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TEMPORAL REPRESENTATION OF WHITE NOISE

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A signal being a model of white noise in the time domain is proposed. It has the form of an infinite sequence of Dirac's impulses with random, Poisson-distributed distances between them. Then, the influence of two parameters, existing during a realization, on the form of the two-sided power spectral density is discussed. These quantities are the length of impulses and the average distance between them. Finally, the form of the power spectral density for a signal being a realization of white noise in the time domain, performed on the basis of the presented definition, is shown.

1. Introduction

In the analysis and simulation of real mechanical systems, when the frequency spectrum of excitations is not known in detail, a signal of white noise type is often used. For example, while designing a new vehicle, the frequency spectrum of excitations is known only after a prototype is made. Moreover, for slowly varying processes the power spectral density is "almost" constant with respect to frequency. Therefore the modelling of such a signal by white noise seems to be a sufficiently good approximation, which radically simplifies the analysis.

A characteristic property of white noise is the same energy for each frequency. Therefore its whole energy is infinite. One of the main disadvantages of this signal is only its frequency representation. Consequently, in applications of white noise to the description of excitations during the analysis of system vibrations, these excitations could not be defined in the time domain. But in some applications it is useful to make an analysis in such a domain. Thus the question about a possibility of defining of a new signal which could be a temporal representation of white noise arises. This signal should be defined in such a way that its frequency characteristics are the same (or very similar) to those for white noise.

Lyon (1975) introduces an "alternative time domain definition of white noise as a series of impulses of strength $\pm a$, occuring randomly in time". But he does not define such a signal in more detail.

Roberts (1965) defines a discrete process, called the shot noise, being some sequence of impulses, which can be in the limit case a temporal representation of white noise. Such a discrete process has a non-stationary character, because the strength of impulses and the average distance between impulses can vary in the time domain.

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Then he applies this signal in the analysis of the response of a linear vibration system having a single degree of freedom to random impulse excitation. The problem of the response of dynamical systems to the excitations in the form of a sequence of impulses is analysed, among others, by Tylikowski and Marowski (1986), Iwankiewicz and Nielsen (1992), Śniady (1989), Gładysz and Śniady (1992). The signal of Poisson's sequence of impulses is discussed by Murzewski (1993). The papers (Gładysz and Śniady, 1992; Śniady, 1989) contain a bibliography connected with the problem of the response of dynamical systems to the excitation in the form of a sequence of impulses.

Levin (1966) discusses a few different signals of the form of impulses randomly occuring in time and characterized by random parameters (amplitudes, distances between impulses, shapes of impulses). Some of them are discussed by Sołodownikow (1964), too.

In the present paper, some aspects of the proposed model of the white noise signal are discussed. This model is a sequence of impulses. It has a stationary character, because the strength of impulses and the average distance between impulses are constant in the time domain.

The following notation regarding the characteristics of the signal $\xi(t)$ is adopted in the analysis: $\overline{\xi}$ denotes the expected value in the time domain, R_ξ stands for the autocorrelation function, and S_ξ is the two-sided power-spectral density function.

2. Modelling White Noise in the Time Domain

2.1. Definition of White Noise

By white noise we mean a stationary, random, Gaussian-distributed signal ξ_{BS} characterized by the parameters given by

$$\overline{\xi_{BS}} = 0, \quad R_{\xi_{BS}}(\tau) = N\delta(\tau), \quad S_{\xi_{BS}}(\omega) = N, \quad N > 0 \tag{1}$$

Szabatin (1990) states that it is "the most 'random' among all the continuous signals with continuous time". Due to its form, this signal has not any direct temporal representation.

2.2. Definition of the Poisson Series of Dirac's Impulses

Let us consider a sequence of points, randomly distributed in the time domain. We call it the Poisson series. Szabatin (1990) states that this series "is characterized by the highest 'randomness' among all the series with randomly distributed points". If λ denotes the density of distribution of the points on the time axis, then the random variable η describing the distance between randomly distributed points has exponential distribution and its probability density is given by

$$f_\eta = \lambda\, e^{-\lambda t}\, H(t) \tag{2}$$

where $H(t)$ is the Heaviside function.

