difficult problem and well understood under special assumptions only, cf. (Birk and Zeitz, 1988). In general, the separation principle does not hold which implies that the design of a stable closed loop control by using state observers is difficult.

Being aware of these problems the question arises if there exist alternative methods to design a feedback control for non-linear dynamic systems avoiding the abovementioned disadvantages.

4. Alternative Control Design Methods

To demonstrate effective alternative control design methods we assume the following state space description of the non-linear control problem:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{N}\mathbf{n}(\mathbf{x}, t) + \mathbf{B}\mathbf{u}(t) \tag{9}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{10}$$

Here $\mathbf{x}$, $\mathbf{u}$, $\mathbf{y}$ denote the n-dimensional state vector, the r-dimensional control vector and the m-dimensional measurement vector, respectively. The vector $\mathbf{n}(\mathbf{x}, t)$ represents p more or less unknown functions which are generally non-linear, but which may be in special cases linear functions with unknown parameters or external disturbances depending only on time. The matrices $\mathbf{A}$, $\mathbf{N}$, $\mathbf{B}$, $\mathbf{C}$ are of related dimensions and represent the system matrix, non-linearity and control input matrices, and the measurement matrix, respectively. To avoid redundant formulations, the conditions

$$\operatorname{rank} \mathbf{N} = p, \qquad \operatorname{rank} \mathbf{B} = r, \qquad \operatorname{rank} \mathbf{C} = m \tag{11}$$

are assumed to be satisfied.

The simplest control design would exist, if the non-linearity vector $\mathbf{n}$ were known as a function of the measurements $\mathbf{y}$,

$$\mathbf{n} = \mathbf{n}(\mathbf{y}) \tag{12}$$

and if the matching condition

$$\operatorname{rank}\mathbf{B} = \operatorname{rank}[\mathbf{B}\ \mathbf{N}], \quad \text{i.e.} \quad \mathbf{N} = \mathbf{BT} \tag{13}$$

held. Then the direct counteraction

$$\mathbf{u} = -\mathbf{Tn}(\mathbf{y}) + \mathbf{v} \tag{14}$$

would compensate the non-linearities and a common linear control design problem would remain:

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{Bv}(t), \qquad \mathbf{y}(t) = \mathbf{Cx}(t) \tag{15}$$

But generally assumptions (12), (13) are not satisfied. Therefore, we can ask how to use these simple ideas in general cases. For this, three methods have been developed: (i) reverse linearization method, (ii) non-linearity estimation, (iii) non-linearity compensation.

4.1. Reverse Linearization Method

The "reverse linearization method" has been proposed in (Tahboub, 1993). It is related to the problem of the matching condition (13). If eqn. (13) does not hold,

$$\mathbf{N} \neq \mathbf{BT} \tag{16}$$

then a non-linear coordinate transformation is looked for to reach the matching condition (13) in the new coordinates. We assume

$$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{Sm} \tag{17}$$

with a suitable matrix $\mathbf{S}$ and a suitable non-linear vector function $\mathbf{m}$. Introducing (17) into eqns. (9), (10) we have

$$\dot{\overline{\mathbf{x}}} = \mathbf{A}\overline{\mathbf{x}} + \mathbf{BT}\overline{\mathbf{r}}(\overline{\mathbf{x}}) + \mathbf{Bu} \tag{18}$$

$$\mathbf{y} = \mathbf{C}\overline{\mathbf{x}} \tag{19}$$

if the conditions

$$(\dot{\mathbf{Sm}}) = \mathbf{A}(\mathbf{Sm}) - \mathbf{B}(\mathbf{T}\overline{\mathbf{r}}) + \mathbf{Nn} \tag{20}$$

$$0 = \mathbf{C}(\mathbf{Sm}) \tag{21}$$

are satisfied. A solution of eqns. (20), (21) for the unknowns $(\mathbf{Sm})$, $(\mathbf{T}\,\overline{\mathbf{r}})$ exists if and only if the condition

$$\operatorname{rank}\begin{bmatrix} s\mathbf{I}-\mathbf{A} & \mathbf{B} \\ \mathbf{C} & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} s\mathbf{I}-\mathbf{A} & \mathbf{B} & \mathbf{N} \\ \mathbf{C} & 0 & 0 \end{bmatrix} \quad \text{for all } s \in C \tag{22}$$

holds.

The transfer zeros of the linear input-output transfer function have to coincide with the transfer zeros of the generalized transfer function with respect to the generalized input of controls and non-linearities.

