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A HYBRID APPROACH FOR SCHEDULING TRANSPORTATION NETWORKS

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In this paper, we consider a regulation problem of an urban transportation network. From a given timetable, we aim to find a new schedule of multiple vehicles after the detection of a disturbance at a given time. The main objective is to find a solution maximizing the level of service for all passengers. This problem was intensively studied with evolutionary approaches and multi-agent techniques, but without identifying its type before. In this paper, we formulate the problem as a classical one in the case of an unlimited vehicle capacity. In the case of a limited capacity and an integrity constraint, the problem becomes difficult to solve. Then, a new coding and well-adapted operators are proposed for such a problem and integrated in a new evolutionary approach.

Keywords: transportation systems, traffic regulation, genetic algorithms, multi-criteria optimization

1. Problem Formulation

1.1. Description

Let us consider an urban transportation network which consists of several lines. Each line is represented by a set of successive stations. On each line, a set of vehicles pass by stations according to schedules fixed in the timetable. At every time, passengers arrive at the different stations to board these vehicles. The arrival statistical distributions of passengers at a given station are known.

1.2. Problem

At a given moment, a disturbance occurs in the network and affects a vehicle at its arrival at a station of a certain line. The problem is to correct the timetable (the schedule of the vehicles) so that the quality of service for passengers boarding vehicles is maximized. The service quality can in this case be reduced to the aggregation of several criteria, e.g., the minimization of the waiting time of the passengers caused by the delays, the minimization of the increase in the total travel time and the minimization of the total transit time spent in the connecting nodes (Aloulou, 1999; Fayech, 2000; Fayech *et al.*, 2001).

2. Mathematical Formulation

In this section, we consider the regulation problem as it was treated in (Aloulou, 1999; Fayech, 2000; Fayech *et al.*, 2001). We present the different characteristics of such a problem, its constraints as well as the different criteria.

2.1. Initial Data

- The transportation network consists of N lines. Each line l $(1 \le l \le N)$ contains n_l stations S_k^l $(1 \le k \le n_l)$.
- For each line l $(1 \le l \le N)$, there are m_l vehicles V_i^l $(1 \le i \le m_l)$. Each vehicle can simply represent a journey ($V_i^l = 1$ journey from S_1^l to $S_{n_l}^l$).
- $\forall 1 \le l \le N$, $\forall 1 \le i \le m_l$, $\forall 1 \le k \le n_l$, the departure time of V_i^l on S_k^l is $d(V_i^l, S_k^l) = d_{i,k}^l$.
- $\forall 1 \le l \le N$, $\forall 1 \le i \le m_l$, $\forall 1 \le k \le n_l$, the charge of a vehicle at a station is $C(V_i^l, S_k^l) = C_{i,k}^l$.
- Such departure times were previously calculated by taking account of several factors like the passenger's arrival statistical distributions in order to maximize the service quality. These distributions are supposed to be known and precisely evaluated. Thus, we denote by $\mu_k^l(t)$ the number of passengers per unit of time which arrive at the station S_k^l at the time t. Departure times must satisfy two constraints: the minimal duration constraint and the transit duration constraint.
- The minimal duration is the duration that a vehicle puts to reach S_{k+1}^l from S_k^l: $\forall 1 \le l \le N$, $\forall 1 \le i \le m_l$, $\forall 1 \le k \le n_l - 1$, $d_{i,k+1}^l - d_{i,k}^l \ge dm_{i,k}^l$.

The data $dm^l_{i,k}$ are calculated according to the distance between S^l_k and S^l_{k+1} and to the traffic state in the periods corresponding to the departure of the i-th vehicle.

- As for the transit duration, we suppose that the lines l_1 and l_2 are crossed in a node which corresponds to the station $S^{l_1}_{k_1} = S^{l_2}_{k_2}$. For each vehicle $V^{l_1}_i$ of the line l_1, there is a vehicle $V^{l_2}_{\sigma(i)}$ which arrives after $V^{l_1}_i$ and takes a proportion of passengers equal to $\tau^{l_1,l_2}_{i,\sigma(i)}$, who change from $V^{l_1}_i$ to $V^{l_2}_{\sigma(i)}$ with $\sigma(i) \in \{1,\dots,m_{l_2}\}$. In fact, σ is a mapping depending on the node (l_1,l_2,k_1,k_2). The different nodes are represented in a list $\Im$ and each node is represented by the quadruple (l_1,l_2,k_1,k_2).[1]

2.2. Perturbation Information

- At a given time d_0, a disturbance occurs in the network and affects the vehicle $V^{l_0}_{i_0}$ which will arrive at the station $S^{l_0}_{k_0}$ with a delay of a duration δ. Thus, we can write $d'^{l_0}_{i_0,k_0} = d^{l_0}_{i_0,k_0} + \delta$.
- The problem is to find new departure times $d'^l_{i,k}$ of the vehicles at the different stations such that $d'^l_{i,k} > d_0$.
- Two possibilities can be considered. In the first one, we do nothing. In this case, the delay of the disturbed vehicle will be propagated through the stations according to the equation $d'^{l_0}_{i_0,k} = d^{l_0}_{i_0,k} + \delta$, $\forall\, k \ge k_0$. Clearly, this solution is naive because it does not guarantee the performance. The second solution consists in updating all departure times of the vehicles by following a regulation strategy.

2.3. Constraints

- The constraints of the minimum duration necessary to go from a station to the next one must be obeyed, i.e., $\forall\, 1 \le l \le N$, $\forall\, 1 \le i \le m_l$, $\forall\, 1 \le k \le n_l - 1$, we have $d'^l_{i,k+1} - d'^l_{i,k} \ge dm^l_{i,k}$.
- The constraint of the transit duration is given as $\forall\,(l_1,l_2,k_1,k_2) \in \Im$, $d'^{l_2}_{\sigma(i),k_2} - d'^{l_1}_{i,k_1} \ge tr^{l_1,l_2}_{k_1,k_2}$. This constraint is taken into account in order to ensure a maximum number of transits realized.
- According to the adopted strategy, delaying or advancing the vehicles, we can have constraints like $d'^l_{i,k} - d^l_{i,k} \le \pi^l_{i,k}$ (the maximum delay constraint) and $d'^l_{i,k} \ge \nu^l_{i,k}$ (in the case of regulation only by delay, we have $d^l_{i,k} = \nu^l_{i,k}$).
- As for other constraints, it is obvious that $d'^l_{i,k} = d^l_{i,k}$ for all $d^l_{i,k} \le d_0$.

[1] Note that $(l_1,l_2,k_1,k_2) \neq (l_2,l_1,k_2,k_1)$.

2.4. Criteria

In this section, we also consider the same criteria as those considered in (Aloulou, 1999; Fayech, 2000; Fayech *et al.*, 2001). The service quality can be reduced to an aggregation of the following criteria:

The minimization of the sum of waiting times of the passengers caused by the vehicles' delays. Figure 1 describes an example of arrival distribution $\mu^l_k(t)$ at a station S^l_k between two successive departure moments $d^l_{i,k}$ and $d^l_{i+1,k}$ of two vehicles from the same line. The waiting time A of passengers for the different stations and for all vehicles can be described by

$$A = \sum_{l=1}^{l=N} \sum_{k=1}^{k=n_l} \sum_{i=1}^{i=m_l-1} \int_0^{d^l_{i+1,k}-d^l_{i,k}} \mu^l_k(t)(d^l_{i+1,k} - d^l_{i,k} - t)\,\mathrm{d}t. \tag{1}$$

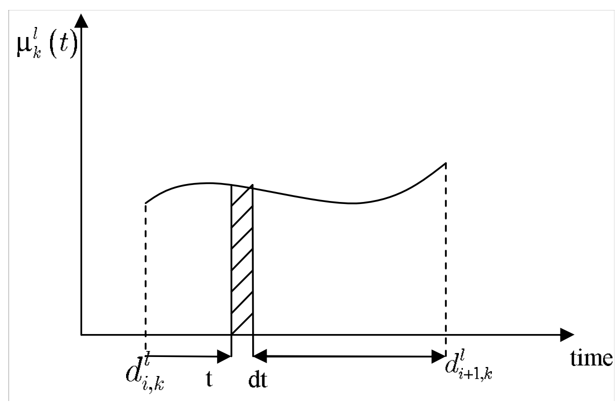

Fig. 1. Arrival distribution between two successive departures.