Let $\{t_i : i = 0, \pm 1, \pm 2, \ldots\}$ denote Poisson's series of points (in practice its realization), with constant density λ. We define the signal ξ_{PSI} in the form below, called the Poisson series of Dirac's impulses with unit strength:

$$\xi_{PSI}(t) = \sum_{i=-\infty}^{+\infty} \delta(t - t_i) \tag{3}$$

It is characterized by the values of the following characteristic quantities (Szabatin, 1990):

$$\overline{\xi_{PSI}} = \lambda, \quad R_{\xi_{PSI}}(\tau) = \lambda^2 + \lambda\delta(\tau), \quad S_{\xi_{PSI}}(\omega) = 2\pi\lambda^2\delta(\omega) + \lambda \tag{4}$$

2.3. Temporal Representation of White Noise

Let us define, by the Poisson series of Dirac's impulses and a binary symmetric random variable γ, a new signal ξ_{WNt} in the form

$$\xi_{WNt} = \gamma\, \xi_{PSI} \tag{5}$$

which is the proposed temporal representation of white noise. Here γ is the variable taking two values +1 and -1 with the same probability 1/2. Because $\overline{\gamma} = 0$ and $\overline{\gamma^2} = 1$, the values of parameters of ξ_{WNt} are as follows:

$$\overline{\xi(t)} = \overline{\gamma\, \xi_{PSI}(t)} = \overline{\gamma}\; \overline{\xi_{PSI}(t)} = 0 \tag{6}$$

$$\begin{aligned} R_{\xi_{WNt}}(t_1, t_2) &= \overline{\gamma\, \xi_{PSI}(t_1)\, \gamma\, \xi_{PSI}(t_2)} \\ &= \overline{\gamma^2}\; \overline{\xi_{PSI}(t_1)\, \xi_{PSI}(t_2)} = R_{\xi_{PSI}}(t_1, t_2) \end{aligned} \tag{7}$$

Finally, the signal defined in (5), discontinuous in the time domain, is characterized by

$$\overline{\xi_{WNt}} = 0, \quad R_{\xi_{WNt}}(\tau) = \lambda^2 + \lambda\delta(\tau), \quad S_{\xi_{WNt}}(\omega) = 2\pi\lambda^2\delta(\omega) + \lambda \tag{8}$$

On account of close values of the characteristic quantities for the white-noise signal (1) and for the modified Poisson series of Dirac's impulses (6), (7) (there is a difference only for the power spectral density function for the zero value), it seems to us that the signal defined in (5) can be considered as a temporal representation of white noise. But the new signal is not of Gaussian type. Therefore the representation is valid in the sense of correlation theory.

An example of the real signal generated on the basis of the above definition is given in Appendix.

3. Real Form of White Noise in the Time Domain

3.1. Introductory Remarks

The proposed model of white noise is characterized by two quantities used for its description in the time domain: the average distance between impulses Δt and the strength of each impulses. In practice, two parameters are included in the quantity of the average distance between impulses: the time of realization T_k and the number of impulses L_{imp}. The presented characteristic quantities are valid for $T_k \longrightarrow \infty$ and $L_{\text{imp}} \rightarrow \infty$ in such a way that $T_k/L_{\text{imp}} \rightarrow \Delta t$. It should be noted that it is not possible to generate this signal in real conditions. Two reasons yield this fact. The first of them is a finite time of its realization and the other is the impossibility of realization of the Dirac's impulse.

The strength of Dirac's impulses, denoted by P, can be introduced using a periodic sequence of rectangular impulses with amplitude A and time of realization τ. The periodic sequence of Dirac's impulses can be interpreted as the limit of the rectangular sequences of impulses as $\tau \longrightarrow 0$, where the strength of impulses has the value of $P = A\tau$. Thus, physically, the strength of Dirac's impulses has a character of the impulse of the quantity describing the amplitude of a rectangular impulse (dimensionally $[P] = [A][\tau]$, where $[\cdot]$ is the dimension of the given quantity).