If assumption (12) still holds, then eqns. (20), (21) are satisfied by some vector functions $\mathbf{m} = \mathbf{m}(\mathbf{y})$, $\bar{\mathbf{r}} = \bar{\mathbf{r}}(\mathbf{y})$ such that the direct counteraction $\mathbf{u} = -\mathbf{T}\,\bar{\mathbf{r}}(\mathbf{y}) + \mathbf{v}$ compensates the non- linearities analogously to (14), (15). But if (12) does not hold, then the method of estimation of non-linearities has to be applied additionally.

4.2. Estimation of Non-linearities

The main idea behind the estimation of $\mathbf{n}(\mathbf{x}(t), t)$ consists in the approximation of its time behaviour by using some basis functions which are solutions of a fictitious linear dynamical system (Müller, 1994a):

$$\mathbf{n}(\mathbf{x}(t), t) \approx \mathbf{H}\mathbf{w}(t) \tag{23}$$

$$\dot{\mathbf{w}}(t) = \mathbf{W}\mathbf{w}(t) \tag{24}$$

System (24) has to be selected suitably with respect to its dimension s as well as to its system matrix $\mathbf{W}$. Later it will be shown that the choice $\mathbf{W} = \mathbf{0}$ is often satisfactory.

Substituting the non-linearities of (9) by model (23), (24) an extended linear system is obtained for which a state observer can be designed. In the case of an identity observer the estimation system runs as

$$\begin{aligned} \begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} &= \begin{bmatrix} \mathbf{A} & \mathbf{N}\mathbf{H} \\ \mathbf{0} & \mathbf{W} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} (\mathbf{y} - \hat{\mathbf{y}}) \\ &= \begin{bmatrix} \mathbf{A} - \mathbf{L}_x\mathbf{C} & \mathbf{N}\mathbf{H} \\ -\mathbf{L}_w\mathbf{C} & \mathbf{W} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_w \end{bmatrix} \mathbf{y} \end{aligned} \tag{25}$$

The choice of the observer gain matrices $\mathbf{L}_x$, $\mathbf{L}_w$ can be realized in such a way that observer (25) is asymptotically stable if and only if the extended system is detectable. Moreover, arbitrary eigenvalues of the observer system matrix can be realized if the extended system is completely observable:

$$\text{rank} \begin{bmatrix} \lambda\mathbf{I}_n - \mathbf{A} & -\mathbf{N}\mathbf{H} \\ \mathbf{0} & \lambda\mathbf{I}_s - \mathbf{W} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} = n + s \qquad \text{for all} \quad \lambda \in C \tag{26}$$

With the estimated signals of eqn. (25) the non-linearities can be reconstructed. Their time behaviour is estimated by

$$\hat{\mathbf{n}}(t) = \mathbf{H}\hat{\mathbf{w}}(t) \tag{27}$$

According to the discussion in (Müller, 1994a) some comments on this method of non-linearity estimation should be mentioned:

1. By (26) the number of measurements must be at least equal to the number of non-linearity inputs:

$$m \geq p \tag{28}$$

2. In general it is difficult to prove the convergence of estimate (27) to the true quantity:

$$\hat{\mathbf{n}}(t) \rightarrow \mathbf{n}\Big(\mathbf{x}(t), t\Big) \tag{29}$$

In the case of

$$\mathbf{CN} = \mathbf{0} \tag{30}$$

a high gain convergence result has been found (Söffker and Müller, 1993). Condition (30) is satisfied e.g. for mechanical control systems with displacement measurements only.

3. The signals of the fictitious model (23), (24) should approximate the time behaviour of the non-linearities as well as possible. A suitable choice of the matrices $\mathbf{H}$, $\mathbf{W}$ requires usually good *a priori* knowledge of the system behaviour. However, in many applications a simple consideration is more efficient. If we consider the true time behaviour of the non-linearities to be approximated by step functions, then we have an approximation by piecewise constant basis functions, i.e. $\mathbf{W} = \mathbf{0}$ piecewise. If observer (25) is fast enough, it will follow the changes of the step functions. Therefore, the choice of the fictitious model can often be simply realized by p integrators:

$$s = p, \qquad \mathbf{H} = \mathbf{I}_p, \qquad \mathbf{W} = \mathbf{0} \tag{31}$$

In (Söffker and Müller, 1993) this result has been confirmed by applying the Laplace transform. The practical experience from the choice of (31) is very positive. In this case observer (25) may be called a PI-observer because the measurements influence the observer in proportional and integral ways.