We suppose that the arrival rate of passengers at a stop is constant and equal to μ^l_k. Hence we obtain

$$A = \sum_{l=1}^{l=N} \sum_{k=1}^{k=n_l} \sum_{i=1}^{i=m_l-1} \frac{\mu^l_k}{2}(d^l_{i+1,k} - d^l_{i,k})^2 = \sum_{l=1}^{l=N} \sum_{k=1}^{k=n_l} \sum_{i=1}^{i=m_l-1} \frac{\mu^l_k}{2} I^2_{i,k,l}. \tag{2}$$

In (Aloulou, 1999; Fayech, 2000; Fayech *et al.*, 2001), the authors defined the gain in the total waiting time of passengers at stations, denoted by $E(\Delta A)$. If the arrival rate of passengers at stops is constant, we have

$$E(\Delta A) = \sum_{l=1}^{l=N} \sum_{k=1}^{k=n_l} \sum_{i=1}^{i=m_l-1} \frac{\mu^l_k}{2}(I^2_{i,k,l} - I'^2_{i,k,l}), \tag{3}$$

where $I_{i,k,l}$ and $I'_{i,k,l}$ are respectively the intervals without and with regulation $(I_{i,k,l} = d^l_{i+1,k} - d^l_{i,k})$.

The minimization of the increase in the expected total travel time. For the passengers aboard the vehicles, the increase in the expected total travel time induced by the regulation is calculated according to the charges of the vehicles at the different stations:

$$E(\Delta T) = \sum_{l=1}^{l=N} \sum_{k=2}^{k=n_l} \sum_{k=1}^{i=m_l} r^l_{i,k} C'^l_{i,k-1}, \qquad (4)$$

where $r^l_{i,k}$ is the delay imposed by the regulation of the vehicle V^l_i at the station S^l_k (in fact, $r^l_{i,k} = d'^l_{i,k} - d^l_{i,k}$ and $C'^l_{i,k} = C^l_{i,k} + \mu^l_k(d'^l_{i,k} - d^l_{i,k}) - \mu^l_k(d'^l_{i-1,k} - d^l_{i-1,k})$).

The minimization of the transit time. To minimize the transit duration of passengers in a node from the line l_1 to the line l_2 or the opposite, a quality indicator which measures the gain induced by the regulation on the total duration of the transit was proposed in (Aloulou, 1999; Fayech, 2000; Fayech *et al.*, 2002a). It is supposed that the number of passengers carrying out the correspondence from $V^{l_1}_i$ to $V^{l_2}_{\sigma(i)}$ is proportional to the charge of $V^{l_1}_i$ at its arrival at the node (l_1, l_2, k_1, k_2) with a rate equal to $\tau^{l_1 l_2}_{i,\sigma(i)}$. We calculate the number of passengers in transit in each node in the following way: $np(V^{l_1}_i \rightarrow V^{l_2}_{\sigma(i)}) = \tau^{l_1 l_2}_{i,\sigma(i)} C'^{l_1}_{i,k_1-1}$ with $C'^{l_1}_{i,k_1-1}$, the charge of $V^{l_1}_i$ on its arrival at the node, i.e., at its departure from the station $S^{l_1}_{k_1-1}$. We assume that the rates $\tau^{l_1 l_2}_{i,\sigma(i)}$ can be considered constant for all vehicles (Fayech *et al.*, 2002b). The total waiting time of passengers in transit can be given by

$$A_{\text{transit}} = \sum_{(l_1,l_2,k_1,k_2)\in\Im} \sum_i \tau^{l_1 l_2}_{k_1,k_2} C'(V^{l_1}_i, S^{l_1}_{k_1-1}) \times (d'^{l_2}_{\sigma(i),k_2} - d'^{l_1}_{i,k_1}). \qquad (5)$$

Thus, the quality indicator can be deduced by comparing the values of the transit durations without and with regulation. The gain in the total transfer time is then equal to $E(\Delta A_{\text{transit}}) = A_{\text{transit}}$ (without regulation) $-A_{\text{transit}}$ (with regulation). We aim, therefore, to maximize this gain to reduce the durations of transit at the nodes.

2.5. Global Evaluation Function

In order to aggregate the three quality indicators previously presented, $E(\Delta A)$, $E(\Delta A_{\text{transit}})$ and $E(\Delta T)$, in a one global function, the authors of (Fayech, 2000; Fayech *et al.*, 2001; 2002a) defined weights for the different criteria. In fact, an importance degree could be fixed to each criterion, according to the different constraints which are present. The cost function to be maximized can be reduced to the following one:

$$f = \alpha E(\Delta A) + \beta E(\Delta A_{\text{transit}}) - \gamma E(\Delta T),$$

where α, β and γ are positive parameters fixed by the regulator. Such parameters assign a weight to each criterion.

3. Transportation Systems with Unlimited Capacity

As we can notice, the problem can be reduced to an optimization problem with several variables. If we assimilate the set of $d'^l_{i,k}$ to a vector $z = (z_1, z_2, \dots, z_q, \dots, z_r)$ from $(\mathbb{R}_+)^r$ with $r = \sum_{l=1}^{l=N} n_l m_l$, the regulation problem is reduced to the following problem:

$$(II) : \begin{cases} \text{Maximize } f(z) \\ \text{such that } g_h(z) \le 0 \\ \text{with } h \in \{1, 2, \dots, p\}, \end{cases}$$

where $f(\cdot)$ is a second-degree polynomial function in z, p is the total number of constraints on $d'^l_{i,k}$ and $g_h(\cdot)$ are linear forms in z. The problem (II) is then a classical optimization problem. Note that the problem **is not combinatorial** as was proposed by Fayech (2000). In the literature, we can find many solvers able to solve efficiently such a problem. As an example, we can cite the Lancelot© software based on analytical approaches like the Karush-Kuhn-Tucker method.

3.1. Illustrative Example

Consider a vehicle network composed of $N = 3$ lines (Fig. 2). These three lines are crossed in a node. Each line l contains $n_l = 5$ stops and $m_l = 3$ vehicles. Let us study a disturbance that affects the second vehicle V^1_2 of the first line. This disturbance is detected at 10h:23 and it

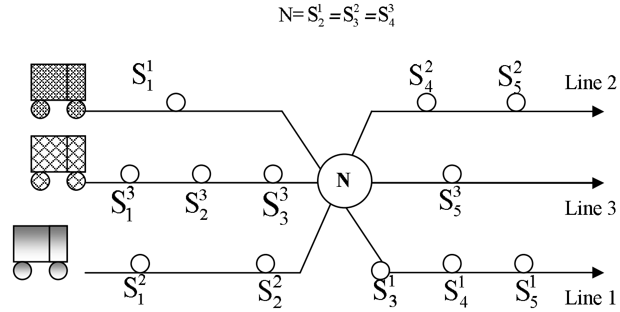

Fig. 2. Structure of the network studied.

is caused by a technical problem which obliges the vehicle V_2^1 to have a standstill of 3 minutes at the stop S_2^1. All lines have a passage frequency of 20 minutes. The maximal delay is $\pi_{i,k}^l = 5$ minutes, the minimal duration between two stops is $dm_{i,k}^l = 3$ minutes and the transit minimal duration is $tr_{k_1,k_2}^{l_1,l_2} = 1$ minute. Theoretical time-tables of three lines are given in Table 1.

