THE PROBLEMS OF COLLISION AVOIDANCE AT SEA IN THE FORMULATION OF COMPLEX MOTION PRINCIPLES

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The paper presents a mathematical model of a collision situation for objects afloat based on the rules of a multiple complex motion. It also contains an analysis of the presented model and draws some conclusions from it. The method used to determine the minimum-time control of ships in a situation of colliding with other objects afloat is presented for a mathematical model of a collision situation. It also includes the results of a simulation study conducted by means of this method. A parallel approach of a ship to an encountered object was studied, i.e., a situation generating a critical case which is the collision of two ships.

Keywords: collision situation, modelling, anti-collision systems, time-optimal control

1. Introduction

The methods used in maritime navigation are connected to the position of an observation line and, in particular, to a change in the position of the point and the direction of observation. These methods share one common feature: they are classical examples of a complex motion in general mechanics (Suslov, 1960). The float motion is the motion of the observation line and the relative movement is the movement of the encountered object afloat.

In many cases, the treatment of the control of objects afloat in a collision situation as a constrained complex movement with generalized constraints (Dubiel, 1973; 1993) not only simplifies the analysis of the dynamics of the collision situation and the synthesis of a controller but, first of all, makes the method very clear. This approach allows for an ideal separation of the controlled movement from the movement in the deviation space which represents a transient control process (Dubiel, 1995a; 1995b; 1995c).

The equations of an ideally controlled motion describe the required position of an object afloat in a collision situation. In order to determine the best movement conditions, these equations are optimized. The optimal control program obtained in this way becomes the basis for selecting the right method to avoid the collision situation, and the rule how to use it. It is equivalent to establishing the motion of a floating system in which one axis is the observation line (Dubiel, 1997; 1999).

In the literature dealing with the methods of guidance and self-guidance, the equations which describe the position of the observation line are commonly referred to as kinematic equations of motion. In fact, these equations are examples of constraints derived from mutually varying positions of two points: the observation pole, which is moving in a general case, and the point which determines the position of the moving object afloat.

2. Model of a Collision

The current situation of two objects afloat which are going to collide, i.e., a ship at speed V_0 and course ψ_0, and an encountered object at speed V_j and course ψ_j is presented in Fig. 1 (Żak, 2001; 2002b; 2002c).

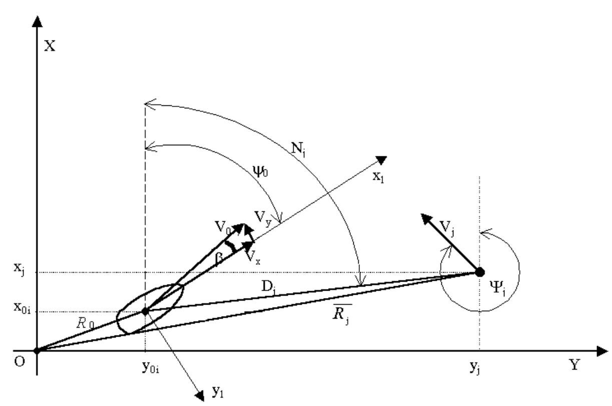

Fig. 1. Movement of the analysed objects as a complex motion.

The equations of the ship motion can be formulated as follows:

$$\begin{aligned}
\frac{\mathrm{d}V_0}{\mathrm{d}t} &= \frac{\cos\beta}{\cos 2\beta}\frac{1}{m_x}F_x - \frac{\sin\beta}{\cos 2\beta}\frac{1}{m_y}F_y \\
&\quad + \frac{\cos\beta}{\cos 2\beta}V_0\omega_z\left(\frac{m_x}{m_y}\cos\beta - \frac{m_y}{m_x}\sin\beta\right), \\
\frac{\mathrm{d}\psi_0}{\mathrm{d}t} &= \omega_z, \\
\frac{\mathrm{d}\beta}{\mathrm{d}t} &= \frac{\sin\beta}{\cos 2\beta}\frac{1}{m_x}\frac{1}{V_0}F_x - \frac{\cos\beta}{\cos 2\beta}\frac{1}{m_y}\frac{1}{V_0}F_y \\
&\quad + \omega_z\left(\frac{m_y}{m_x}\frac{\sin^2\beta}{\cos 2\beta} - \frac{m_x}{m_y}\frac{\cos^2\beta}{\cos 2\beta}\right), \\
\frac{\mathrm{d}\omega_z}{\mathrm{d}t} &= \frac{1}{I_{zz}}M_z - (m_x - m_y)\sin\beta\cos\beta,
\end{aligned} \tag{1}$$

where

- $F_x = f_1(V_0, \psi_0, \omega_x, \beta, \alpha, \dot\alpha, n)$ – external forces which have an effect on the x axis of the ship,
- $F_y = f_2(V_0, \psi_0, \omega_x, \beta, \alpha, \dot\alpha, n)$ – external forces which have an effect on the y axis of the ship,
- $M_z = f_3(V_0, \psi_0, \omega_x, \beta, \alpha, \dot\alpha, n)$ – moment with respect to the z axis,
- m_x, m_y – mass of the ship together with the mass of the accompanying water with respect to the x and y axes, respectively,
- V_0, ψ_0 – speed and course of the ship, respectively,
- ω_x, β – angular speed of the turn and angle of ship drift, respectively,
- n – rotational speed of the driving motor,
- $\alpha, \dot\alpha$ – angle and angular speed of the rudder fin deflection, respectively.

The equations of two-sided constraints can be written as follows:

$$\begin{aligned}
\frac{\mathrm{d}D_j}{\mathrm{d}t} &= -V_0\cos(N_j - \psi_0 + \beta) + V_j\cos(N_j - \psi_j), \\
\frac{\mathrm{d}N_j}{\mathrm{d}t} &= \frac{V_j[\sin\psi_j + \cos(N_j - \psi_j)\sin N_j]}{D_j\cos N_j} \\
&\quad - \frac{V_0[\sin(\psi_0 - \beta) + \cos(N_j - \psi_0 + \beta)\sin N_j]}{D_j\cos N_j},
\end{aligned} \tag{2}$$

where

- D_j – distance from the j-th object afloat,
- N_j – bearing on the j-th object afloat,
- $\psi_j,\ V_j$ – course and speed of the j-th encountered object, respectively.

The equations of one-sided constraints result from technical limitations:

$$\begin{aligned}
\alpha^2 - \alpha^2_{\max} &\le 0, \\
0 \le \dot\alpha &\le \dot\alpha_{\max},
\end{aligned} \tag{3}$$

and the limitations imposed on the control:

$$\begin{aligned}
0 \le \overline{h} &\le 1, \\
u^2_\alpha - 1 &\le 0.
\end{aligned} \tag{4}$$

The coupling relations are

$$\begin{aligned}
\frac{\mathrm{d}n}{\mathrm{d}t} &= \frac{K_h}{2\pi I_\omega}\overline{h} - \frac{M_s}{2\pi I_\omega}, \\
\frac{\mathrm{d}\alpha}{\mathrm{d}t} &= \dot\alpha, \\
\frac{\mathrm{d}\dot\alpha}{\mathrm{d}t} &= -\frac{1}{T_{MS}}\dot\alpha + \frac{k_{MS}}{T_{MS}}u_\alpha,
\end{aligned} \tag{5}$$

where

- n – rotational speed of the driving motor,
- K_h – coefficient connecting the setting of the fuel slat with the effective rotational moment of the motor reduced to the line of the drive shaft,
- I_ω – inertial moment of the driving unit,
- M_s – rotational moment taken by the propeller,
- $k_{MS},\ T_{MS}$ – gain coefficient and time constant of the control machine, respectively,
- $\alpha,\ \dot\alpha$ – angle and angular speed of the rudder fin deflection, respectively,
- $\overline{h},\ u_\alpha$ – control signals sent to the fuel slat of the main driving engine and the steering unit, respectively.

