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RELATIONSHIP BETWEEN ENERGY AND INFORMATION

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Two ways of speed stabilization of the D.C. motor are considered. One way consists in the use of additional kinetic energy accumulated in a wheel with a large moment of inertia J. The other consists in the use of additional information supplied by a feedback loop with gain K. In both the cases the motor is under the influence of the same white Gaussion noise. These two ways of stabilization are compared under the assumption of the same value of the speed error in the steady state.

Keywords: entropy of information, energy, stabilization, D.C. motor, white noise

1. Introduction

Speed stabilisation of a direct-current motor with external excitation is considered. Two methods for speed stabilisation are analysed: one way is to use a wheel with a high moment of inertia J (a flywheel), while the other uses a feedback with a static controller of gain K. The motor is supplied by a constant voltage u and under the influence of a white Gaussian disturbance (Marshall *et al.*, 1992). The same level of accuracy of these two methods of stabilisation is assumed. A relation between the kinetic energy of the flywheel and the amount of information in the feedback will be found.

2. Two Methods of Stabilisation

2.1. Stabilisation Method Using Additional Energy

In Fig. 1 a scheme of motor stabilisation is presented. For the sake of simplicity the nonlinear effects and time constant T_0 of the main circuit are neglected. It is assumed that the speed of the motor is disturbed by zero mean white Gaussian noise $z(t)$. A stabilisation effect is attained due to the additional kinetic energy of the flywheel.

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Let us denote by $\delta\omega(t)$ the increment in speed caused by the noise $z(t)$. The state equation can be written as follows:

$$(T_1 + T_2)\frac{\mathrm{d}\delta\omega(t)}{\mathrm{d}t} = -\delta\omega(t) + \frac{1}{k_2}z(t), \tag{1}$$

where $T_1 = J_1 R/k_1 k_2 i_e$ is the electromechanical time constant of the motor, $T_2 = J_2 R/k_1 k_2 i_e$ stands for the electromechanical time constant of the flywheel, J_1 and J_2 signify the corresponding moments of inertia, R denotes the resistance of the main circuit, i_e is the steady-state current in the excitation circuit, and k_1, k_2 stand for some coefficients.

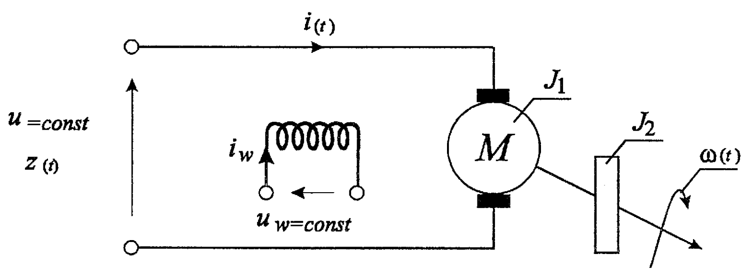

Fig. 1. Nomenclature for the D.C. motor considered.

The additional average kinetic energy $\overline{\Delta E}$ of the flywheel is given by

$$\overline{\Delta E} = \frac{1}{2}J_2\big(\omega_n + \overline{\delta\omega}\big)^2 - \frac{1}{2}J_2\omega_n^2 = \frac{1}{2}J_2\overline{\delta^2\omega}, \tag{2}$$

where ω_n is the nominal value of the speed. From the assumption that the expected value of the noise is equal to zero we have that so is the mean value $\overline{\delta\omega}$.

Using the method described in (Marshall *et al.*, 1992), we can calculate the variance of the speed of the motor with flywheel:

$$\overline{\delta^2\omega_c} = \frac{1}{2k_2^2(T_1 + T_2)}. \tag{3}$$

The variance of the speed without flywheel is obtained from (3) by setting $T_2 = 0$. We have

$$\overline{\delta^2\omega_1} = \frac{1}{2k_2^2 T_1}. \tag{4}$$

2.2. Stabilisation Method Using a Feedback Loop

In Fig. 2 a scheme of stabilisation of the motor speed using a tachometer T_G and a gain K in the feedback loop is presented. The corresponding state equation is

$$\frac{T_1}{1+K}\frac{\mathrm{d}\delta\omega(t)}{\mathrm{d}t} = -\delta\omega(t) + \frac{1}{k_2(1+K)}z(t). \tag{5}$$

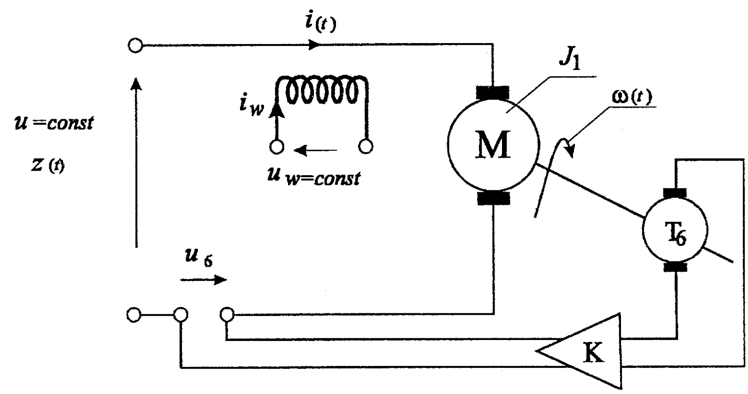

Fig. 2. Stabilisation scheme of the motor speed.

