Int. J. Appl. Math. Comput. Sci., 2009, Vol. 19, No. 1, 59–68
DOI: 10.2478/v10006-009-0005-7

ROBUST PARAMETER DESIGN USING THE WEIGHTED METRIC METHOD—THE CASE OF 'THE SMALLER THE BETTER'

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In process robustness studies, it is desirable to minimize the influence of noise factors on the system and simultaneously determine the levels of controllable factors optimizing the overall response or outcome. In the cases when a random effects model is applicable and a fixed effects model is assumed instead, an increase in the variance of the coefficient vector should be expected. In this paper, the impacts of this assumption on the results of the experiment in the context of robust parameter design are investigated. Furthermore, two criteria are considered to determine the optimum settings for the control factors. In order to better understand the proposed method and to evaluate its performances, a numerical example for the case of 'the smaller the better' is included.

Keywords: multiobjective decision making, regression estimation, response surface methodology, robust parameter design, weighted metric method.

1. Introduction

Response surface methodology (RSM) has become an important tool in process and product development. RSM consists of mathematical and statistical optimization techniques that are used to improve existing processes or develop new ones. In both of these situations, RSM can be used to obtain optimal conditions resulting in a better overall product or service. Applications of RSM can be found in many industrial settings where several variables influence the desired outcome. Moreover, there are many excellent discussions and practical examples illustrating RSM and its applications in the literature, and many authors have contributed to this area including Vining and Myers (1990), Lucas (1994), Pledger (1996), Khattree (1996), Myers *et al.* (1992, 1997, 2004), Montgomery (1999).

Taguchi (1987) introduced the idea of Robust Parameter Design (RPD). He argued that not only the controllable factors of interest in a process, $\mathbf{x}$, should be considered, but also those uncontrollable or noise factors, $\mathbf{z}$, that often cause variation in the response. It is desirable to find settings of the controllable factors such that the process or product is robust or insensitive to variability transmitted from these variables. His approach consists of placing the controllable factors in one design called the *inner array* and the noise factors in a second design called the *outer array*, and then running the set of experiments given by the Cartesian product of these two designs. This strategy produces a *crossed array* design. Then an analysis is performed and the signal-to-noise ratios are computed across the outer array observations. References on this issue include Box *et al.* (1988), Pignatiello and Ramberg (1991), Nair (1992), Box and Jones (1992), and Kunert *et al.* (2007).

In general, RSM is well suited to the RPD problem and process robustness studies. Myers *et al.* (2004) believe that the solution of the robust parameter design problem in the RSM framework would be one of the most important areas of research. In RPD problems, authors usually assume that the levels of the noise variables are fixed in the experiment and random in the process. Noise variables in experimentation could and, in many cases, should be considered random and not fixed as is commonly used. Myers and Montgomery (2002) discussed the *bias* that can occur where estimates of the response variance may exceed the actual variance due to treating the noise variables as fixed. Estimates of this variance depend on estimates of the model parameters, and in some cases they are positively biased; i.e., the mean variance estimates may exceed the actual variance. Emphasizing noise factors and their related assumptions in the estimating process of the response(s) obtained from RSM is necessary. For instance, Khuri (1996) pointed out that in many situations the noise effects in the experiment are random and should be treated as such in the modeling and analysis. Drain *et al.* (2005) used numeric and visual displays to assess these effects given the presence of noise variable correlation. Del Castillo *et al.* (2007) tried to achieve robustness in parameter design based on the appropriate estimation of the parameters.

The major aspect of this research is to investigate the effects of different estimation methods related to noise variables on final optimum solutions in RPD. In other words, we measure the impact of regression estimation affected by the noise variables on the robustness of the product. In this regard, two optimization techniques, namely, Copeland and Nelson's method and the L_p method, are applied for the purpose of a fruitful illustration.