The motion equations for the j-th encountered object can be formulated as follows:

$$\begin{aligned}
\frac{\mathrm{d}V_j}{\mathrm{d}t} &= \frac{\cos\beta_j}{\cos 2\beta_j}\left(\frac{1}{m_{jx}}F_{jx} + \frac{m_{jx}}{m_{jy}}V_j\omega_{jz}\cos\beta_j\right) \\
&\quad - \frac{\sin\beta_j}{\cos 2\beta_j}\left(\frac{1}{m_{jy}}F_{jy} + \frac{m_{jy}}{m_{jx}}V_j\omega_{jz}\cos\beta_j\right), \\
\frac{\mathrm{d}\psi_j}{\mathrm{d}t} &= \omega_{j\,z}, \\
\frac{\mathrm{d}\beta_j}{\mathrm{d}t} &= \frac{\sin\beta_j}{\cos 2\beta_j}\frac{1}{m_{jx}}\frac{1}{V_j}F_{jx} - \frac{\cos\beta_j}{\cos 2\beta_j}\frac{1}{m_{jy}}\frac{1}{V_j}F_{jy} \\
&\quad + \omega_{jz}\left(\frac{m_{jy}}{m_{jx}}\frac{\sin^2\beta_j}{\cos 2\beta_j} - \frac{m_{jx}}{m_{jy}}\frac{\cos^2\beta_j}{\cos 2\beta_j}\right), \\
\frac{\mathrm{d}\omega_{jz}}{\mathrm{d}t} &= \frac{1}{I_{jzz}}M_{jz} - (m_{jx} - m_{jy})\sin\beta_j\cos\beta_j.
\end{aligned} \tag{6}$$

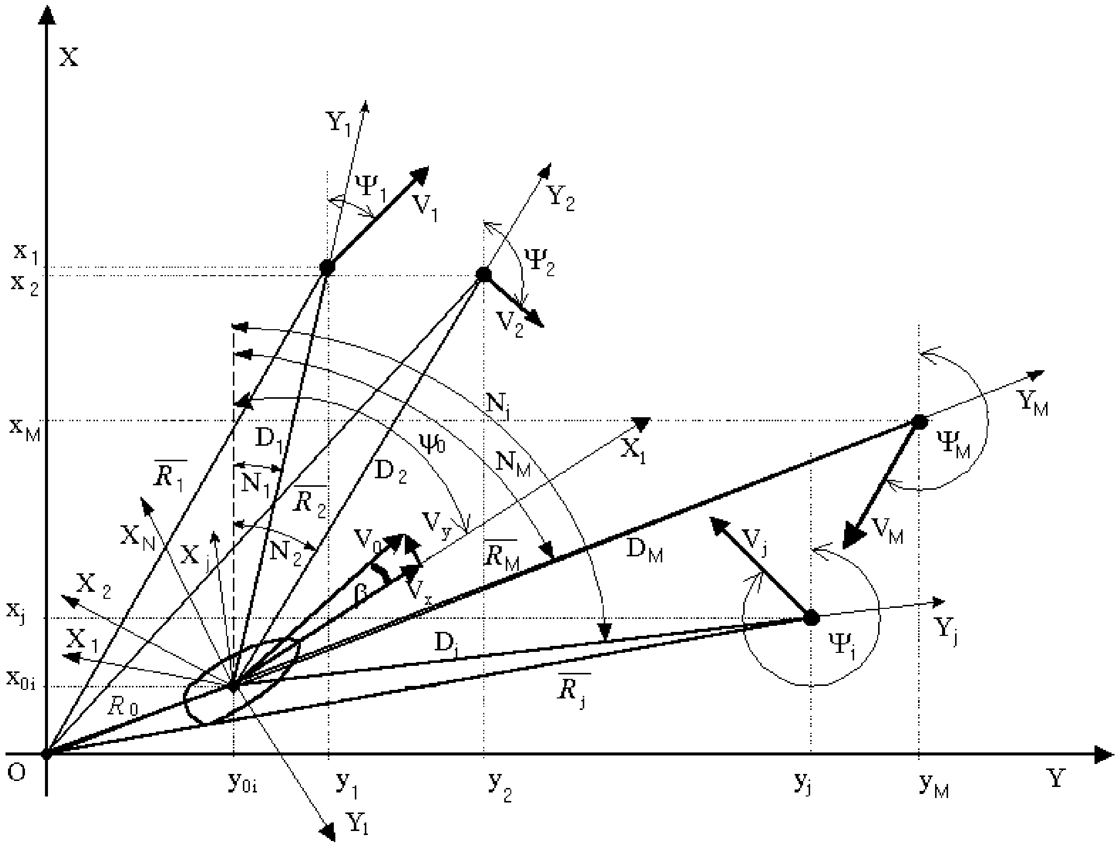

Fig. 2. Navigational situation in a collision area with j encountered objects.

The symbols are the same as in (1) and the index 'j' stands for the j-th encountered object afloat.

In a general case, the number of the encountered objects afloat in the collision area can be M $(j = 1, 2, \ldots, M)$. The current situation of the ship which moves at speed V_0 and course ψ_0 passing j objects afloat, moving at speeds $V_1, V_2, \ldots, V_j, \ldots, V_M$ and courses $\psi_1, \psi_2, \ldots, \psi_j, \ldots, \psi_M$, respectively, is presented in Fig. 2.

The equations of motion for these objects can be written as follows:

$$\begin{cases} \dot{X}_1 = f_1(X_1, U_1), \\ \dot{X}_2 = f_2(X_2, U_2), \\ \quad\vdots \\ \dot{X}_j = f_j(X_j, U_j), \\ \quad\vdots \\ \dot{X}_M = f_M(X_M, U_M). \end{cases} \tag{7}$$

Two-sided constraints for this situation can be formulated as follows:

$$\begin{cases} \dot{D}_1 = f_D(X_0, V_1, \psi_1), \\ \dot{N}_1 = f_N(X_0, D_1, N_1, V_1, \psi_1), \\ \dot{D}_2 = f_D(X_0, V_2, \psi_2), \\ \dot{N}_2 = f_N(X_0, D_2, N_2, V_2, \psi_2), \\ \quad\vdots \\ \dot{D}_j = f_D(X_0, V_j, \psi_j), \\ \dot{N}_j = f_N(X_0, D_j, N_j, V_j, \psi_j), \\ \quad\vdots \\ \dot{D}_M = f_D(X_0, V_M, \psi_M), \\ \dot{N}_M = f_N(X_0, D_M, N_M, V_M, \psi_M). \end{cases} \tag{8}$$

Equations (7) describe the motion of a moving system whose D axis is the observation axis of the encountered object. The ship has an observation system on board, and therefore its position corresponds to the beginning of the moving system O_0. Thus the ship with its moving system implements a float movement.