3.2. Fourier Series Expansion of Periodic Series of Rectangular Impulses

Let us consider a signal, denoted by $f(t)$, in the form of a series of rectangular impulses with amplitude A, duration τ and period T (see Fig. 1a). The exponential Fourier series expansion of this signal takes the form

$$f(t) = \sum_{n=-\infty}^{\infty} F_n \, e^{jn\omega_0 t} \tag{9}$$

where $\omega_0 = 2\pi/\tau$ and

$$F_n = A\frac{\tau}{T}\frac{\sin(n\pi\frac{\tau}{T})}{n\pi\frac{\tau}{T}} \tag{10}$$

Therefore the amplitude spectrum of this signal consists of spectral lines with the values F_n occuring at the points with circular frequencies $\omega = 0, \pm\omega_0, \pm 2\omega_0, \dots$. The form of the spectrum envelope of the signal $F(\omega)$ and its normalized quantity $\mathcal{F}(\omega)$ are related by the following relationship:

$$F(\omega) = A\frac{\tau}{T}\underbrace{\frac{\sin(\frac{\tau}{2}\omega)}{\frac{\tau}{2}\omega}}_{\mathcal{F}(\omega)} = A\frac{\tau}{T}\mathcal{F}(\omega) \tag{11}$$

The amplitude envelope of the signal under consideration is the function $|F(\omega)|$ whose plot is shown in Fig. 1b.

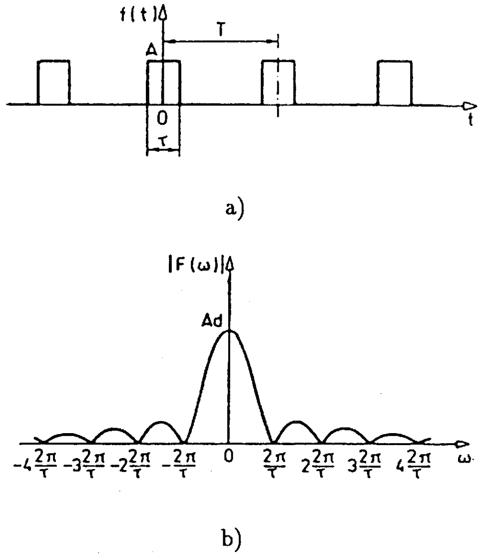

Fig. 1. Periodic stream of rectangular impulses (Ozimek, 1985):
a) form of the signal,
b) amplitude envelope for the exponential Fourier series ($d = \frac{\tau}{T}$).

It should be noted that as $\tau \to 0$ (i.e. for the signal consisting of periodic Dirac's impulses), the amplitude envelope tends to a constant value ($|F(\omega)| \to A\frac{\tau}{T}$) and the normalized amplitude envelope tends to one ($|\mathcal{F}(\omega)| \to 1$).

3.3. Influence of Average Distance Between Impulses

Finite duration of a signal realization in the time domain means in practice a finite time of realization T_k and a finite number of impulses L_{imp} when the condition $\Delta t = L_{\text{imp}}/T_k$ is satisfied. The following question arises: how does the value of the average distance between impulses influence the frequency characteristics of the signal, e.g. the two-sided power spectral density? In the case of a realization of finite duration this question takes the following form: how do the time of realization and the number of impulses influence the two-sided power spectral density?

Let us consider first (Levin, 1966; Sołodownikow, 1964) a signal in the form of a sequence of rectangular impulses with a constant amplitude A, length τ and random distances between impulses. We write Δt for the average time of impulse repetition, and δt_k for the deviation of the k-th distance between the adjacent impulses subject to the condition $\overline{\delta t} = 0$. Let $f_{\delta t}$ be the probability density of the random variable δt. If χ denotes the random variable with probability density f_χ, then its characteristic function $W_\chi(j\omega)$ is defined by the following relationship (Sołodownikow, 1964):

$$W_\chi(j\omega) = \int_{-\infty}^{\infty} f_\chi(t)\, e^{-j\omega t}\, \mathrm{d}t \tag{12}$$

The two-sided power spectral density of the signal under analysis is given by (Levin, 1966; Sołodownikow, 1964)

$$S(\omega) = \frac{(A\tau)^2}{\Delta t} |\mathcal{F}(j\omega)|^2 \Big[1 - |W_{\delta t}(j\omega)|^2 + \frac{|W_{\delta t}(j\omega)|^2}{\Delta t} 2\pi \sum_{k=-\infty}^{\infty} \delta\Big(\omega - \frac{2\pi k}{\Delta t}\Big)\Big] \tag{13}$$

where $|\mathcal{F}(j\omega)|$ is the normalized amplitude spectrum envelope of the sequence of rectangular impulses with amplitude A and length τ, periodically repeated with period T (see (11), Fig. 1b).