The rest of the paper is organized as follows: A brief introduction to RPD and its different approaches are presented in Section 2. Section 3 contains the two optimization techniques: of Copeland and Nelson, and L_p. In Section 4, we provide a brief review on the ordinary least squares (OLS) and weighted least squares (WLS) methods. A numerical example comparing the performance of the optimization criteria is presented in Section 5. Finally, our concluding remarks are given in Section 6.

2. Robust parameter design

RPD is an engineering methodology intended as a cost-effective approach for improving the quality of products and processes. If we refer to a product and process as a system, there are two types of inputs that operate on a system: control factors and noise factors. Control factors are easily controlled and manipulated, whereas noise factors are difficult or costly to control and considered uncontrollable in the experiment. Common examples of noise factors are environmental qualities such as temperature, humidity, or properties of incoming materials. In a mathematical perspective, RPD is a special case of the multiple response problem, where two responses, the mean μ_y and the standard deviation σ_y of the characteristics of interest (y), are measured simultaneously for each setting of a group of design or control variables ($\mathbf{x}$s) during the experiment. After estimating these two responses as $\hat{\boldsymbol{\mu}}_y$ and $\hat{\boldsymbol{\sigma}}_y$, we should apply optimization techniques to determine the levels of the control factors that lead to an optimum solution. In other words, control variables should be chosen such that the noise variables have less effect on the process or product; and, consequently, the process or product is insensitive to noise variables. In order to find the responses related to the mean and standard deviation of the process, there are two well-known approaches in the literature that follow.

2.1. Dual-response method. The dual-response system assumes that the two responses of interest can be categorized as primary and secondary. It then solves the decision variables that produce an optimum value for one response while the other one is considered as a constraint. These responses are assumed to be second-order polynomial regressions of $\hat{\mu}_y$ and $\hat{\sigma}_y$. The sample means and variances of the responses from what is called the *outer array* are taken as the data for fitting the responses. After estimating the location and dispersion parameters, Lagrangian multipliers are used to find the optimum solution(s). Del Castillo and Montgomery (1993) solved the dual-response problem by a nonlinear programming approach. Fan and Del Castillo (1999) proposed a method to find the optimal solution for a dual-response system fitted from experimental data. Their method further considered the inherent sampling variability of the fitted response by using the Monte Carlo simulation. Vining and Myers (1990) proposed a response surface approach to solve the dual response model with the added constraint $\mathbf{x}'\mathbf{x} = \rho^2$ for restricting the search area to a spherical region of radius ρ. However, since the constraints in the optimization problem all involve equalities, we cannot often find a feasible solution. To overcome this obstacle, one can replace the equality in the constraint with an inequality and apply the method proposed by Del Castillo and Montgomery (1993), which uses the generalized reduced gradient (GRG) algorithm to optimize the problem. Fathi (1991) also proposed using nonlinear programming techniques to solve the dual response problem assuming the functional form of the response is known. Kim and Lin (1998) proposed an optimization method to optimize dual problems in a fuzzy environment. A membership function in fuzzy set theory is used to measure the decision-maker's degree of satisfaction concerning the mean and standard deviation of the responses. Kim and Cho (2002), Tang and Xu (2002), Koksoy and Doganaksoy (2003) presented various extensions for solving the dual response approach.

2.2. Single-response model. A significant departure from the dual-response approach suggested by Welch *et al.* (1990) is to simultaneously consider both the control and the noise factors in a single design called the *combined array*. Useful references on the *combined array* and its applications include Montgomery (1990), Shoemaker *et al.* (1991), Lucas (1994), Borkowski and Lucas (1997), Borror and Montgomery (2000), Romano *et al.* (2004). These designs typically require fewer runs than Taguchi's *crossed arrays* used in the dual-response model and also allow the experimenter to estimate potentially important interactions.