3. Analysis of the Collision Model

The equations of ship movement (1) with the constraints (2) and (3) and the coupling relations (4) allow the determination of the parameters of the controlled movement provided that the parameters of the encountered object are known. For further deliberations the simplest movement type of the encountered object is adopted, i.e., the rectilinear movement with constant speed. Therefore the movement of the encountered object can be described as follows:

$$\begin{aligned} V_j &= \text{const}, \\ \psi_j &= \text{const}. \end{aligned} \tag{9}$$

In further deliberations we will analyze the constraints (2) with regard to safety at see, which will allow us to classify the encountered objects into safe objects, threat objects and dangerous objects. In a situation when a ship is moving past an encountered object, the following cases can be set (Żak 2001; 2002a):

$$\begin{aligned} &\text{(a)} \quad \begin{cases} \dot{D}_j > 0 \\ \dot{N}_j < 0, \text{ or } \dot{N}_j = 0, \text{ or } \dot{N}_j > 0; \end{cases} \\ &\text{(b)} \quad \begin{cases} \dot{D}_j = 0 \\ \dot{N}_j t < 0, \text{ or } \dot{N}_j = 0, \text{ or } \dot{N}_j > 0; \end{cases} \\ &\text{(c)} \quad \begin{cases} \dot{D}_j < 0 \\ \dot{N}_j < 0, \text{ or } \dot{N}_j = 0, \text{ or } \dot{N}_j > 0. \end{cases} \end{aligned} \tag{10}$$

The first case takes place when the positions of the encountered objects afloat with respect to the ship are outside the circle of radius D_j after time Δt (Fig. 3). Thus they are moving-away objects. In the second case, the distance between the ship and the encountered objects is constant. Thus they will lie on the circle of radius D_j (Fig. 3). The objects which meet these conditions are safe according to the safe navigation rules. The third case takes place when the encountered objects are close to the ship. Their positions are inside the circle of radius D_j (Fig. 3). This case is most interesting since the encountered objects are dangerous and for this reason it will be analyzed in detail.

For $\dot{D}_j < 0$ and $\dot{N}_j > 0$ as well as for $\dot{D}_j < 0$ and $\dot{N}_j < 0$, there are two cases:

Case 1. The positions of the encountered objects with respect the ship are inside the circle of radius D_j and outside the circle of radius D_b. We then have

$$D_j + \int_0^{T^j_{D\min}} \dot{D}_j \mathrm{d}t \geq D_b,$$

where $T^j_{D\min}$ is the time required to achieve the smallest distance $D^j_{\min}$. The ship and the encountered object will pass each other at a distance no less than the safe distanceD_b under the given hydrodynamic and navigational conditions, so the collision situation does not exist.

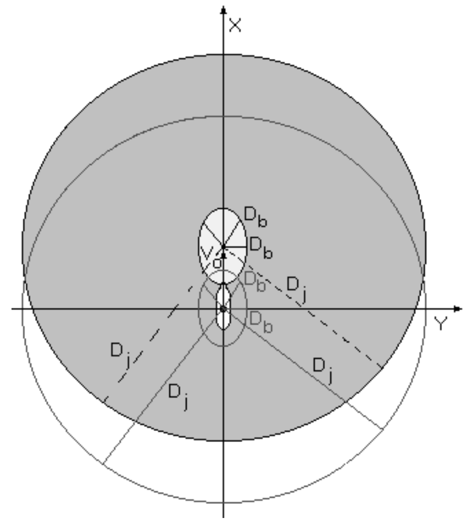

Fig. 3. Area of mutual positions of objects afloat.

Case 2. The positions of the objects afloat with respect to the ship are inside the circle of radius D_b. Then we have

$$D_j + \int_0^{T^j_{D\min}} \dot{D}_j \mathrm{d}t < D_b.$$

The ship will pass the encountered object at a distance smaller than the safe distance D_b, so they will be in a collision situation being too close to each other. Such an object will be dangerous. This situation does not have to yield a direct collision, yet, due to the assumed criteria, it is a dangerous situation and therefore a decision concerning appropriate control measures has to be made in order to avoid it.

For $\dot{D}_j < 0$ and $\dot{N}_j = 0$ the situation is dangerous and leads to a direct collision of the ship with the encountered object. This situation corresponds to the proportional approach of two moving objects and for this reason, a decision of taking appropriate control measures has to be made.

Analyzing possible situations and taking into account the ILRM rules, we come to the following conclusions:

- if the encountered object is on the board side, $\dot{D}_j < 0$ and $\dot{N}_j > 0$, then it is necessary to make a manoeuvre of passing the encountered object ahead of its bow;
- if the encountered object is on the board side, $\dot{D}_j < 0$ and $\dot{N}_j < 0$, then it is necessary to make a manoeuvre of passing the encountered object behind its stern;

- if the encountered object is on the port side, $\dot{D}_j < 0$ and $\dot{N}_j > 0$, then it is necessary to make a manoeuvre of passing the encountered object behind its stern;
- if the encountered object is on the port side, $\dot{D}_j < 0$ and $\dot{N}_j < 0$, then it is necessary to make a manoeuvre of passing the encountered object ahead of its bow;
- if $\dot{D}_j < 0$ and $\dot{N}_j = 0$, it is necessary to make a manoeuvre of passing the encountered object behind its stern.

From the above analysis, it follows that when the shortest distance of approaching the j-th object afloat $D^j_{\min}$ is less than D_b, it is necessary to make a decision concerning further control such that $D^j_{\min} \geq D_b$. This constitutes the basic criterion of ship control in a collision situation, which ensures a safe passage of objects. Additionally, we take account of the optimal criterion in the form of the minimal time loss on the anti-collision manoeuvre, which leads to time-optimal control. Adopting the starting time of the manoeuver as $t_0 = 0$ and $t_k = T^j_{D\min}$, the quality criterion can be expressed as follows:

$$I = \int_0^{T^j_{D\min}} \mathrm{d}t. \tag{11}$$

The problem formulated above can be reduced to searching for time-optimum control that will not to lead to a collision with encountered objects. To solve this problem, it is necessary to take into account the safe distance D_b (Fig. 4) in the kinematic model. Therefore one of the constraints will be changed. From Fig. 4 it follows that when the ship is passing an object encountered ahead of its bow at a safe distance, the relation connecting the bearing with the safe distance can be formulated as follows:

$$N^1_j = N_j - \arcsin\frac{D_b}{D_j}, \tag{12}$$

whereas, in the case of passing behind its stern, this relationship can be expressed as

$$N^2_j = N_j + \arcsin\frac{D_b}{D_j}. \tag{13}$$

The above relations will be referred to as advanced bearings. Differentiating these functions with respect to time, we obtain

$$\dot{N}^1_j = \dot{N}_j + \frac{D_b \dot{D}_j}{D_j\sqrt{D^2_j - D^2_b}}, \tag{14}$$

$$\dot{N}^2_j = \dot{N}_j - \frac{D_b \dot{D}_j}{D_j\sqrt{D^2_j - D^2_b}}. \tag{15}$$