In the case of the signal considered, the random variable δt is defined by the following relationship:

$$\delta t = \eta - \Delta t \tag{14}$$

where η is a random variable describing the distance between the points with exponential distribution (see Section 2.2, $\lambda = 1/\Delta t$), and Δt has a character of a deterministic quantity. Because the probability density of the deterministic quantity Δt satisfies

$$f_{\Delta t}(t) = \delta(t - \Delta t) \tag{15}$$

the probability density of the random variable η takes the form (2), so their characteristic functions are of the form

$$W_{\Delta t}(j\omega) = e^{-j\,\omega\,\Delta t}, \qquad W_\eta(j\omega) = \frac{\lambda^2}{\lambda^2 + \omega^2} + j\,\frac{\lambda\,\omega}{\lambda^2 + \omega^2} \tag{16}$$

It is known (Sołodownikow, 1964) that the characteristic function of the sum of two independent random variables is the product of their characteristic functions. Hence the characteristic function of the random variable δt, according to (14) is calculated based on the relationship $W_{\delta t}(j\omega) = W_\eta(j\omega)/W_{\Delta t}(j\omega)$, and the square of its modulus is of the form

$$|W_{\delta t}(\omega)|^2 = \frac{\lambda^2}{\lambda^2 + \omega^2} \tag{17}$$

To determine the signal which is a temporal representation of white noise (that is to say, the sequence of Dirac's impulses defined in Section 2.3) let us take the limit as $\tau \to 0$. Then $A\tau \to P$. In this case the amplitude envelope tends to a constant value ($|F(\omega)| \to P/\Delta t$, see Section 3.2 for $T = \Delta t$). After some calculations, the two-sided power spectral density of this signal can be written down as the sum of the components $S_1(j\omega)$ and $S_2(j\omega)$ as follows:

$$S_{\delta t}(j\omega) = \underbrace{\frac{P^2}{\Delta t}(1 - \frac{1}{1+(\omega\,\Delta t)^2})}_{S_1(j\omega)} \\ \underbrace{+\underbrace{\frac{P^2}{(\Delta t)^2}\frac{1}{1+(\omega\,\Delta t)^2}}_{s_2(\omega)} 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{\Delta t})}_{S_2(j\omega)} \tag{18}$$

The component $S_1(j\omega)$ of the two-sided power spectral density has a continuous character in the frequency domain, whereas the component $S_2(j\omega)$ is a sequence of pulses of strength $s_2(\omega)$ occuring at the points of the circular frequencies $\omega = 0,\, 2\pi/\Delta t,\, 4\pi/\Delta t, \ldots$.

At this moment, two common conventions of graphic representation of the amplitude spectrum should be discussed. These are distributional and classic (based on the complex Fourier series expansion of a signal). The Dirac's impulse of strength $2\pi|X_k|$ at the point $k\omega_0$ in the distributional representation corresponds to a pulse of strength $|X_k|$ occuring at the same frequency point for the classic representation (Szabatin, 1990).

Two measures of errors are introduced for estimating the difference between the two-sided power spectral density functions for two signals: the theoretical one (in the form (8) but for the strength of impulses P), and the real one. The first error coefficient (the relative one), denoted by $\varepsilon_{S1}(\omega)$, describes a deviation of the continuous component of the real signal $S_1(j\omega)$ from the corresponding component of the theoretical signal, i.e. $P^2/\Delta t$, through the following relationship:

$$\varepsilon_{S1}(\omega) = \frac{\frac{P^2}{\Delta t} - S_1(j\omega)}{\frac{P^2}{\Delta t}} = \frac{1}{1+(\omega\,\Delta t)^2} \tag{19}$$

The other error coefficient (the relative one) is connected with the component $S_2(j\omega)$. It is denoted by $\varepsilon_{S2}(\omega)$ and defined as follows:

$$\varepsilon_{S2}(\omega) = \frac{s_2(\omega)}{\frac{P^2}{\Delta t}}\frac{\Delta t}{2\pi} = \frac{1}{2\pi}\frac{1}{1+(\omega\,\Delta t)^2} \tag{20}$$

The value of the error coefficient answers the following question: how big is the strength of impulses divided by the distance between impulses in the frequency domain (this is done by multiplying the whole expression by $\Delta t/2\pi$) in comparison with a constant value for the two-sided power spectral density of the theoretical signal?