The single-response model is generally given in the following form:

$$y(\mathbf{x},\mathbf{z}) = \beta_0 + \mathbf{x}'\boldsymbol{\beta} + \mathbf{x}'\mathbf{B}\mathbf{x} + \mathbf{z}'\boldsymbol{\gamma} + \boldsymbol{x}'\boldsymbol{\Delta}\boldsymbol{z} + \varepsilon, \quad (1)$$

where $y(\mathbf{x},\mathbf{z})$, $\mathbf{x}\,(r_x \times 1)$, and $\mathbf{z}\,(r_z \times 1)$ denote the response, the control factors and the noise vectors, respectively. The quantity β_0 is the intercept, $\boldsymbol{\beta}$ is a vector of coefficients for the linear effects in control variables, $\mathbf{B}$ contains the coefficients for the quadratic effects in control factors and the control $\times$ control interactions, $\boldsymbol{\gamma}$ is a vector of coefficients for the linear effects in the noise variables, and $\boldsymbol{\Delta}$ contains the coefficients of the interaction effects between control and noise factors, which is critical for the success of RPD. The experimental error, i.e., the error due to the inability of the model to explain the real physical phenomenon, is defined by $\boldsymbol{\varepsilon}$. It is assumed that $\mathrm{Var}(\boldsymbol{\varepsilon}) = \boldsymbol{\sigma}_\varepsilon^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and $E(\boldsymbol{\varepsilon}) = \mathbf{0}$. The response surface model for the mean, assuming $E(\mathbf{z}) = \mathbf{0}$, is given by $\mathrm{E}\left[y(\mathbf{x},\mathbf{z})\right] = \beta_0 + \mathbf{x}'\boldsymbol{\beta} + \mathbf{x}'\mathbf{B}\mathbf{x}$. The response surface model for the process variance is given by $\mathrm{Var}\left[y(\mathbf{x},\mathbf{z})\right] = (\boldsymbol{\gamma} + \boldsymbol{\Delta}'\mathbf{x})'\,\boldsymbol{\Sigma}_z\,(\boldsymbol{\gamma} + \boldsymbol{\Delta}'\boldsymbol{x}) + \boldsymbol{\sigma}_\varepsilon^2$, where $\mathrm{Var}(\mathbf{z}) = \boldsymbol{\Sigma}_\mathrm{z}$ is usually assumed to be $\boldsymbol{\sigma} z^2\,\mathbf{I}$. In much the same may as in the dual model approach, here two responses are also considered, a location response corresponding to $\hat{\mu}_y$ and a dispersion response corresponding to $\hat{\sigma}_y$, which are necessary to formulate the standard RPD problems.

In order to determine the factor settings that lead to an optimum solution, there are various optimization techniques that are available in the multi-objective decision making (MODM) area. According to Jeong and Kim (2005), different multi-objective optimization methods are classified into three major categories by the timing of a decision maker (DM)'s preference information articulation into the model: prior, progressive, and posterior preference articulation. Prior preference articulation methods require that all the preference information of a DM be extracted prior to solving the problem. Progressive preference articulation methods require that a DM input his or her preference information into a model during the problem solving process. Posterior preference articulation methods do not need any substantial articulation of a DM's preference before or during the problem solving process. Some of these optimization techniques are discussed in the next section.

3. Some available optimization methods

In dealing with several objective functions, due to the contradiction of objective functions, it is very unlikely to find a single solution which could optimize all the objective functions simultaneously. Consequently, we are usually interested in finding Pareto optimal solutions. In a Pareto optimal solution, any criterion cannot be improved without deteriorating the value of at least one other criterion. As has been mentioned earlier, there exist many optimization methods in the MODM framework which could be considered for optimization purposes. In order to bring the effects of a wrong estimation problem to light, we need to apply optimization methods through simulation. A brief discussion of the methods used in this study is given below.

3.1. Weighted metric method ($\boldsymbol{L_p}$ method). The L_p method belongs to the first category of MODM problems, i.e., the case where a DM gives all required information before solving the problem. It is discussed in MODM references such as Hwang and Masud (1979), Asgharpour (1998) and Deb (2001), and it combines multiple objective functions into a single one. This method is considered for two main reasons. The first one is that this method requires less information from a DM, and the second one is its ease of implication in practice.