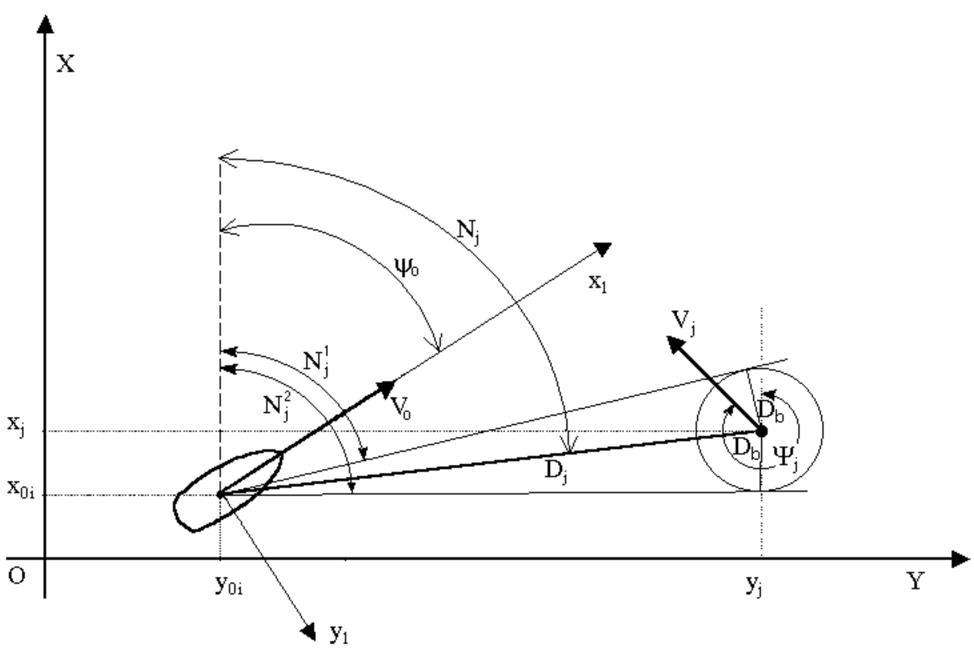

Fig. 4. Kinematic relations in a complex motion which accounts for a safe passing distance.

Substituting (2) into (14) and (15) and making appropriate transformations, the following equations are obtained (Żak 2002b; 2003):

$$\dot{N}_j^1 = \left(\frac{1}{D_j}\tan N_j + \frac{D_j D_b\sqrt{D_j}}{D_j^2 + D_b^2}\right) \times \left[V_j \cos\left(N_j - \psi_j\right) - V_0 \cos\left(N_j - \psi_0\right)\right] + \frac{V_j \sin\psi_j - V_0 \sin\psi_0}{D_j \cos N_j}, \tag{16}$$

$$\dot{N}_j^2 = \left(\frac{1}{D_j}\tan N_j - \frac{D_j D_b\sqrt{D_j}}{D_j^2 + D_b^2}\right) \times \left[V_j \cos\left(N_j - \psi_j\right) - V_0 \cos\left(N_j - \psi_0\right)\right] + \frac{V_j \sin\psi_j - V_0 \sin\psi_0}{D_j \cos N_j}. \tag{17}$$

The task is to find the minimum of the functional (1) subject to the constraints derived from (1)–(5), (16) or (17) (Kitowski and Żak, 2002).

4. Optimization of the Ship Trajectory in the Situation of a Collision

For further deliberations, it is assumed that the angle of drift is $\beta = 0$ and the ship is not affected by any disturbances. The equations of the ship movement related to velocities are

$$\begin{aligned} \frac{\mathrm{d}V_0}{\mathrm{d}t} &= a_2 n V_0 + a_3 V_0^2 + a_4 n^2, \\ \frac{\mathrm{d}\psi_0}{\mathrm{d}t} &= \omega_z, \\ \frac{\mathrm{d}\omega_z}{\mathrm{d}t} &= c_5 V_0^2 \omega_z + c_6 V_0^2 \alpha + c_8 V_0 \omega_z, \end{aligned} \tag{18}$$

where

$a_2,\ a_3,\ a_4,\ c_5,\ c_6,\ c_8$ – coefficients depending on ship dimensions,

$V_0,\ \psi_0,\ \omega_x,\ n,\ \alpha$ – speed and course of the ship, angular speed of turn, rotational speed of the propeller, and angle of rudder blade deflection, respectively.

In the speed frame, the equations of the motion of encountered objects are described by means of differential equations of the form (18).

In a general case, the ship is in a colliding situation with M encountered objects whose current positions with respect to the ship are known, i.e., bearing N_j, distance D_j and current parameters of movement speed V_j and course ψ_j.

An attempt is made to find control for which the minimum distance $D^j_{\min}$ of approaching the j-th encountered object is greater than the safe distance D_b resulting from the geometric dimensions of the objects which are in the collision situation, and from the dynamics of the navigational situation, i.e., the control for which the condition $D_b \le D^j_{\min}$ is satisfied.

In the process of searching for the control, the optimization criterion is taken into account in the form of the smallest loss of time, which leads to time-optimal control. Assuming the initial time for the manoeuvre as $t_0 = 0$ and $t_k = T^j_{D\,\min}$, the quality criterion can be formulated as (11).

The task is to find the minimum of functional (11) subject to the constraints

$$\begin{aligned}
\varphi_1 &= \dot{V}_0 - a_2 n V_0 - a_3 V_0^2 - a_4 n^2 = 0, \\
\varphi_2 &= \dot{\psi}_0 - \omega_z = 0, \\
\varphi_3 &= \dot{\omega}_z - c_5 V_0^2 \omega_z - c_6 V_0^2 \alpha - c_8 V_0 \omega_z = 0, \\
\varphi_4 &= \dot{D}_j + V_0 \cos\left(N_j - \psi_0\right) - V_j \cos\left(N_j - \psi_j\right) = 0, \\
\varphi_5 &= \dot{N}_j^1 - \left(\frac{1}{D_j}\tan N_j + \frac{D_j D_b\sqrt{D_j}}{D_j^2 + D_b^2}\right) \times \left[V_j \cos\left(N_j - \psi_j\right) - V_0 \cos\left(N_j - \psi_0\right)\right] - \frac{V_j \sin\psi_j - V_0 \sin\psi_0}{D_j \cos N_j} = 0,
\end{aligned}$$

or (19)