The form of the two-sided power spectral density of the real signal is approaching the form of the same function for the theoretical signal when the error coefficients $\varepsilon_{S1}(\omega)$ and $\varepsilon_{S2}(\omega)$ attain small values. Taking into account the fact that $\varepsilon_{S2}(\omega) =$

$\varepsilon_{S1}(\omega)/2\pi$ (see (19), (20)), it is sufficient to analyse only the dependence of $\varepsilon_{S1}(\omega)$ on the parameters Δt and ω.

Equation (19) describes the relationship between the circular frequency ω, the average distance between impulses Δt and the error coefficient $\varepsilon_{S1}(\omega)$ for a realization of a signal infinite in the time domain. In practice, the time of realization is finite and the average distance between impulses Δt is equal to its value T_k divided by the number of impulses L_{imp}. When we assume a sufficiently large number of impulses or a sufficiently long time of realization, it is possible, by applying relationship (19), to estimate the value of the circular frequency above which the frequency response of the real signal is close to the same characteristic for the theoretical signal, in the sense of the assumed error coefficient $\varepsilon_{S1}(\omega)$ and its value. For such an interpretation relationship (19) which includes four quantities ($\varepsilon_{S1}(\omega)$, T_k, L_{imp}, ω) can be used to estimate the value of one of them when the values of the others are given. But during calculations of the parameters of the real signal by applying the relationship mentioned above one should remember the fundamental assumptions ($T_k \to \infty$, $L_{\text{imp}} \to \infty$ in such a way that $T_k/L_{\text{imp}} \to \Delta t$). These include the requirement of taking not too small numbers of impulses.

The error coefficient $\varepsilon_{S1}(\omega)$ as a function of circular frequency is shown in Fig. 2. The distance between the impulses Δt changes in such a way that the time of realization of the signal takes the constant value $T_k = 0.25$ s, and the number of impulses has the following four values $L_{\text{imp}} = 50, 100, 250, 500$.

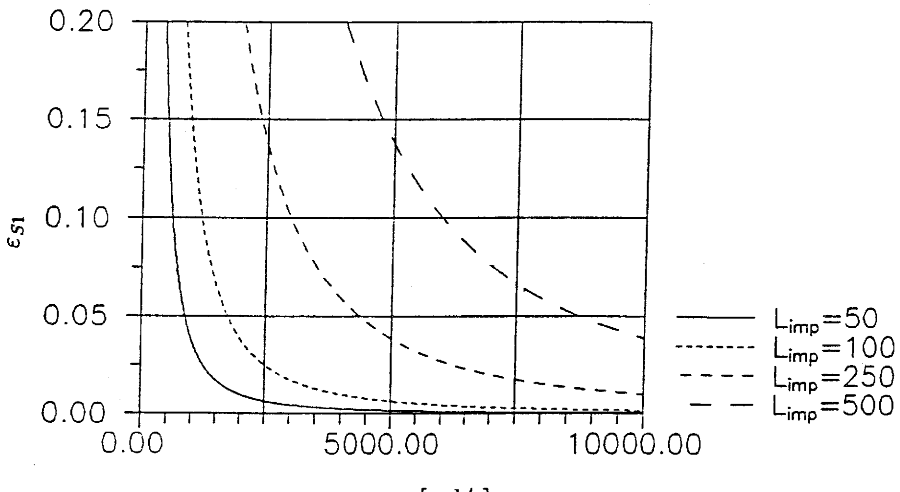

Fig. 2. Error coefficient $\varepsilon_{S1}(\omega)$ v. frequency for signals with the same numbers of impulses and the same time of realization.