The variance of the speed in this case is described by the relation

$$\overline{\delta^2 \omega_K} = \frac{1}{2k_2^2(1+K)T_1}. \tag{6}$$

The cornerstone of this report is that these two systems of stabilisation are equivalent if the variances of the speed increments are the same, i.e.

$$\overline{\delta^2 \omega_C} = \overline{\delta^2 \omega_K}. \tag{7}$$

The substitution of the relations (3) and (6) into (7) gives the basic relation connecting these two methods of stabilisation:

$$T_2 = KT_1, \tag{8}$$

or

$$J_2 = KJ_1. \tag{9}$$

The average increment in the entropy of information in the system with feedback is (Górecki, 1996)

$$\overline{\Delta H} = \overline{H_2} - \overline{H_1}, \tag{10}$$

where $\overline{H_2}$ and $\overline{H_1}$ denote the average entropies of information of the system with and without feedback, respectively.

The average entropy of information for continuous systems is

$$H(p) = -\int_{-\infty}^{\infty} p(x) \ln\left[p(x)\right] \mathrm{d}x \tag{11}$$

where $p(x)$ is a probability density function. In our case,

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{12}$$

where σ^2 is the variance.

Substituting (12) into (11), after some calculations (Górecki, 1996), we obtain

$$\overline{\Delta H} = \ln\sqrt{\frac{\overline{\delta^2\omega_K}}{\overline{\delta^2\omega_1}}}. \tag{13}$$

Combining (6) and (4) with (13) gives

$$\overline{\Delta H} = -\ln\sqrt{1+K}, \tag{14}$$

or, equivalently,

$$\exp\left(-2\overline{\Delta H}\right) - 1 = K. \tag{15}$$

Taking account of the relation (9) in (15), we obtain

$$\exp\left(-2\overline{\Delta H}\right) - 1 = \frac{J_2}{J_1}. \tag{16}$$

From (2) we have

$$J_2 = 2\frac{\overline{\Delta E}}{\overline{\delta^2\omega_c}}. \tag{17}$$

The substitution of (17) into (16) gives

$$\exp\left(-2\overline{\Delta H}\right) - 1 = 2\frac{\overline{\Delta E}}{J_1\overline{\delta^2\omega_c}}. \tag{18}$$

In turn, the substitution of (3) in (18) and taking into account that

$$J_1 = 2\frac{E_1}{\omega_n^2}, \tag{19}$$

where E_1 is the kinetic energy of the motor, gives

$$\exp\left(-2\overline{\Delta H}\right) = 1 + \frac{\overline{\Delta E}}{E_1\dfrac{\overline{\delta^2\omega_c}}{\omega_n^2}}. \tag{20}$$

Introducing the relative increment in energy

$$\overline{\Delta E_r} = \frac{\overline{\Delta E}}{E_1} \tag{21}$$

and the relative variance of the speed

$$\overline{\delta^2\omega_{cr}} = \frac{\overline{\delta\omega^2}}{\omega_n^2}, \tag{22}$$

we finally obtain the fundamental relation between the average relative increment in energy $\overline{\Delta E_r}$ and the average increment in information $\overline{\Delta H}$:

$$\exp\left(-2\overline{\Delta H}\right) = 1 + \frac{\overline{\Delta E_r}}{\overline{\delta^2\omega_{cr}}} \tag{23}$$

or, equivalently,

$$\overline{\Delta H} = -\frac{1}{2}\ln\left[1 + \frac{\overline{\Delta E_r}}{\overline{\delta^2\omega_{cr}}}\right]. \tag{24}$$

It is possible to generalise these considerations to systems with time delays. If there is a time delay $\tau > 0$ in the feedback loop, then the state equation takes the form

$$T_1\frac{\mathrm{d}\delta\omega(t)}{\mathrm{d}t} + \delta\omega(t) + K\delta\omega(t-\tau) = \frac{1}{k_2}z(t-\tau). \tag{25}$$