If the method of estimating non-linearities is used to realize the control by the reverse linearization method of Section 4.1, then it is usually not necessary to solve eqns. (20), (21) explicitly. Based on (18), (19) we only have to estimate the term $(\mathbf{T}\,\overline{\mathbf{r}}(\overline{\mathbf{x}}))$ by observer (25) defining $\mathbf{T}\,\overline{\mathbf{r}} \approx \mathbf{Hw}$. Therefore, if condition (22) holds, it is not necessary to know $\mathbf{n}(\mathbf{x}(t), t)$ of (9) explicitly and it is not necessary to solve (20), (21). Only observer (25) has to be applied to obtain the estimates of $\mathbf{T}\,\overline{\mathbf{r}}$. This fact follows by (19), (21), i.e. by the assumption that the measurements are invariant with respect to the coordinate transformation (17).

4.3. Compensation of Non-linearities

Besides the control design by the reverse linearization method, in many applications the feedback control can be designed according to the method of disturbance rejection (Müller and Lückel, 1977):

$$\mathbf{u}(t) = -\mathbf{K}_x \hat{\mathbf{x}}(t) - \mathbf{K}_w \hat{\mathbf{w}}(t) \tag{32}$$

The analysis shows that the gain matrix $\mathbf{K}_x$ of the state feedback can be designed as usual if stabilizability or complete controllability is assured:

$$\text{rank}\,[\lambda\mathbf{I}_n - \mathbf{A} \quad \mathbf{B}] = n \qquad \text{for all} \quad \lambda \in C \tag{33}$$

The gain matrix $\mathbf{K}_w$ has to be calculated in a specific manner to meet the requirement that a q-dimensional error vector (of interesting variables to be controlled)

$$\mathbf{z}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{u}(t) \tag{34}$$

is controlled independently of the influence of $\mathbf{n}(\mathbf{x}, t)$:

$$\mathbf{z}(t) \to \mathbf{0} \qquad \text{for} \qquad t \to \infty \tag{35}$$

Essentially, $\mathbf{K}_w$ has to be determined in such a way that the non-linearity effects are not observable by the error variables. This leads to linear equations

$$(\mathbf{A} - \mathbf{B}\mathbf{K}_x)\mathbf{X} - \mathbf{X}\mathbf{W} - \mathbf{B}\mathbf{K}_w = -\mathbf{N}\mathbf{H} \tag{36}$$

$$(\mathbf{F} - \mathbf{G}\mathbf{K}_x)\mathbf{X} - \mathbf{G}\mathbf{K}_w = \mathbf{0} \tag{37}$$

Besides $\mathbf{K}_w$ there is another unknown matrix $\mathbf{X}$ which characterizes the stationary behaviour of $\mathbf{x}$(t) depending on $\mathbf{w}(t)$. A general solvability condition of (36), (37) has been discussed in (Müller and Lückel, 1977) based on modal representation. But in the special case of (31) the discussion simplifies essentially by cancelling the term $\mathbf{XW}$. Then the compensation of the non-linearities is possible if and only if

$$\text{rank} \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{F} & \mathbf{G} \end{bmatrix} = \text{rank} \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{N} \\ \mathbf{F} & \mathbf{G} & \mathbf{0} \end{bmatrix} \tag{38}$$

holds. Then the solutions of (36), (37) are determined by

$$[(\mathbf{F} - \mathbf{G}\mathbf{K}_x)(\mathbf{A} - \mathbf{B}\mathbf{K}_x)^{-1}\mathbf{B} - \mathbf{G}]\mathbf{K}_w = (\mathbf{F} - \mathbf{G}\mathbf{K}_x)(\mathbf{A} - \mathbf{B}\mathbf{K}_x)^{-1}\mathbf{N} \tag{39}$$

$$\mathbf{X} = (\mathbf{A} - \mathbf{B}\mathbf{K}_x)^{-1}(\mathbf{B}\mathbf{K}_w - \mathbf{N}) \tag{40}$$

The solvability condition (38) does not depend on the gain matrix $\mathbf{K}_x$ but the solutions $\mathbf{K}_w$, $\mathbf{X}$ depend on it in general.

The model approximation (23), (24) with the choice (31) has been proved to be useful for many applications in the case of the non-linearity estimation (25), (27) and compensation (32), (39). The proposed method is a powerful tool to compensate non-linear effects by means of linear system theory.