Table 1. Timetables of the three lines.

	V_1^1	V_2^1	V_3^1
S_1^1	10h:00	10h:20	10h:40
$N = S_2^1$	10h:05	10h:25	10h:45
S_3^1	10h:10	10h:30	10h:50
S_4^1	10h:05	10h:35	10h:55
S_5^1	10h:20	10h:40	11h:00

	V_1^2	V_2^2	V_3^2
S_1^2	09h:56	10h:16	10h:36
S_2^2	10h:01	10h:21	10h:41
$N = S_3^2$	10h:06	10h:26	10h:46
S_4^2	10h:11	10h:31	10h:51
S_5^2	10h:16	10h:36	10h:56

	V_1^3	V_2^3	V_3^3
S_1^3	09h:52	10h:12	10h:32
S_2^3	09h:57	10h:17	10h:37
S_3^3	10h:02	10h:22	10h:42
$N = S_4^3$	10h:07	10h:27	10h:47
S_5^3	10h:12	10h:32	10h:52

We assume that the investigated horizon is included in a homogenous period of the day. Then the arrival distributions of passengers at stops are constant and equal to $\mu_k^l = 1$ passenger per minute. We also suppose that the number of passengers in a vehicle from a line arriving at a node and willing to take a vehicle of another line is proportional to the charge of the vehicle with a rate of 10%. These rates between the concerned three lines are supposed to be constant. Hence the nonlinear optimization problem is as follows: Minimize

$$f(z)=\alpha E(\Delta A)(z)+\beta E(\Delta A_{\text{transit}})(z)-\gamma E(\Delta T)(z),$$

where

$$z = (d'^{1}_{1,1}, d'^{1}_{2,1}, \dots, d'^{N}_{m_N-1,n_N}, d'^{N}_{m_N,n_N})^T,$$

$$E(\Delta A)(z) = \sum_{l=1}^{l=N}\sum_{k=1}^{k=n_l}\sum_{i=1}^{i=m_l-1}\frac{\mu_k^l}{2}\Big[(d_{i+1,k}^l - d_{i,k}^l)^2 - (d'^l_{i+1,k} - d'^l_{i,k})^2\Big],$$

$$E(\Delta T)(z) = \sum_{l=1}^{l=N}\sum_{k=2}^{k=n_l}\sum_{i=1}^{i=m_l}(d'^l_{i,k} - d^l_{i,k})\Big[C^l_{i,k-1} + \mu^l_{k-1}(d'^l_{i,k-1} - d^l_{i,k-1}) - \mu^l_{k-1}(d'^l_{i-1,k-1} - d^l_{i-1,k-1})\Big],$$

$$E(\Delta A_{\text{transit}})(z) = \sum_{(l_1,l_2,k_1,k_2)\in\Im}\sum_i \tau^{l_1,l_2}_{k_1,k_2}\Big[C^{l_1}_{i,k_1-1} + \mu^{l_1}_{k_1-1}(d'^{l_1}_{i,k_1-1} - d^{l_1}_{i,k_1-1}) - \mu^{l_1}_{k_1-1}(d'^{l_1}_{i-1,k_1-1} - d^{l_1}_{i-1,k_1-1})\Big] \times (d'^{l_2}_{\sigma(i),k_2} - d'^{l_1}_{i,k_1}),$$

subject to

$$\forall\, 1\le l\le N,\quad \forall\, 1\le i\le m_l,\quad \forall\, 1\le k\le n_l-1,\quad d'^l_{i,k+1} - d'^l_{i,k} \ge dm^l_{i,k},$$

$$\forall\,(l_1,l_2,k_1,k_2)\in\Im,\quad d'^{l_2}_{\sigma(i),k_2} - d'^{l_1}_{i,k_1} \ge tr^{l_1,l_2}_{k_1,k_2},$$

$$\forall\, 1\le l\le N,\quad \forall\, 1\le i\le m_l,\quad \forall\, 1\le k\le n_l-1,\quad d'^l_{i,k} - d^l_{i,k} \le \pi^l_{i,k},$$

$$\forall\, d^l_{i,k}\le d_0,\quad d'^l_{i,k} = d^l_{i,k}.$$

A comparison between the solution which uses a genetic algorithm (as was studied in (Fayech *et al.*, 2001)) and analytical methods (AM) is given in Table 2.

Table 2. Comparison between the AM and the GA.

	AM		Genetic algorithms	
	f	Time(s)	f	Time(s)
Ins1	327.9	0	260.6	10
Ins2	167.2	0	120.3	11
Ins3	247.5	0	190.2	9
Ins4	407.0	0	330.1	10
Ins5	487.9	0	400.98	9

We notice that solving the problem with genetic algorithms is not suitable since they require a lot of time to obtain an approximate solution as shown in Table 2 and the value of the objective function is always smaller than the value given by the analytic method. In the next section, we consider other constraints in order to integrate some real practical conditions. The problem becomes difficult to solve and heuristic methods will be studied to solve it.

4. Introducing Practical Constraints

In this section, we deal with practical constraints which should be taken into account to obtain feasible schedules. Such constraints concern the capacities of the vehicles which are limited (which corresponds to reality) and the integrity of the numbers of passengers. In fact, when disturbances occur, the number of passengers who wait at a station can exceed the vehicle capacity ($C_{\max}$). We write $na^l_{i,k}$ and $na'^l_{i,k}$ for the numbers of passengers who cannot board the vehicle V^l_i at the station S^l_k (before and after regulation, respectively) because of the capacity constraint. We denote by $nd^l_{i,k}$ the number of passengers who get off the vehicle V^l_i at the station S^l_k before regulation, and $nd'^l_{i,k}$ is the number of passengers after regulation. For the calculation of the charge, there are two cases:

- The vehicle V^l_i cannot take all passengers who wait at the station S^l_k and then its charge on its departure from S^l_k is $C'^l_{i,k} = C_{\max}$.
- The vehicle can take all passengers who wait at the station S^l_k, and their number is $\mu^l_k(d'^l_{i,k} - d'^l_{i-1,k}) + na'^l_{i-1,k}$. We deduce that the charge of the vehicle V^l_i on its departure from the station S^l_k is given by

$$C'^l_{i,k} = \min\big\{C'^l_{i,k-1} + \mu^l_k(d'^l_{i,k} - d'^l_{i-1,k}) - nd'^l_{i,k} + na'^l_{i-1,k}, C_{\max}\big\}.$$

As the capacity of vehicles is an integer, the quantity $\mu^l_k(d'^l_{i,k} - d'^l_{i-1,k})$ must be approximated by an integer $zd'^l_{i,k}$ such that $\mu^l_k(d'^l_{i,k} - d'^l_{i-1,k}) \le zd'^l_{i,k} \le \mu^l_k(d'^l_{i,k} - d'^l_{i-1,k}) + 1$. Hence the charge of the vehicle V^l_i on its departure from the station S^l_k is given by

$$C'^l_{i,k} = \min\big\{C'^l_{i,k-1} + zd'^l_{i,k} - nd'^l_{i,k} + na'^l_{i-1,k}, C_{\max}\big\}.$$