$$\begin{aligned}
\varphi_5 &= \dot{N}_j^2 - \left(\frac{1}{D_j}\tan N_j - \frac{D_j D_b\sqrt{D_j}}{D_j^2 + D_b^2}\right) \times \left[V_j \cos\left(N_j - \psi_j\right) - V_0 \cos\left(N_j - \psi_0\right)\right] - \frac{V_j \sin\psi_j - V_0 \sin\psi_0}{D_j \cos N_j} = 0, \\
\varphi_6 &= \dot{n} - \frac{\eta_w \eta_r N_e}{2\pi I_\omega n i_r}\overline{h} + \frac{K_M \rho n^2 D^5}{2\pi I_\omega} = 0, \\
\varphi_7 &= \ddot{\alpha} + \frac{1}{T_{MS}}\dot{\alpha} - \frac{k_{MS}}{T_{MS}} u_\alpha = 0, \\
\varphi_8 &= \dot{\alpha}\left(\dot{\alpha}_{\max} - \dot{\alpha}\right) + \zeta_1^2 = 0, \\
\varphi_9 &= \alpha\left(\alpha + \alpha_{\max}\right) + \zeta_2^2 = 0, \\
\varphi_{10} &= \alpha\left(\alpha - \alpha_{\max}\right) + \zeta_3^2 = 0, \\
\varphi_{11} &= \overline{h}\left(1 - \overline{h}\right) - \zeta_4^2 = 0, \\
\varphi_{12} &= u_\alpha\left(u_\alpha + 1\right) + \zeta_5^2 = 0, \\
\varphi_{13} &= u_\alpha\left(u_\alpha - 1\right) + \zeta_6^2 = 0.
\end{aligned}$$

The function minimizing the functional (11) is sought with respect to variables $V_0,\ \psi_0,\ \omega_z,\ n,\ D_j,\ \zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6,\ N_j^1$ or N_j^2. To this end, the function $F\left(y,\dot{y},t\right)$ is assumed in the form

$$F\left(\cdot\right)=1+\sum_{i=0}^{n}\lambda_i\varphi_i, \tag{20}$$

for which the Euler-Lagrange equations are determined from the relation

$$\frac{\partial F}{\partial y_i}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial F}{\partial \dot{y}_i}=0. \tag{21}$$

The movement control of the ship is executed by means of changing its speed or course. For further deliberations, it will be assumed that the collision avoidance manoeuvre will be executed by changing the ship course. In such a case, it is assumed that V_D and n are constant and the limitations imposed by the change in the position of the fuel slat $\bar{h}$ are not taken into account. Thus the Euler-Lagrange equations are obtained for individual variables as follows:

1. For variable ψ_0:

$$\begin{aligned}&-\lambda_4V_0\sin\left(N_j-\psi_0\right)\\&-\lambda_5V_0\left(\frac{1}{D_j}\tan N_j\pm\frac{D_jD_b\sqrt{D_j}}{D_j^2+D_b^2}\right)\sin\left(N_j-\psi_0\right)\\&\qquad+\frac{\lambda_5V_0\cos\psi_0}{D_j\cos N_j}-\dot{\lambda}_2=0.\end{aligned}$$

2. For variable ω_z: $\quad -\lambda_2-\lambda_3c_5V_0^2-\lambda_3c_8V_0-\dot{\lambda}_3=0.$

3. For variable D_j:

$$\begin{aligned}&-\lambda_5\left(\frac{-1}{D_j^2}\tan N_j\pm\frac{1.5D_b\sqrt{D_j}-0.5D_jD_b\sqrt{D_j}}{\left(D_j^2+D_b^2\right)^2}\right)\\&\times\left[V_j\cos\left(N_j-\psi_j\right)-V_0\cos\left(N_j-\psi_0\right)\right]\\&\qquad-\lambda_5\frac{V_j\sin\psi_j-V_0\sin\psi_0}{D_j^2\cos N_j}-\dot{\lambda}_4=0.\end{aligned}$$

4. For variables N_j^1 or N_j^2: $\quad -\dot{\lambda}_5=0.$

5. For variable $\dot{\alpha}$: $\quad \dfrac{\lambda_7}{T_{MS}}+\lambda_8\dot{\alpha}_{\max}-2\lambda_8\dot{\alpha}-\dot{\lambda}_7=0.$

6. For variable α:

$$\begin{aligned}&-\lambda_3c_6V_0^2+\lambda_9\left(2\alpha+\alpha_{\max}\right)+\lambda_{10}\left(2\alpha-\alpha_{\max}\right)\\&\qquad-\frac{\dot{\lambda}_7}{T_{MS}}+\dot{\lambda}_8\alpha_{\max}-2\dot{\alpha}\dot{\lambda}_8=0.\end{aligned}$$

7. For variable ζ_1: $\quad \lambda_8\zeta_1=0.$

8. For variable ζ_2: $\quad \lambda_9\zeta_2=0.$

9. For variable ζ_3: $\quad \lambda_{10}\zeta_3=0.$

10. For variable ζ_5: $\quad \lambda_{12}\zeta_5=0.$

11. For variable ζ_6: $\quad \lambda_{13}\zeta_6=0.$

12. For variable u_α:

$$-\lambda_7\frac{k_{MS}}{T_{MS}}+2\lambda_{12}u_\alpha+\lambda_{12}+2\lambda_{13}u_\alpha-\lambda_{13}=0.$$

The best way to solve the Euler-Lagrange equations is to start from Eqns. (20) and (21), which have alternative solutions

$$\lambda_{12}=0\quad\text{or}\quad\zeta_5=0, \tag{22}$$

$$\lambda_{13}=0\quad\text{or}\quad\zeta_6=0. \tag{23}$$

The first set of solutions, in accordance with the Euler-Lagrange equations, leads to $\lambda_{12}=0$ and $\lambda_{13}=0$, which, due to the arbitrariness of λ_{12} and λ_{13}, is excluded. At the same time, this excludes the solution of $u_\alpha(t)$. But the second set of solutions yields two boundary values:

$$u_\alpha=0\quad\text{as well as}\quad u_\alpha=-1 \tag{24a}$$

and

$$u_\alpha=0\quad\text{as well as}\quad u_\alpha=1. \tag{24b}$$

Depending on the sign of the initial angular speed of the advanced bearing, this solution can be written as follows:

$$u_\alpha=\operatorname{sign}\frac{\mathrm{d}N_j^{1,2}}{\mathrm{d}t}. \tag{25}$$

Since only the sign of the angular velocity of the advanced bearing is important, it is sufficient to treat (16) or (17) as the switching function:

$$\begin{aligned}\delta(t)=&\left(\frac{1}{D_j}\tan N_j\pm\frac{D_jD_b\sqrt{D_j}}{D_j^2+D_b^2}\right)\\&\times\left[n\cos\left(N_j-\psi_j\right)-\cos\left(N_j-\psi_0\right)\right]\\&+\frac{n\sin\psi_j-\sin\psi_0}{D_j\cos N_j},\end{aligned} \tag{26}$$

where $D_j,\ N_j$ and ψ_0 can be derived from Eqs. (2), (16) and the coupling relations (5).