3.4. Influence of Non-Zero Time of Realization of Dirac's Impulses

Let us consider a signal (infinite in the time domain) in the form of a sequence of Dirac's impulses defined in Section 2.3. For realization, an impulse has a finite length and finite strength. Hence, to estimate of the influence of finite length of the impulse on the two-sided power spectral density, it is possible to use this function for a signal in the form of a sequence of rectangular impulses with a randomly varying distance between them as considered in the previous section. It follows from this form that the impulse length affects the normalized amplitude envelope $|\mathcal{F}(\omega)|$ defined in Section 3.2. Based on the analysis of the form of the envelope it is possible to determine the highest circular frequency ω_{max} below which (i.e. for $\omega \in [0, \omega_{max}]$) the real realization of a signal with impulse length τ is valid for the assumed error coefficient $\varepsilon_i(\omega)$. If the impulse length tends to zero ($\tau \to 0$), then the normalized amplitude envelope reaches one ($|\mathcal{F}(\omega)| \to 1$).

Let us assume that $\omega < 2\pi/\tau$. This means that we consider the frequency range from zero to the value for which the normalized amplitude envelope reaches for the first time the zero value. Then we define the mesure of the difference between the envelope obtained for the signal with non-zero length and the envelope for the sequence of Dirac's impulses (the straight line with the ordinate value AP/T), in the following form:

$$\varepsilon_i = \frac{|F(\omega_{max}) - A\frac{\tau}{T}|}{A\frac{\tau}{T}} = \left| \frac{\sin(\frac{\tau}{2}\omega_{max})}{\frac{\tau}{2}\omega_{max}} - 1 \right| \tag{21}$$

After some manipulation, we obtain a non-linear equation for the product $\tau\omega_{max}$ with parameter ε_i in the form

$$\frac{\tau}{2}\omega_{max}(1 - \varepsilon_i) - \sin(\frac{\tau}{2}\omega_{max}) = 0 \tag{22}$$

It is possible to estimate the value of the product $\tau\omega_{max}$ from the relationship

$$\tau\omega_{max} = 2\sqrt{6}\sqrt{\varepsilon_i} \tag{23}$$

when the Maclaurin series expansion is applied, taking into account only the first two terms.

Numerical calculations show that it is possible to apply the approximate relationship (23) to estimate the value of the product $\tau\omega_{max}$ when the value of ε_i is known. Comparison of the exact and approximate values of the product $\tau\omega_{max}$, made for some values of the error coefficient ε_i, is shown in Table 1.

Tab. 1. Exact and approximate (from the Maclaurin series) values of $\tau\omega_{max}$ for some values of error coefficient ε_i

ε_i	VALUE OF $\tau\omega_{max}$	
	EXACT	APPROXIMATE
0.01	0.490636	0.489898
0.05	1.103822	1.095445
0.10	1.573366	1.549193
0.20	2.262205	2.190890

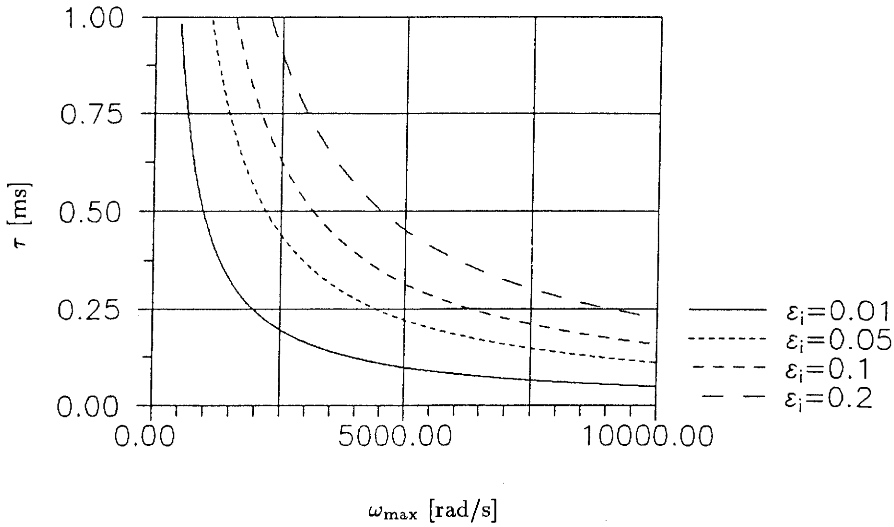

Fig. 3. Influence of the impulse length γ on the maximum frequency $\omega_{\max}$ for the assumed values of error coefficients ε_i.