For the calculation of $na'^l_{i,k}$, there are also two cases:

- All passengers who wait for the vehicle V^l_i at the station S^l_k can board the vehicle V^l_i, $na'^l_{i,k} = 0$ (no one is waiting for V^l_{i+1}).
- Only a part of the passengers who wait for the vehicle V^l_i at the station S^l_k can board the vehicle V^l_i, the number of passengers who cannot board V^l_i and wait for V^l_{i+1} being $na'^l_{i,k} = C'^l_{i,k-1} + zd'^l_{i,k} - nd'^l_{i,k} + na'^l_{i-1,k} - C_{\max}$. Consequently, the number of passengers who cannot board the vehicle V^l_i at the station S^l_k is then given by

$$na'^l_{i,k} = \max\big\{0, C'^l_{i,k-1} + zd'^l_{i,k} - nd'^l_{i,k} + na'^l_{i-1,k} - C_{\max}\big\}.$$

In order to simplify the problem, we can suppose that the number of passengers $nd^l_{i,k}$ who get off the vehicle V^l_i at the station S^l_k is proportional to the charge $C'^l_{i,k-1}$ of the vehicle V^l_i at the station S^l_{k-1} with $nd'^l_{i,k} = \alpha^l_{i,k-1}C'^l_{i,k-1}$. Also, here the quantity $nd'^l_{i,k}$ must be an integer, and so we take $nd'^l_{i,k}$ such that $\alpha^l_{i,k-1}C'^l_{i,k-1} \le nd'^l_{i,k} \le \alpha^l_{i,k-1}C'^l_{i,k-1} + 1$. In this case, we obtain

$$C'^l_{i,k} = \min\big\{(1-\alpha^l_{i,k-1})C'^l_{i,k-1} + zd'^l_{i,k} + na'^l_{i-1,k}, C_{\max}\big\},$$

$$na'^l_{i,k} = \max\big\{0, (1-\alpha^l_{i,k-1})C'^l_{i,k-1} + zd'^l_{i,k} + na'^l_{i-1,k} - C_{\max}\big\},$$

$$C_{\max} - C'^l_{i,k} = \max\big\{0, C_{\max} - (1-\alpha^l_{i,k-1})C'^l_{i,k-1} - zd'^l_{i,k} - na'^l_{i-1,k}\big\}.$$

The new constraints induce an additional difficulty. The preceding analytical formulation will not be able to solve it because of the 'min-max' and the integrity constraints. The problem becomes nonlinear and difficult to solve. In such a case, we have to update the formulation of the different criteria:

(a) $$E(\Delta A) = \sum_{l=1}^{l=N}\sum_{k=1}^{k=n_l}\sum_{i=1}^{i=m_l-1}\Big[\frac{\mu^l_k}{2}(I^2_{i,k,l} - I'^2_{i,k,l}) + (H^l_{i,k} - H'^l_{i,k})\Big]$$

with

$$H^l_{i,k} = na^l_{i,k}(d^l_{i+1,k} - d^l_{i,k})$$

and

$$H'^l_{i,k} = na'^l_{i,k}(d'^l_{i+1,k} - d'^l_{i,k}),$$

(b) $$E(\Delta T) = \sum_{l=1}^{l=N}\sum_{k=2}^{k=n_l}\sum_{i=1}^{i=m_l} r^l_{i,k}C'^l_{i,k-1}$$

so that $C'^l_{i,k}$ is calculated according to the preceding formula,

(c) $$E(A_{\text{transit}}) = A_{\text{transit}}(\text{without regulation}) - A_{\text{transit}}(\text{with regulation}).$$

That is why we choose genetic algorithms to solve it.

4.1. Genetic Algorithms

Genetic algorithms enable us to make an initial set of solutions evolve into a final set of solutions bringing a global improvement according to a criterion fixed at the beginning (Banzhaf *et al.*, 1998; Burke and Smith, 2000). These algorithms function with the same usual genetic mechanisms (crossover, mutation, selection). In this section, we present the different elements of our genetic algorithm and we illustrate them with some examples.

4.2. Coding for the Constrained Problem

The selection of a representational scheme of the solution is a basic and essential prerequisite step for a successful application of genetic algorithms. Aloulou (1999) developed a coding which is presented in Table 3. When a perturbation is detected for a vehicle $V_{i_0}^{l_0}$ at $S_{k_0}^{l_0}$, we should determine a set of variables corresponding to $d_{i,k}^{l} > d_{i_0,k_0}^{l_0}$. In Table 3, the grey cells correspond to the variables such that $d_{i,k}^{l} \leq d_{i_0,k_0}^{l_0}$ and the white cells correspond to the variables such that $d_{i,k}^{l} > d_{i_0,k_0}^{l_0}$.

Table 3. Aloulou's encoding.

	V_{i-N}^l	···	V_{i-1}^l	V_j^l	V_{i+1}^l	···	V_{i+N}^l
S_{k-N}^l					D_0		D_2
⋮	⋮		⋮	⋮	⋮		⋮
S_{k-1}^l					D_2		D_0
S_k^l				D_0	D_1		D_0
S_{k+1}^l			D_0	D_0	D_0		D_1
⋮				⋮	⋮		⋮
S_{k+N}^l	D_1		D_1	D_0	D_2		D_0

Each cell (S_k^l, V_i^l) contains the decision to be taken when the vehicle V_i^l reaches the station S_k^l or when it is on the way toward the station. For example, a D_1 decision can consist in delaying the vehicle by a unit of time when it arrives at S_k^l. Another decision D_2 encourages the driver to accelerate until S_k^l is reached. The cell which contains D_0 implies that nothing will be done. The decisions which can be made are:

D_0: do nothing,

D_1: stop a vehicle for some time at the station,

D_2: accelerate, i.e., pass from the initial speed to a higher speed, if possible.

This type of coding has the inconvenience of a reduced exploration of the search space in terms of exchange possibilities. We present a new coding which enhances the exploration of the search space. The solution is a multi-dimensional vector $z = (z_1, z_2, \ldots, z_q, \ldots, z_r)^T$ with $r = \sum_{l=1}^{l=N} n_l m_l$. The advantage of such a coding is the capability of exploring more the search space and enhancing the genetic exchange possibilities by applying some fine crossover operators. Figure 3(a) describes the important elements of this coding. As an example, Fig. 3(b) describes the encoding for the example treated in Section 3.1.

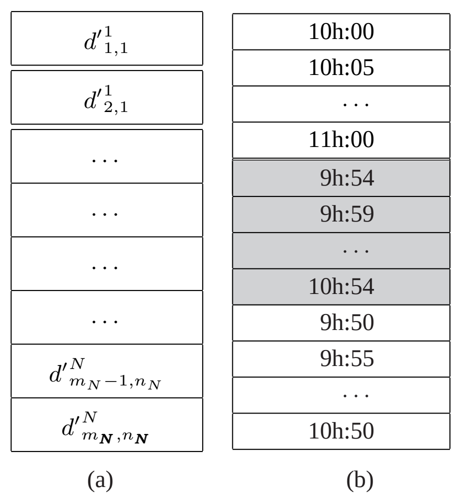

Fig. 3. (a) Chromosome encoding; (b) Example of chromosome encoding.

4.3. Crossover Operator

Crossover is a basic operator of GAs, and the performance of GAs depends on it considerably. Crossover is the process of creating two children by the combination of two parents. The crossover allows us to explore the search space. It will be carried out depending on the crossover probability p_{cross} (Goldberg, 1989; Dasgupta and Michalewicz, 1997).