The sought control in the general form can be written as follows:

$$u_\alpha=\operatorname{sign}\delta\left(t\right)\left[H\left(t-t_0\right)-H\left(t-t_p\right)\right]. \tag{27}$$

The dependence (27) contains two solutions: $u_\alpha=\operatorname{sign}\delta\left(t\right)$ for $t\in\left(t_0,t_p\right)$ and $u_\alpha=0$ for $t\geq t_p$, but t_p

corresponds to $\delta(t) = 0$. The change in the direction of the ship motion requires a control impulse, which can be calculated from (5), substituting the control (26), and taking into account the technical limitations (3). As a result, for $t_p < \alpha/\dot{\alpha}_{\max}$ we obtain

$$\begin{aligned} &\alpha(t_p) - \alpha(t_0) \\ &\quad = K_{MS}(t_p - t_0) \operatorname{sign} \delta(t) \big[H(t - t_0) - H(t - t_p)\big] \\ &\qquad - T_{MS}\alpha_{\max}(t_p - t_0), \end{aligned} \tag{28}$$

and for $t_p > \alpha/\dot{\alpha}_{\max}$ we get

$$\alpha(t_p) - \alpha(t_0) = \pm\alpha_{\max}, \tag{29}$$

where $\alpha(t_p)$ is the angle of the rudder blade deflection affecting the ship movement so as to achieve the parallel approaching of the point of safe passing.

The ship's course ensuring a parallel approaching can be calculated from (16) or (17), and the second and third equations from the set of equations (18). Thus the duration time of the control impulse executing time-optimum control is calculated.

5. Simulation Study

The same ship was used for testing purposes. Its displacement was $V = 213,758$ [m^3], the length on waterline $L = 36,3$ [m], the width of the midship section $B = 7$ [m] and the draught $T = 1,742$ [m]. The ship has two main propellers and two fin rudders, which are situated in the shaft line.

A dangerous navigational situation was simulated for a relative speed $D_j < 0$ and the speed of the change in the bearing angle $N_j = 0$, i.e., for a critical case of the parallel approach. The solution to the collision situation was sought under the assumption that the encountered object did not make any manoeuvres and was moving rectilinearly at a constant speed. For this situation a time-optimal control determining the safe trajectory for the ship was determined. The control of the ship movement was carried out through changing the course during the manoeuvre of safe passing behind the stern of the encountered object at a safe distance of $D_b = 250$ [m]. For this manoeuvre we calculated the trajectories of the encountered object and the ship, the distribution of their state coordinates, the corresponding control signals and the solutions to the constraint equations. The simulation results are presented in Figs. 5–11. The control signal and force acting on the rudder are calculated for time-optimal control.

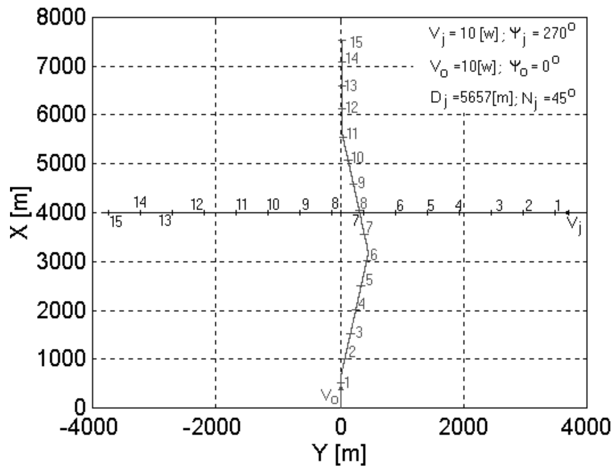

Fig. 5. Trajectories of the encountered object and the ship when passing the encountered object behind the stern.

6. Conclusions

The relative motion of an encountered object with respect to a floating system is of special importance. The equations of a relative motion are a very useful model to study the dynamics of objects afloat. Subject to the process of optimization, they produce especially valuable results to be used in the synthesis of a control system of the most useful structure. Such a structure offers the fastest disappearance of transient processes, i.e., the fastest approach of the real movement to an ideally controlled movement.

The introduction of moving coordinates is of special importance for control because it decomposes the motion of the controlled object into a float movement and a relative movement. The float movement includes the strategy of navigation and thus it allows us to define a proper program for ideal control. The relative movement permits the control because its parameters are exactly the control errors. These movements should be used as quantities to form the control signal, which in turn allows a proper execution of the control program.

Making use of multiple complex motion rules to describe the collision situation of objects afloat shows a close relationship between theoretical mechanics and a controlled movement of the object afloat. This concept constitutes a special case of a more general definition of the controlled movement of the object afloat as a constrained movement with generalized constraints. The reduction of the concept to a two-dimensional geographic plane leads to specific constraints, which are: segment equations of the D_j axis in the moving coordinates system linking the point of the observation system with the mass centre of the observed object, and the angle of the observation line N_j contained between the north and the direction towards the mass centre of the observed object.

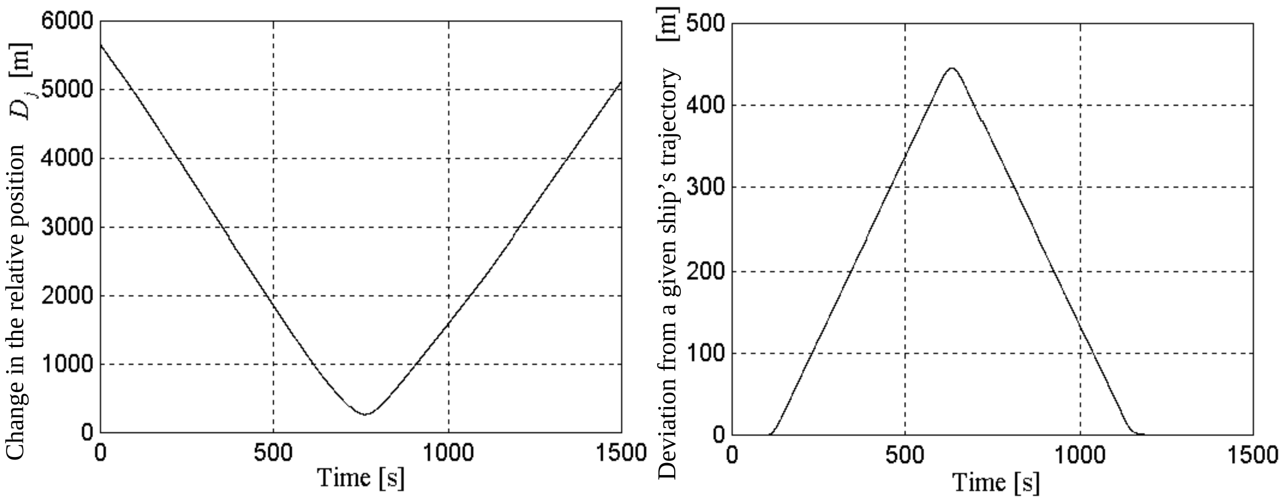

Fig. 6. Change in the relative position D_j and deviation from a preset trajectory of the ship during collision avoidance.