The dynamics of the closed loop control system can be described by

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}}_x \\ \dot{\hat{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K}_x & -\mathbf{B}\mathbf{K}_x & -\mathbf{B}\mathbf{K}_w \\ \mathbf{0} & \mathbf{A} - \mathbf{L}_x\mathbf{C} & \mathbf{N}\mathbf{H} \\ \mathbf{0} & -\mathbf{L}_w\mathbf{C} & \mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e}_x \\ \hat{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{N}\mathbf{n}(\mathbf{x}, t) \\ -\mathbf{N}\mathbf{n}(\mathbf{x}, t) \\ \mathbf{0} \end{bmatrix} \tag{41}$$

Here, $\mathbf{e}_x = \hat{\mathbf{x}} - \mathbf{x}$ is the state estimation error. The linear part of (41) pretends the algebraic separation of the eigenvalues of the state controller and of the observer but there is a coupling by the non-linear effects. This coupling of state feedback control and observer has been observed in applications. Therefore, the closed loop control design must take this fact into consideration. The above-mentioned comment 2 guarantees the successful control design.

5. Robust Independent Joint Control of Robots

The proposed method for estimation and compensation of non-linearities has been successfully applied to the design of robust independent joint control of robots (Hu and Müller, 1994; Müller, 1994b). For this, the complete model (1)–(3) of the robot dynamics is written down relative to the single axes. The mass matrix is divided into a constant diagonal matrix of the mean values of the moments of inertia and a remaining position-dependent part:

$$\mathbf{M}(\mathbf{q}) = \mathbf{M}_0 + \Delta\mathbf{M}(\mathbf{q}) \tag{42}$$

The mean values are chosen with regard to a typical work space of the robot or along a desired trajectory. Summarizing later for each axis i all non-linear terms in n_i,

$$n_i = \sum_j \Delta M_{ij}(\mathbf{q})\ddot{q}_j + h_i(\mathbf{q}, \dot{\mathbf{q}}) \tag{43}$$

equations (1)–(3) can be separately considered for the single axis:

$$M_{0i}\ddot{q}_i + n_i + K_i(q_i - p_i) = 0 \tag{44}$$

$$J_i\ddot{p}_i + D_i\dot{p}_i - K_i(q_i - p_i) = m_i \tag{45}$$

$$T_i\dot{m}_i + m_i = G_i u_i \tag{46}$$

This one-axis model can be described correspondingly in state space. Leaving out the index i for the sake of brevity, description (9), (10),

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{N}n + \mathbf{B}u \tag{47}$$

$$y = \mathbf{C}\mathbf{x} \tag{48}$$

is obtained with the state vector $\mathbf{x} = [\,q \quad \dot{q} \quad p \quad \dot{p} \quad m\,]^T$ and the matrices

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{K}{M_0} & 0 & \frac{K}{M_0} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{K}{J} & 0 & -\frac{K}{J} & -\frac{D}{J} & \frac{1}{J} \\ 0 & 0 & 0 & 0 & -\frac{1}{T} \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 0 \\ -\frac{1}{M_0} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{G}{T} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{49}$$

Here again the measurement of the joint coordinate q is assumed.

The objective of the position control of robots is the tracking of joint coordinate $q(t)$ along a desired trajectory $r(t) = q_d(t)$ which is determined by path planning. The control error is

$$z = \mathbf{F}\mathbf{x} + Rr \tag{50}$$

where

$$\mathbf{F} = [-1 \quad 0 \quad 0 \quad 0 \quad 0], \qquad R = 1 \tag{51}$$

The design of each joint controller is based on model (47)–(51). The non-linearities and coupling effects, which are contained in n, will be compensated applying the methods of non-linearity estimation and compensation of Section 4. Additionally, we have to look for the tracking control which is reached by a feedforward control based on the method of disturbance rejection control (Müller and Lückel, 1977).

According to (23), (24) the time signal n of the non-linearities and couplings is approximated by

$$n(t) \approx H_1 w_1(t), \quad \dot{w}_1(t) = W_1 w_1(t) \quad \text{with} \quad H_1 = 1, \quad W_1 = 0 \tag{52}$$

The desired trajectory $r(t) = q_d(t)$ is assumed to be known. For the feedforward control the variables $\dot{q}_d(t)$, $\ddot{q}_d(t)$, $q_d^{(3)}(t)$, $q_d^{(4)}(t)$ and $q_d^{(5)}(t)$ are needed additionally. Theoretically they are available but usually they do not exactly correspond to the derivatives of the actually requested trajectory due to possible disturbances. Therefore, they are estimated by an observer which is constructed like (23)–(25) as well:

$$r(t) \approx \mathbf{H}_2 \mathbf{w}_2(t), \qquad \dot{\mathbf{w}}_2(t) = \mathbf{W}_2 \mathbf{w}_2(t) \tag{53}$$

with

$$\mathbf{H}_2 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \qquad \mathbf{W}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{54}$$

representing the relations among the derivatives.