The procedure consists in choosing randomly two feasible parents (individuals) $P1$ and $P2$. Then, we choose from the two individuals a common portion $[k_i, k_f]$ between the stations $S_{k_i}^{l_1}$ and $S_{k_f}^{l_1}$. This portion must not contain a connecting zone (in order to avoid perturbations of the transit operation).

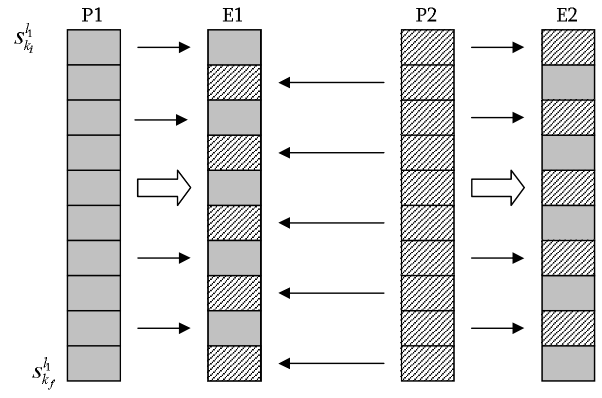

Fig. 4. Crossover.

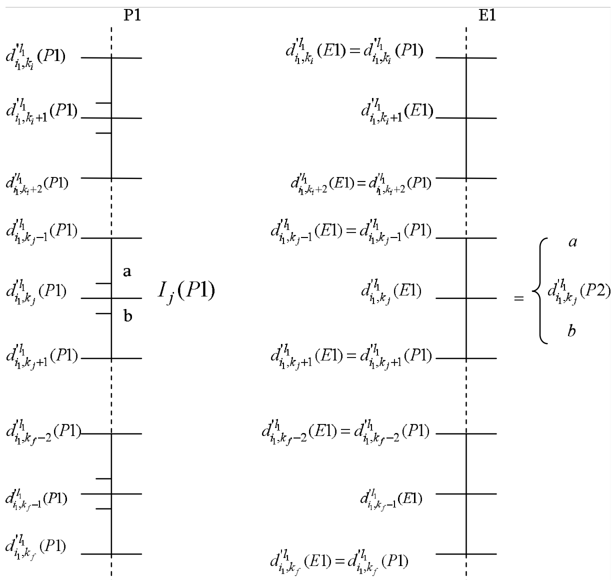

Fig. 5. Transmission of genes from the parent $P1$ to the child $E1$.

The child $E1$ keeps the same schedules as the parent $P1$ for the 'even' stations $s^{l_1}_{k_i}$, $s^{l_1}_{k_i+2}$, $s^{l_1}_{k_i+4}, \ldots, s^{l_1}_{k_f}$ (Fig. 4). For the 'odd' stations, it keeps the schedules as the parent $P2$, only if these schedules assure a feasible individual. So, for each 'odd' station of the individual $P1$, we determine the admissible interval I_j for k_j located between $k_j - 1$ and $k_j + 1$ as shown in Fig. 5. $I_j = [a, b]$ with $a = d'^{l_1}_{i_1,k_j-1}(P1) + dm^{l_1}_{i_1,k_j-1}(P1)$ and $b = d'^{l_1}_{i_1,k_j+1}(P1) - dm^{l_1}_{i_1,k_j}(P1)$. The objective is to replace the schedule of $P2$ (which leads to a nonfeasible individual by a limit of the interval (a or b). The rule we have to apply is as follows:

- If $d'^{l_1}_{i_1,k_j}(P2) \in I_j$ then $d'^{l_1}_{i_1,k_j}(E1) = d'^{l_1}_{i_1,k_j}(P2)$,
- If $d'^{l_1}_{i_1,k_j}(P2) \notin I_j$ then
  - If $d'^{l_1}_{i_1,k_j}(P2) < a$ then $d'^{l_1}_{i_1,k_j}(E1) = a$,
  - If $d'^{l_1}_{i_1,k_j}(P2) > b$ then $d'^{l_1}_{i_1,k_j}(E2) = b$.

In this way, we are sure that the obtained child $E1$ contains feasible schedules. For the construction of the child $E2$, we proceed in the same way with replacing $P1$ by $P2$.

Example 1. Consider a portion which contains 3 stations. We suppose that $\forall\, i, k, l \;\; dm^l_{i,k} = 3$ (Fig. 6). The admissible intervals for the two individuals are:

$$\begin{aligned} I_j(P1) &= [\text{10h:00} + \text{00h:03}, \text{10h:11} - \text{00h:03}] \\ &= [\text{10h:03}, \text{10h:08}], \\ I_j(P2) &= [\text{10h:01} + \text{00h:03}, \text{10h:10} - \text{00h:03}] \\ &= [\text{10h:04}, \text{10h:07}], \end{aligned}$$

$$\begin{aligned} d'^{l_1}_{i_1,k_j}(P2) &= \text{10h:05} \in I_j(P1) \Rightarrow d'^{l_1}_{i_1,k_j}(E1) \\ &= \text{10h:05}, \\ d'^{l_1}_{i_1,k_j}(P1) &= \text{10h:06} \in I_j(P2) \Rightarrow d'^{l_1}_{i_1,k_j}(E2) \\ &= \text{10h:06}. \qquad \blacklozenge \end{aligned}$$

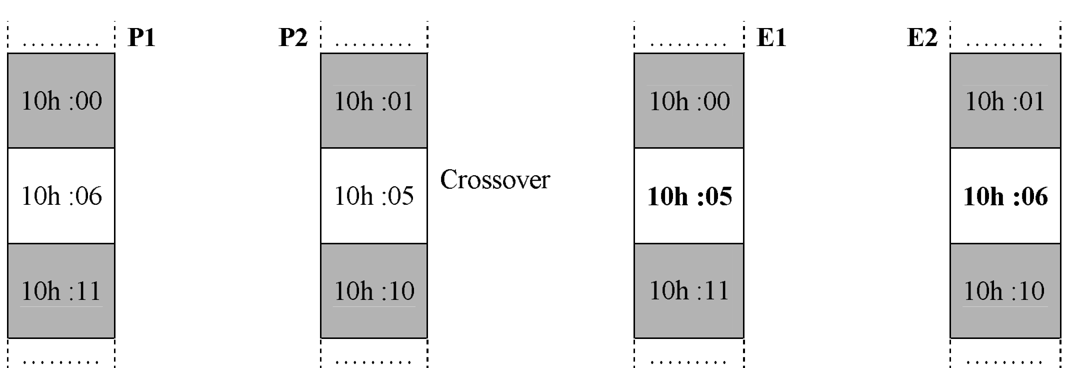

Fig. 6. Crossover: Example 1.

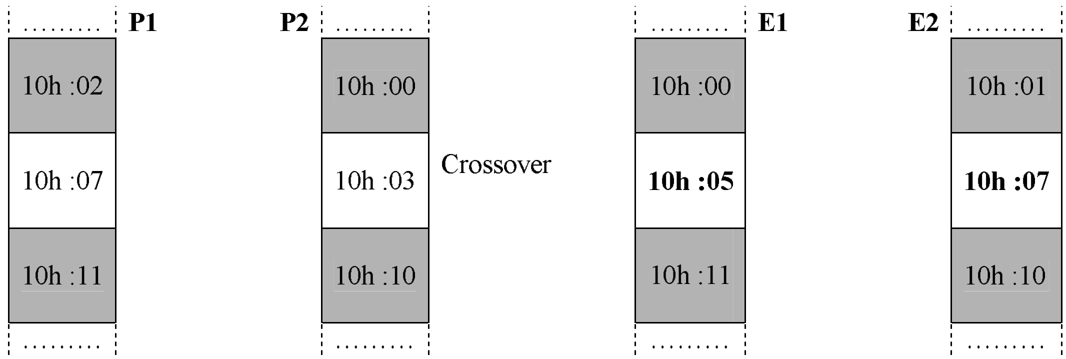

Fig. 7. Crossover: Example 2.