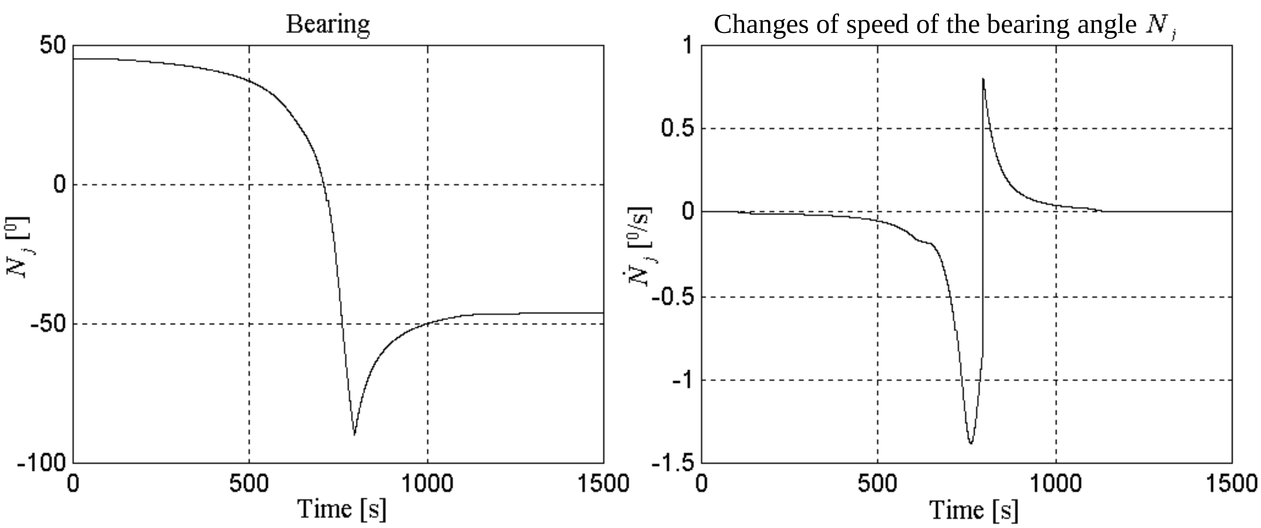

Fig. 7. Distribution of changes in the bearing and its derivative.

The performed analysis of various situations of mutual positions of objects afloat allowed a quick assessment of a collision risk based on the speed of changes in mutual object positions and the speed of changes in the bearing angle. Dangerous objects are the ones for which $\dot{D}_j < 0$ and $\dot{N}_j = 0$. In this situation it is necessary to make a collision avoidance manoeuver. The quick assessment of the collision risk and the separation of dangerous objects allows us to reduce the number of objects to be considered, and at the same time, to shorten the time needed to determine the optimum safe manoeuvre and a new trajectory for the ship. This is especially important in the case of a large number of vessels in a water region.

The programs for time-optimal control have never been used in dealing with collision avoidance problems. In these problems, the parallel approach to an encountered object generates a dangerous situation, and thus appropriate control has to be executed to avoid it. However, while formulating the problem with constraints with respect to the advanced bearing angle $N_{j1,2}$ depending on the safe distance of the approach D_b, time-optimal control programs can be used to solve collision situations at sea. The boundary process of time-optimal control, i.e., the parallel approach is possible only for angles for which the condition of the parallel approach is met as early as at time t_0. Obviously, this occurs when the angular speed of the observation line is $\dot{N}_j = 0$. In the case of employing the presented method to avoid a collision at sea and to calculate time-optimal control, there is always a possibility of executing this control, since for a danger-posing object the angular speed of the observation line is $\dot{N}_j = 0$, which means that a parallel approach occurs. Then the

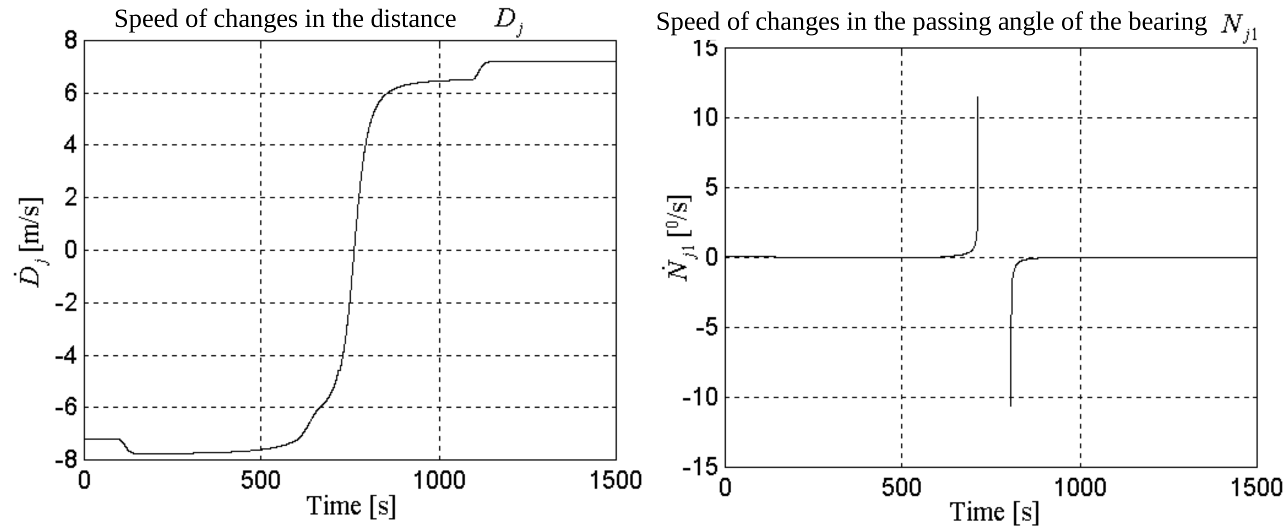

Fig. 8. Distribution of derivatives of the distance and passing angles of the bearing.

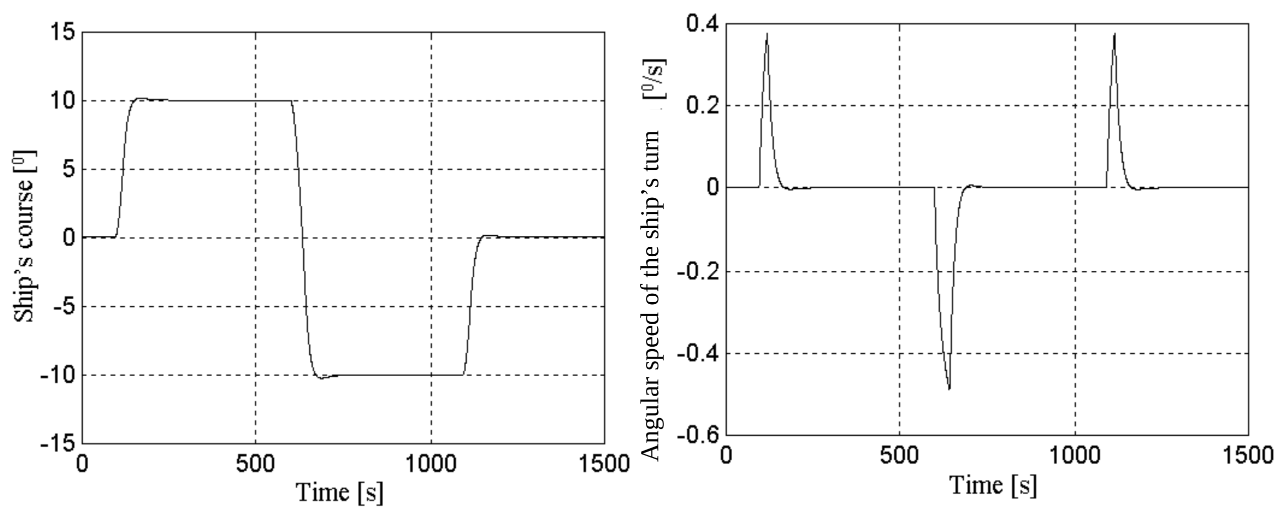

Fig. 9. Course and angular speed of the ship turn during collision avoidance.