By the exact value we mean the value calculated based on the non-linear equation (22), subject to the condition that the values $\frac{\tau}{2}\omega_{max}(1-\varepsilon_i)$ and $\sin(\frac{\tau}{2}\omega_{max})$ are equal with the precision of 14 significant digits. By the approximate value we mean the exact value calculated based on the approximate relationship (23).

Therefore, to choose the parameters of a signal, from the point of view of the non-zero length of the realized impulses it is necessary to assume arbitrarily a value of the error coefficient ε_i. Then, based on the exact relationship (22) or, as it seems to us good enough, on the approximate relationship (23), the value of the product $\tau\omega_{max}$ is obtained. Based on this value and assuming the impulse length τ, it is possible to estimate the upper circular frequency limit ω_{max}. When the value of the product $\tau\omega_{max}$ is known, it is alternatively possible to estimate the maximum impulse length τ_{max} for the assumed circular frequency band (by the assumed upper circular frequency limit ω_{max}).

The relationships between the impulse length τ and the upper frequency limit ω_{max} for four values of the error coefficient $\varepsilon_i = 0.01, 0.05, 0.1$ and 0.2, calculated based on the exact equation (22), are shown in Fig. 3.

4. Final Remarks

A stationary temporal representation of white noise is proposed in the paper. The representation is valid in the sense of correlation theory. The fact that white noise is of infinite energy appears in its temporal representation by the infinite time of realization. The temporal representation is defined by two quantities: the strength of impulses and the average distance between impulses. Because of the stationarity of the signal, these quantities are constant in time. The estimation of the influence of the average distance between impulses and the time of realization of Dirac's impulses on the form of the two-sided power spectral density has been carried out by introducing error coefficients. The real realization of the temporal representation of white noise is not infinite in the time domain, so the estimation of the average distance between the impulses is in practice the estimation of the time of realization and the number of impulses, whose values, taken arbitrarily, should be large enough.

Some remarks result from the analysis. The first one refers to the average distance between impulses. If its value is large, then the two-sided power spectral density of the signal approaches the form of the same function for white noise, starting from low frequencies. The second remark is connected with the length of impulses (the time of realization of the impulses). This quantity has an important influence on the form of the two-sided power spectral density and its value should be as small as possible.

We hope that the analysis presented here facilitates the modelling of the signal defined in the time domain characterized approximately by the same energy for each frequency. It should be noted once again that the two-sided power spectral density for the proposed temporal representation of white noise in the form (8) is theoretically valid only as $T_k \to \infty$ and $L_{\text{imp}} \to \infty$, in such a way that $T_k/L_{\text{imp}} \to \Delta t$.

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References

Bendat J.S. and Piersol A.G. (1976): *Random Data: Analysis and Measurement Procedures.* — Warsaw: Polish Scientific Publishers, (in Polish).

Gładysz M. and Śniady P. (1992): *Stochastic vibration of linear structures due to sample discontinuous load.* — J. Sound and Vibration, v.158, No.1, pp.1–10.

Iwankiewicz R. and Nielsen S.R.K. (1992): *Dynamic response of non-linear systems to Poisson-distributed random impulses.* — J. Sound and Vibration, v.156, No.3, pp.407–423.

Levin B.R. (1966): *Theoretical Foundations of Statistical Radio Engineering.* — Moscow: Sovietskoe Radio, (in Russian).

Lyon R.H. (1975): *Statistical Energy Analysis of Dynamical Systems — Theory and Applications.* — Cambridge, Massachusetts: MIT Press.

Murzewski J.W. (1993): *The Poisson process of actions and their combinations* — Zeitschrift fur Angewandte Mathematik und Mechanik (ZAMM), v.73, No.1, pp.27–33.

Ozimek E. (1985): *Theoretical Foundations of Signal Frequency Analysis.* — Warsaw: Polish Scientific Publishers, (in Polish).

Roberts J.B. (1965): *The response of linear systems to random impulses.* — J. of Sound and Vibration, v.2, No.4, pp.375–390.