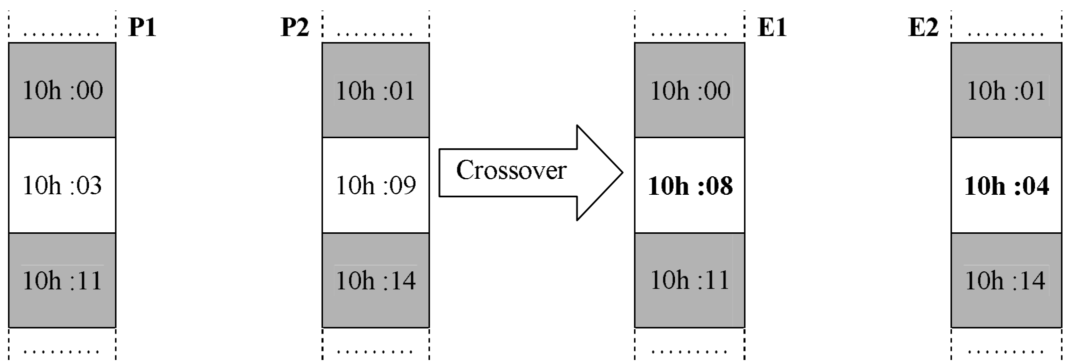

Fig. 8. Crossover: Example 3.

Example 2. Here also we have a portion which contains 3 stations. We suppose that $\forall\, i,k,l \quad dm^{l}_{i,k} = 3$ (Fig. 7). It follows that

$$\begin{aligned} I_j(P1) &= [\text{10h:02} + \text{00h:03}, \text{10h:11} - \text{00h:03}] \\ &= [\text{10h:05}, \text{10h:08}], \\ I_j(P2) &= [\text{10h:00} + \text{00h:03}, \text{10h:10} - \text{00h:03}] \\ &= [\text{10h:03}, \text{10h:07}], \end{aligned}$$

$$\begin{aligned} d'^{l_1}_{i_1,k_j}(P2) &= \text{10h:03} \notin I_j(P1) \Rightarrow d'^{l_1}_{i_1,k_j}(E1) \\ &= \text{10h:05}, \\ d'^{l_1}_{i_1,k_j}(P1) &= \text{10h:07} \in I_j(P2) \Rightarrow d'^{l_1}_{i_1,k_j}(E2) \\ &= \text{10h:07}. \quad \blacklozenge \end{aligned}$$

Example 3. Here also we have a portion which contains 3 stations. We suppose that $\forall\, i,k,l$ $dm^{l}_{i,k} = 3$ (Fig. 8). It follows that

$$\begin{aligned} I_j(P1) &= [\text{10h:00} + \text{00h:03}, \text{10h:11} - \text{00h:03}] \\ &= [\text{10h:03}, \text{10h:08}], \\ I_j(P2) &= [\text{10h:01} + \text{00h:03}, \text{10h:14} - \text{00h:03}] \\ &= [\text{10h:04}, \text{10h:11}], \end{aligned}$$

$$\begin{aligned} d'^{l_1}_{i_1,k_j}(P2) &= \text{10h:09} \notin I_j(P1) \Rightarrow d'^{l_1}_{i_1,k_j}(E1) \\ &= \text{10h:08}, \\ d'^{l_1}_{i_1,k_j}(P1) &= \text{10h:03} \notin I_j(P2) \Rightarrow d'^{l_1}_{i_1,k_j}(E2) \\ &= \text{10h:04}. \quad \blacklozenge \end{aligned}$$

4.4. Mutation

The mutation operator represents a random exchange on a gene. It will be carried out depending on the mutation probability p_{mut}. An illustration of the proposed method is given in Fig. 9. We choose a station $S^{l_1}_{i_1,k_1}$ of

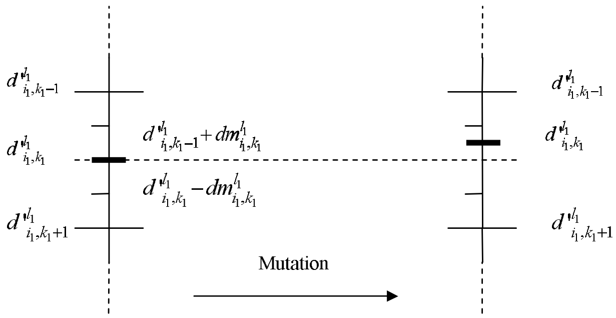

Fig. 9. Proposed mutation.

a line l_1 and a vehicle $V^{l_1}_{i_1}$ such that the previous and the next station are not concerned by any transit (so as not to perturb the transit operation). Here $d'^{l_1}_{i_1,k_1}$, $d'^{l_1}_{i_1,k_1-1}$ and $d'^{l_1}_{i_1,k_1+1}$ are the passage times of the vehicle $V^{l_1}_{i_1}$ at the stations $S^{l_1}_{i_1,k_1}$, $S^{l_1}_{i_1,k_1-1}$ and $S^{l_1}_{i_1,k_1+1}$. The procedure consists in changing the value of $d'^{l_1}_{i_1,k_1}$ by a random value in the feasible interval $I_1(P1) =[d'^{l_1}_{i_1,k_1-1} + dm^{l_1}_{i_1,k_1-1}$, $d'^{l_1}_{i_1,k_1+1} - dm^{l_1}_{i_1,k_1}]$.

Example 4. Here also we have a portion which contains 3 stations. We suppose that $\forall\, i,k,l \quad dm^l_{i,k} = 3$ (Fig. 10). It follows that

$$\begin{aligned} I_1(I) &= [\text{10h:00} + \text{00h:03}, \text{10h:10} - \text{00h:03}] \\ &= [\text{10h:03}, \text{10h:07}] \\ &\quad \text{10h:07} \in [\text{10h:03}, \text{10h:07}]. \end{aligned}$$

So, we can take

$$d'^{l_1}_{i_1,k_1}(I) = \text{10h:07}. \qquad \blacklozenge$$

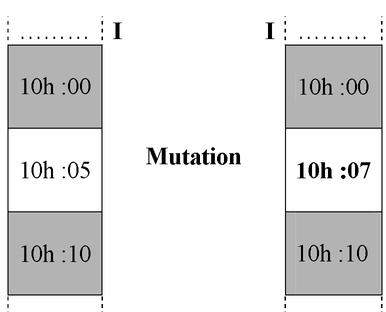

Fig. 10. Example of mutation.

4.5. Initial Population

The initial population is generated by using a constructive heuristic which allows us to build a starting solution. Such a solution will be randomly mutated to obtain a set of individuals (Fig. 11). The different steps of such a method must satisfy different temporal constraints of the studied problem. In addition to that, the different genetic operators are conceived such that the different temporal constraints will be integrated in the generated offspring. The algorithm parameters are fixed in a classical way. In fact, the mutation probability is equal to $P_{\text{mut}} = 0.05$ and the crossover probability is equal to $P_{\text{cross}} = 0.95$.