task for controls is to obtain $\dot{N}_j \neq 0$ and to reduce to zero the speed of the advanced observation line $N_{j1,2}$ at the moment of switching the control, i.e., for $t = t_p$. At this moment both switching functions are cleared. Further control is executed with the use of the control $u_1 = 0$ and $u_2 = 0$. It implements the so-called parallel approaching of the advanced point, whose position is determined by the safe distance of passing D_b determined for given hydrodynamics and navigational conditions.

The distribution of time-optimal control is rectangular and constitutes ideal control, whereas the time to lead the ship to a parallel approach to advanced point depends mainly on the ship manoeuvrability, the initial value of the angular speed of the observation line and the value of the safe distance of the approach D_b.

From the simulation results it follows that the objects were approaching parallelly, and therefore they lead to critical cases where $\dot{N}_j = 0$ and $\dot{D}_j < 0$. After the collision avoidance manoeuvre, in accordance with the calculated time-optimal control, the angular speed of the approach becomes different from zero ($\dot{N}_j \neq 0$), whereas the speed of the advanced observation line becomes zero ($\dot{N}_{j1,2} = 0$). Thus the case occurs where the ship approaches parallelly the advance point determined by D_b. The trajectory which ensures passing the encountered object at a safe distance is achieved by the ship within a short time period because the executed manoeuvres are weak, i.e., the change in the course is performed within a small range.

At the moment when the encountered object has been passed, we have a change in the speed sign of the ad-

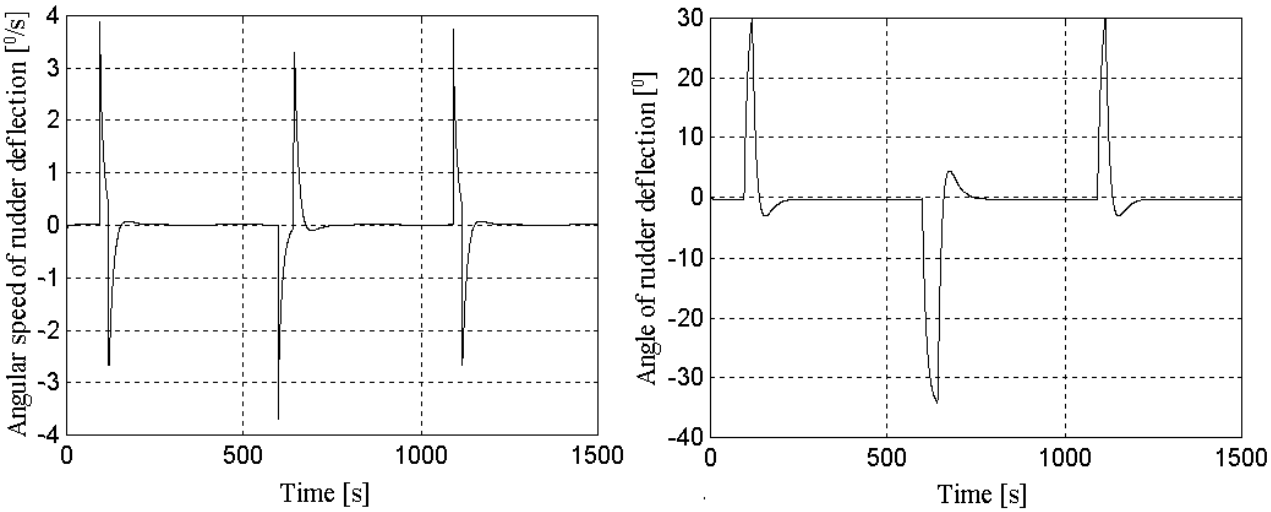

Fig. 10. Angular speed and angle of rudder blade deflection.

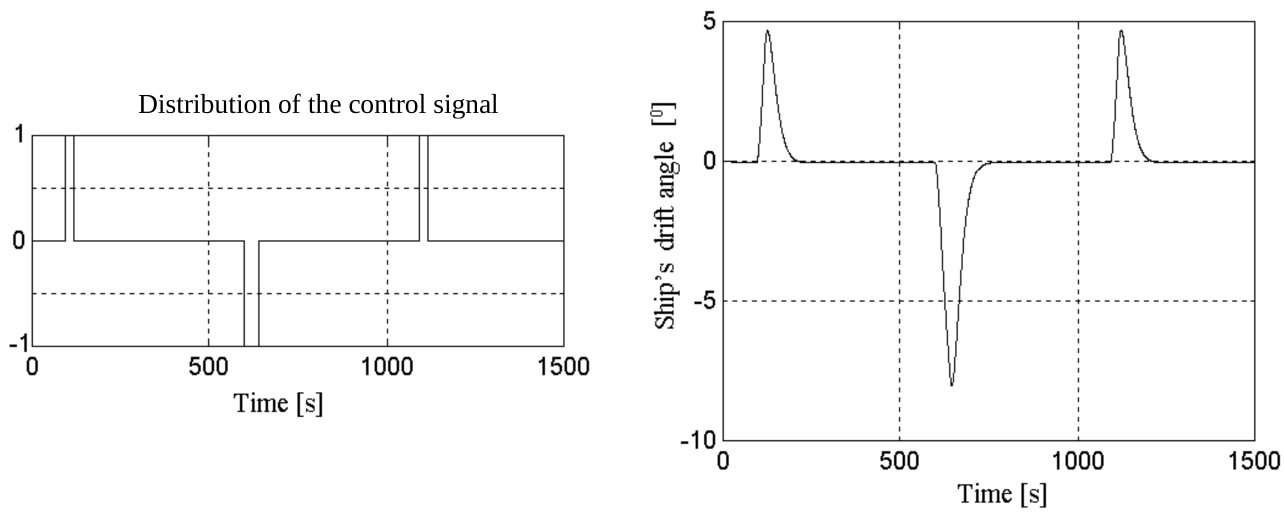

Fig. 11. Distribution of the control signal sent to the steering unit and the angle of the ship drift.

vanced observation line $N_{j1,2}$ and in the speed of approaching D_j. Then the encountered object becomes a safe object. At this time the control signal is sent, bringing the ship onto the preset trajectory. This signal has a value opposite to the signal calculated for the collision avoidance manoeuvre and its duration time is twice as long. Such a signal causes the ship to turn and approach the preset trajectory along which it had been moving before the collision situation occurred.

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