Sołodownikow W.W. (1964): *Statistical Dynamics of Linear Automatic Control Systems.* — Warsaw: Scientific and Technical Publishers, (in Polish)

Szabatin J. (1990): *Foundations of Signal Theory.* — Warsaw: Communication Publishers, (in Polish).

Śniady P. (1989): *Dynamic response of linear structures to a random stream of pulses* — J. Sound and Vibration, v.131, No.1, pp.91–102.

Tylikowski A. and Marowski W. (1986): *Vibration of a non-linear single-degree-of-freedom system due to Poissonian impulse excitation.* — Int. J. Non-Linear Mechanics, v.21, pp.229–238.

Appendix

An Example of the Real Signal Generated on the Basis of the Definition Given in the Paper

Based on the definition of the temporal representation of white noise discussed in the paper, a new signal, finite in the time-domain and piecewise constant, has been generated. The amplitude strength for each time segment whose length is not constant and equal to a randomly varying distance between impulses is equal to the value of a single impulse divided by the distance between the appropriate adjacent impulses. The following values of the signal parameters have been assumed: the time of realization $T_k = 2.5\,10^{-1}$ s, and the average distance between impulses $\tau = 2.5\,10^{-3}$ s, the strength of impulses $P = 1$. The form of the signal defined in this way is shown in Fig. 4. The values of the signal are not zero for small values of time variable, as it appears in Fig. 4, but are only small in comparison with their maximum value in the considered range.

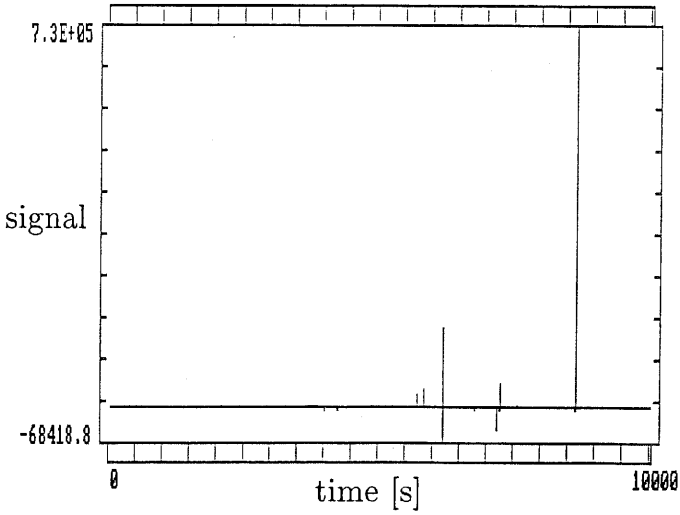

Fig. 4. Form of the real signal.

The power spectral density of this signal, shown in Fig. 5, has been determined by the Blackman-Tukey method (assuming the value of the relative standard error ε_r as equal to 0.1)(Bendat and Piersol, 1976) The calculations have been carried out with the use of the program *SPECTRA v.3.1* from the package *CADEX*©*

Such a new signal (of continuous form in the time domain) was required by the applied package.

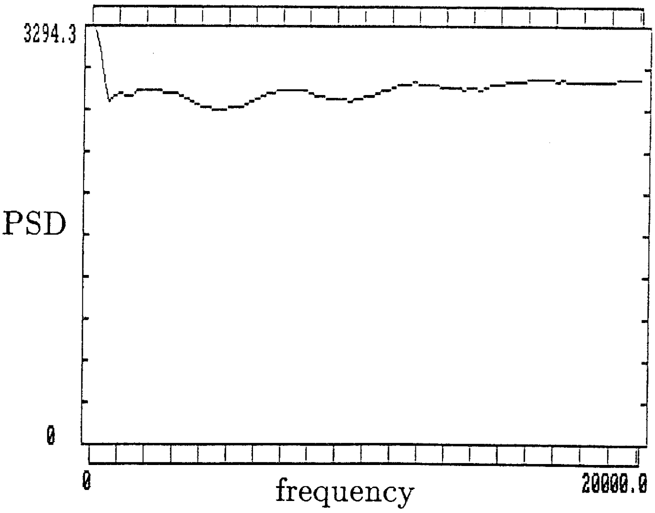

Fig. 5. Power spectral density for the signal shown in Fig. 4.

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