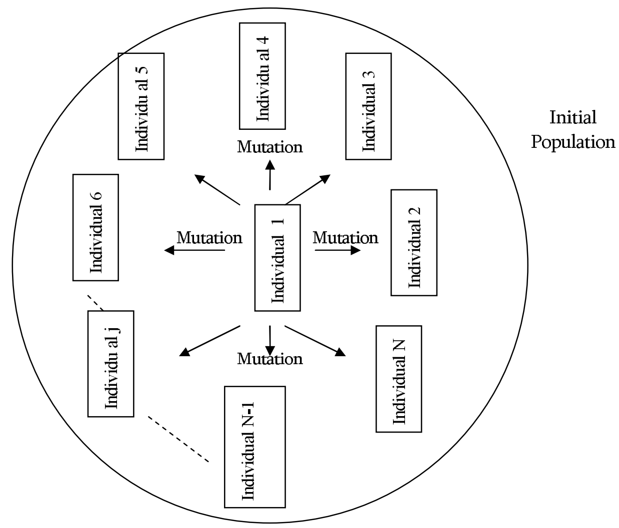

Fig. 11. Creating initial population.

4.5.1. Starting solution

Taking account of the temporal constraints, the starting solution must ensure the maximum of transit operations in nodes. For that, the first step for the construction of the solution would be to allow the delayed vehicle $v^{l_1}_{i_0}$ of the line l_1 to correspond at each time to the same vehicle $v^{l_2}_{\sigma(i_0)}$ of the line with which it is envisaged to make the correspondence. It is thus a question of delaying the vehicle $v^{l_2}_{\sigma(i_0)}$ by a duration equal to the delay undergone by $v^{l_1}_{i_0}$. In Table 4, we illustrate the starting solution for the example of Section 3, with a delay equal to 3 minutes.

4.6. Estimation of the Maximum Value for Each Criterion

In multiobjective optimization, we are often to estimate limits for each criterion studied. The aim is to be able to compare the solution given by the approach with the value of these limits and thus to conclude on the effectiveness of the developed approach (Fonseca, 1998). Here, we also propose to estimate such values for the three criteria (the gain in the total waiting time at the stations, the gain in the waiting time in transit nodes and the increase in the expected total travel time). The determination of these

Table 4. Illustration of the starting solution.

	V_1^1	V_2^1		V_3^1
S_1^1	10h:00	10h:20	10h:20	10h:40
$N = S_2^1$	10h:05	10h:25	10h:28	10h:45
S_3^1	10h:10	10h:30	10h:33	10h:50
S_4^1	10h:15	10h:35	10h:38	10h:55
S_5^1	10h:20	10h:40	10h:33	11h:00

	V_1^2	V_2^2	V_3^2
S_1^2	09h:56	10h:16	10h:36
S_2^2	10h:01	10h:21	10h:41
$N = S_3^2$	10h:06	10h:29	10h:46
S_4^2	10h:11	10h:34	10h:51
S_5^2	10h:16	10h:39	10h:56

	V_1^3	V_2^3	V_3^3
S_1^3	09h:52	10h:12	10h:32
S_2^3	09h:57	10h:17	10h:37
S_3^3	10h:02	10h:22	10h:42
$N = S_4^3$	10h:07	10h:30	10h:47
S_5^3	10h:12	10h:35	10h:52

limits enables us to define thereafter a method of evaluation based on the principle of evolutionary algorithms.

In Section 3, we proved that the solution of the problem considered with unlimited vehicle capacity (denoted by $z_f^* = (z_f^{1*},\ z_f^{2*}, \dots,\ z_f^{q*}, \dots,\ z_f^{r*})^T$) can be well estimated using well-adapted solvers (according to a fixed precision). This solution is calculated by optimizing the aggregation of the three criteria under the function $f(z) = \alpha f_1(z) + \beta f_2(z) - \gamma f_3(z)$ with $f_1(z) = E(\Delta A)(z)$, $f_2(z) = E(\Delta A_{\mathrm{transit}})(z)$ and we write $f_3(z) = E(\Delta T)(z)$.

By considering the hypothesis that all the passengers waiting at stations can all board the vehicle, i.e., the hypothesis that the capacity of the vehicles is unlimited (without integrity constraint), the problem will be reducible to the initial one as explained above. To estimate the maximum value for the criterion $E(\Delta A)$, it is enough to solve the same problem while assuming that $\alpha = 1$, $\beta = 0$ and $\gamma = 0$. Thus, we obtain the solution $z_{f_1}^* = (z_{f_1}^{1*}, z_{f_1}^{2*}, \dots, z_{f_1}^{q*}, \dots, z_{f_1}^{r*})^T$ and $f_1(z_{f_1}^*) = f_1^*$.

In the same way, a maximum value estimated for the criterion $E(\Delta A_{\mathrm{transit}})$ is obtained by solving the problem with taking $\alpha = 0$, $\beta = 1$ and $\gamma = 0$. The solution is $z_{f_2}^* = (z_{f_2}^{1*}, z_{f_2}^{2*}, \dots, z_{f_2}^{q*}, \dots, z_{f_2}^{r*})^T$ and we set $f_2(z_{f_2}^*) = f_2^*$.

For the criterion $E(\Delta T)$, an estimated minimum value is obtained by solving the problem with taking $\alpha = 0$, $\beta = 0$ and $\gamma = 1$. The solution is $z_{f_3}^* = (z_{f_3}^{1*}, z_{f_3}^{2*}, \dots, z_{f_3}^{q*}, \dots, z_{f_3}^{r*})^T$ and we set $f_3(z_{f_3}^*) = f_3^*$.

The objective of the estimation of the maximum criterion value is to automatically find a direction of search when running the algorithm as will be explained in the next section.

4.7. Fuzzy Evolutionary Optimization

To solve multiobjective optimization problems, it is interesting to find solutions in a correct computational time. Kacem *et al.* (2003) propose the application of fuzzy logic to compute different weights for each objective function and measure the quality of each solution. In this section, we use this work to overcome the problem of the direction determination.

In order to make the evaluation more efficient, we must avoid the demand that some objective function be always dominated by others. So we use a fuzzy logic application based on the following steps as is done in (Kacem *et al.*, 2003): The fuzzy evaluation is started by the estimation of a maximum value for the two objectives "*waiting time at stops and transit time in nodes*" and the estimation of a minimum value for the objective "*total travel time*". Each feasible solution z will be characterized by its values for the three objectives $(f_1(z), f_2(z)$ and $f_3(z))$. For each criterion i, we compute f_i^H : the best value given by a heuristic H. The fuzzification of each $f_i(z)$ is made by comparing it with f_i^* and f_i^H. In (Kacem *et al.*, 2003), the authors considered two fuzzy subsets $NEAR$ and FAR for each criterion. Indeed, a solution belongs to $NEAR(i)$ if its value according to criterion i is close to f_i^* and belongs to $FAR(i)$ otherwise. Afterwards, one defines $\overline{f}_i^k$ as the mean of the i-th objective function value of the solutions at the k-th iteration of the genetic algorithm. The membership function value is computed using the same fizzification process (i.e., we compare $\overline{f}_i^k$ with f_i^* and f_i^H, and we assign a membership value to the subset $NEAR(i)$). Thus, the different weights w_i^{k+1} are calculated dynamically according to the distance between the estimated limits and the average of the individuals of each generation (i.e., if $\overline{f}_i^k$ is close to f_i^* then, at the next iteration, the weight w_i^{k+1} must decrease, and increase otherwise). For more details, the reader is referred to (Kacem *et al.*, 2003).