The variance $\overline{\delta^2\omega_c}$ is the same as indicated by formula (3). The variance of the speed corresponding to the relation (6) is much more complicated:

$$\overline{\delta^2\omega_K} = \frac{\left(K^2-1\right)\cos\frac{\tau}{T_1}\sqrt{K^2-1} + \sqrt{K^2-1}\sin\frac{\tau}{T_1}\sqrt{K^2-1}}{2K_2^2T_1\left(K^2-1\right)\left[K - \sqrt{K^2-1}\sin\frac{\tau}{T_1}\sqrt{K^2-1} + \cos\frac{\tau}{T_1}\sqrt{K^2-1}\right]}. \tag{26}$$

According to (7) we have

$$T_2 = \frac{\left(K^2-1\right) - K\sqrt{K^2-1}\sin\frac{\tau}{T_1}\sqrt{K^2-1}}{\left(K^2-1\right)\cos\frac{\tau}{T_1}\sqrt{K^2-1} + \sqrt{K^2-1}\sin\frac{\tau}{T_1}\sqrt{K^2-1}}KT_1. \tag{27}$$

The basic formula (24) is exactly the same.

In Fig. 3 the dependence between T_2/T_1 and K for different values of τ/T_1 is presented. The region of stability is rendered by the curve $T_2/T_1 = f(K)$ and the horizontal line $T_2/T_1 = -1$.

In Fig. 4 the scheme of stabilisation of the motor using a feedback in the exciting circuit is presented. This scheme is interesting since it is often used in practice. In fact, it is a bilinear system and the relation corresponding to (8) takes the form

$$\frac{J_2}{J_1} = \frac{T_2}{T_1} = \frac{K}{1 + \frac{T_e}{T_1}(1+K)}, \tag{28}$$

where $T_e = L/R$ is the time constant of the exciting circuit and L stands for its inductance. The fundamental relation (24) is the same.

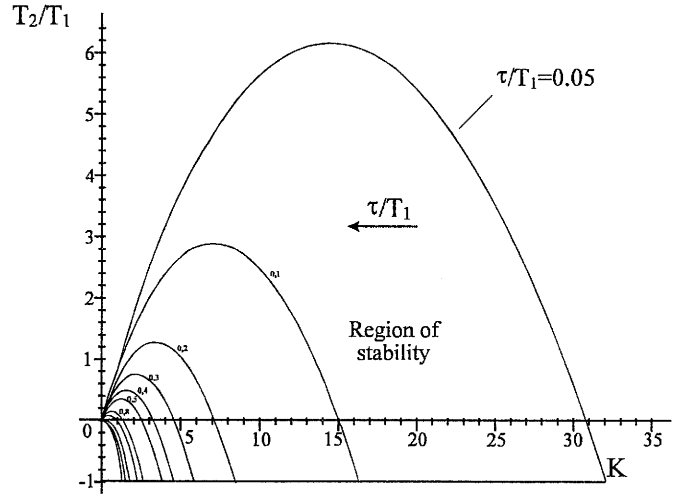

Fig. 3. Relation between T_2/T_1 and K.

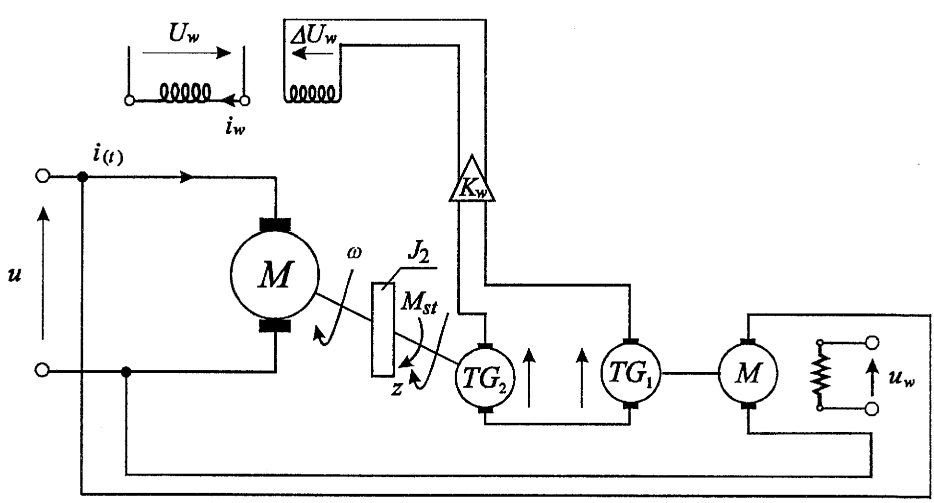

Fig. 4. Stabilisation via the feedback in the exciting circuit.

3. Conclusion

The relation (24) leads to the fundamental statement. The increment in the entropy of information in a control system is proportional to the logarithm of the relative increment in energy divided by the relative increment in the variance of the speed.

References

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