The design of the observer can be separated into two parts for $\hat{\mathbf{x}}, \hat{w}_1$ and for $\hat{\mathbf{w}}_2$. The first observer coincides with (25), the other observer is governed by

$$\dot{\hat{\mathbf{w}}}_2 = (\mathbf{W}_2 - \mathbf{L}_{w2}\mathbf{H}_2)\hat{\mathbf{w}}_2 + \mathbf{L}_{w2}\, r \tag{55}$$

With the estimated values a control

$$u = -\mathbf{K}_x \hat{\mathbf{x}} - K_{w1}\hat{w}_1 - \mathbf{K}_{w2}\hat{\mathbf{w}}_2 \tag{56}$$

has to be designed. According to the results of Section 4.3, the gain matrices

$$K_{w1} = -\frac{1}{K}K_{x3} - (\frac{1}{G} + K_{x5}), \qquad \mathbf{K}_{w2} = \begin{bmatrix} -K_{x1} - K_{x3} \\ -K_{x2} - K_{x4} \\ -\frac{M_0}{K}K_{x3} - (\frac{1}{G} + K_{x5})(M_0 + J) \\ -\frac{M_0}{K}K_{x4} - \frac{T}{G}(M_0 + J) \\ -(\frac{1}{G} + K_{x5})\frac{M_0 J}{K} \\ -\frac{TM_0 J}{GK} \end{bmatrix}^T \tag{57}$$

are obtained depending on the gains K_{xi} of the state feedback. The matrix $\mathbf{K}_x$ is determined by common design methods of linear control theory.

In (Hu and Müller, 1994; Müller, 1994b) simulation results are presented showing the efficiency of this independent joint control. Assuming the exact knowledge of the robot model (1)–(3) the control design by the method of exact linearization and non-linear decoupling of Section 3 gives better results than the proposed method of Section 4. The first method compensates exactly, whereas the other compensates only approximately. But when looking for the results under the more realistic assumptions of parameter uncertainty and unknown friction, the other method is superior to the first one. Because of the mismatching between the mathematical model and the real robot system, the method of exact linearization and non-linear decoupling shows big control errors. However, the method of non-linearity estimation and compensation estimates and counteracts the uncertain or unknown effects so that the control error is sufficiently small.

The robustness of the proposed alternative controller (32) or (56) can be easily verified. If the parameters in the system description (1)–(3) are inaccurate, e.g. due to load variations, or unknown friction torques appear, the real system behaviour is described by a modified model

$$\mathbf{M}'(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}'(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}(\mathbf{q} - \mathbf{p}) = \mathbf{0} \tag{58}$$

$$\mathbf{J}\ddot{\mathbf{p}} + \mathbf{D}\dot{\mathbf{p}} - \mathbf{K}(\mathbf{q} - \mathbf{p}) = \mathbf{m} \tag{59}$$

$$\mathbf{T}\dot{\mathbf{m}} + \mathbf{m} = \mathbf{G}\mathbf{u} \tag{60}$$

showing a mismatching $\mathbf{M}'$ instead of $\mathbf{M}$ and $\mathbf{h}'$ instead of $\mathbf{h}$. The vector function $\mathbf{h}'$ may also include discontinuous non-linearities due to the Coulomb friction.

While controller (7), (8) is still based on the nominal model (1)—(3) so that the mismatching problem appears completely, controllers (32) or (56) obviously have regard to the modified system behaviour (58)–(60). If

$$\mathbf{M}'(\mathbf{q}) = \mathbf{M}_0 + \Delta\mathbf{M}'(\mathbf{q}) \tag{61}$$

then one has only to replace the term n_i in the joint axis description (43) by

$$n_i' = \sum_j \Delta M_{ij}'(\mathbf{q})\ddot{q}_j + h_i'(\mathbf{q}, \dot{\mathbf{q}}) \tag{62}$$

Because the design of controller (56) is based on the fact that n_i (and hence n_i') are interpreted to be unknown functions, n_i' will be estimated by $\hat{w}_1$ and controller (56) fulfils its task also in the case of modified system behaviour. Control (32) or (56) is structurally robust against parameter inaccuracies and unmodelled effects.

The method of non-linearity estimation and compensation has been proved to be a good alternative to the method of exact linearization and non-linear decoupling by state feedback, especially in the case of practical applications. It is an efficient approach to the design of robust position control of robots. In addition to its robustness, its decentralized structure offers additional advantages such that the concept of independent joint control can further be used, even in an improved manner. The requirements on the modelling of the robot dynamics are very low. The control design is easily performed because it is based on the linear system theory only. Practical applications in robot control have shown the efficiency of the proposed control.

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