The aim is to assist regulators of traffic in their decisions when they cannot clearly give preference to some criteria. We can propose, as was studied in (Kacem *et* *al.*, 2003), to find a set of Pareto-optimal solutions without giving any priority to a criterion. In this way, the algorithm finds automatically the search directions and the vector of weights will be dynamically computed when we pass from generation G_k to the next one G_{k+1} according to the distance between the estimated maximum value and the mean of the individuals of the generation (Fig. 12).

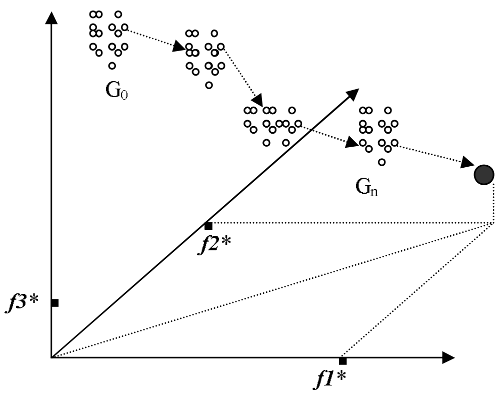

Fig. 12. Fuzzy dynamic control of search directions.

5. Computational Results

As an illustration, we consider the example of the network treated in Section 3.1. After the perturbation, the perturbed time table is shown in Table 5. The perturbation was detected at 10h:23 and caused a delay of 3 minutes in the arrival of the vehicle V_2^1 at the station S_2^1. In this work, decisions which will be applied to vehicles are both delay and advance decisions with the respect of constraints explained in Section 2.3.

The application of one of the solutions given by our evolutionary algorithm is illustrated by the curves of Fig. 13. The bold line represents the disturbed vehicles. The dashed lines are the theoretical schedules, whereas the thin ones represent new schedules resulting from the evolutionary rescheduling algorithm. The regulated timetable is then illustrated in Table 6.

We note that this regulation acts on all vehicles by delaying or advancing in order to optimize the criteria given above and so as to assure that arrival times of vehicles are more regular. Finally, the application of such a regulation scenario supports regulators of traffic in their decisions by giving them a list of feasible solutions which can be applied in order to maximize the level of service.

6. Conclusion

In this paper, we deal with an important transportation problem in two possible versions. In the first case, we

Table 5. Timetables after perturbation.

	V_1^1	V_2^1	V_3^1
S_1^1	10h:00	10h:20	10h:40
$N = S_2^1$	10h:05	10h:28	10h:45
S_3^1	10h:10	10h:33	10h:50
S_4^1	10h:15	10h:38	10h:55
S_5^1	10h:20	10h:33	11h:00

	V_1^2	V_2^2	V_3^2
S_1^2	09h:56	10h:16	10h:36
S_2^2	10h:01	10h:21	10h:41
$N = S_3^2$	10h:06	10h:26	10h:46
S_4^2	10h:11	10h:31	10h:51
S_5^2	10h:16	10h:36	10h:56

	V_1^3	V_2^3	V_3^3
S_1^3	09h:52	10h:12	10h:32
S_2^3	09h:57	10h:17	10h:37
S_3^3	10h:02	10h:22	10h:42
$N = S_4^3$	10h:07	10h:27	10h:47
S_5^3	10h:12	10h:32	10h:52

prove that the unconstrained capacity problem formulation is a classical one and can be solved using many solvers based on analytical methods. In the second case, the integrity and capacity constraints induce an important additional difficulty. The problem becomes a nonlinear one. Therefore, we propose a genetic approach based on a new coding. Such a coding allows us to extend the exploration possibilities and to improve the solution quality thanks to some adapted operators. As perspectives of this research work, the comparison with other methods seems an interesting subject which can offer scientific benefis.

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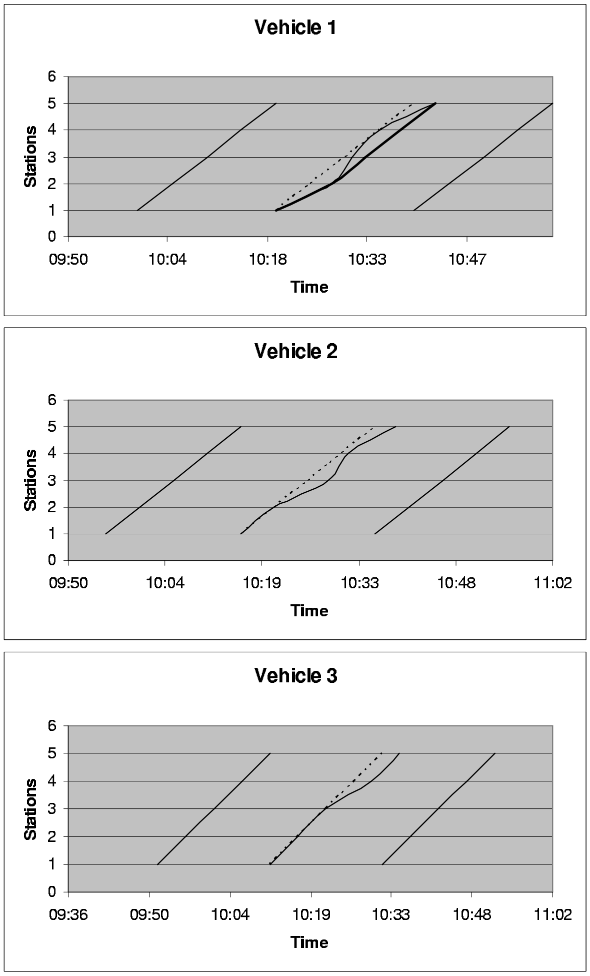

Fig. 13. Vehicle rescheduling after regulation.

Table 6. Timetables after regulation.

	V_1^1	V_2^1	V_3^1
S_1^1	10h:00	10h:20	10h:40
$N = S_2^1$	10h:05	10h:28	10h:45
S_3^1	10h:10	10h:31	10h:50
S_4^1	10h:15	10h:35	10h:55
S_5^1	10h:20	10h:43	11h:00

	V_1^2	V_2^2	V_3^2
S_1^2	09h:56	10h:16	10h:36
S_2^2	10h:01	10h:21	10h:41
$N = S_3^2$	10h:06	10h:29	10h:46
S_4^2	10h:11	10h:32	10h:51
S_5^2	10h:16	10h:39	10h:56

	V_1^3	V_2^3	V_3^3
S_1^3	09h:52	10h:12	10h:32
S_2^3	09h:57	10h:17	10h:37
S_3^3	10h:02	10h:22	10h:42
$N = S_4^3$	10h:07	10h:30	10h:47
S_5^3	10h:12	10h:35	10h:52

Table 7. Experimental results.

N	Nb_sta	Nb_veh	del	dm	Criterion 1		Criterion 2		Criterion 3	
					f_1	f_1^*	f_2	f_2^*	f_3	f_3^*
2	6	3	3	4	29.9	30.99	7.32	8.69	383.0	381.1
2	6	3	4	3	78	79.9	12.6	13.2	736.0	732
3	8	5	3	4	360.3	362.9	110.1	111.3	710.0	708.2
3	10	4	3	4	1110.2	1113.0	81.0	82.49	1097.0	1095.2
3	10	4	5	3	1438.5	1440	221.9	223.49	2250	2248.0
4	7	4	3	4	138.1	139.33	61.23	64.5	539.0	536.5
4	8	3	3	4	346.3	347.5	34.1	35.3	780.0	771.2
4	7	4	5	4	238.65	245.83	65.3	70.7	1150	1123

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