For 'the smaller the better' problems, the L_p method is defined as follows:

$$\begin{aligned}&\mathrm{Min}\, L_p\\ &= \{w(\hat{\sigma}_y - \sigma^{\min})^p + (1-w)(\hat{\mu}_y - \mu^{\min})^p\}^{1/p}, \quad (2)\end{aligned}$$

where the quantities of $\sigma^{\min}$ and $\mu^{\min}$ are the optimum values for the response functions. The quantities w and $1-w$ indicate the importance of the standard deviation and the mean response, respectively, and are determined by the DM as a value between zero and one. Here p indicates the importance of each objective function deviation from its ideal value. When $p = 1$ is used, the problem is changed to a weighted sum of deviations. When $p = 2$ is used, a weighted Euclidean distance of any point in the objective space from the ideal point is minimized. When $p = \infty$ is considered, the largest deviation should be minimized, i.e.,

$$\operatorname*{Min}_x \left(\mathrm{Max}\{w(\hat{\sigma}_y - \sigma^{\min}); (1-w)(\hat{\mu}_y - \mu^{\min})\}\right),$$

which is equivalent to

$$\begin{cases} \underset{x}{\mathrm{Min}}\, \mathrm{z} \\ \text{s.t.} \\ \mathrm{z} \geq \mathrm{w}(\hat{\sigma}_y - \sigma^{\min}), \\ \mathrm{z} \geq (1-\mathrm{w})(\hat{\mu}_{\mathrm{y}} - \mu^{\min}). \end{cases}$$

Chankong and Haimes (1983) showed that when the L_p method is used, all solutions corresponding to $1 \leq p \leq \infty$ and $w > 0$ are efficient solutions. In Eqn. (2), it is assumed that the objective functions have the same scale. Otherwise, each objective function could be made scale-less using either of the following equations:

$$L_p = \left(w(\frac{\hat{\sigma}_y - \sigma^{\min}}{\sigma^{\min}})^p + (1-w)(\frac{\hat{\mu}_y - \mu^{\min}}{\mu^{\min}})^p \right)^{1/p} \tag{3}$$

or

$$L_p = \left(w(\frac{\hat{\sigma}_y - \sigma^{\min}}{\sigma^{\max} - \sigma^{\min}})^p + (1-w)(\frac{\hat{\mu}_y - \mu^{\min}}{\mu^{\max} - \mu^{\min}})^p \right)^{1/p}. \tag{4}$$

Since the values for L_p in Eqn. (4) are between zero and one, i.e., $0 \leq L_p \leq 1$, one can also formulate and solve the problem in a fuzzy environment.

3.2. Copeland and Nelson's method (CN method). Copeland and Nelson (1996) proposed a method that allows the DM to determine the maximum distance $\boldsymbol{\Delta}$ from the desired target, T. They also showed that their method is as effective as the approach proposed by Lin and Tu (1995). They used the Nelder-Mead simplex procedure proposed by Nelder and Mead (1965) for the direct minimization of the problem as

$$\begin{aligned} &\text{Min } \ \hat{\sigma}_y + \varepsilon \\ &\text{such that } \ (\hat{\mu}_y - T)^2 \leq \Delta^2, \end{aligned}$$

where ε is defined as $(\hat{\mu}_y - T)^2$ when $(\hat{\mu}_y - T)^2 > \Delta^2$; otherwise, ε is zero. In order to obtain the best possible solution, one should apply this procedure twice. In Table 1, we summarize optimization methods for 'the smaller the better' scenario. The two criteria, CN and L_p, are formulated in Methods I and II, respectively; see Table 1, where σ_T refers to the target value of $\hat{\sigma}_y$.

Table 1. Two different optimization methods.

Method I	Method II
$\underset{s.t.}{\mathrm{Min}(\hat{\mu}_y)}$ $\hat{\sigma}_y \leq \sigma_T$	$\mathrm{Min}\ L_p = \{w(\hat{\sigma}_y - \sigma^{\min})^p + (1-w)(\hat{\mu}_y - \mu^{\min})^p\}^{1/p}$

4. Estimation methods

This section briefly discusses properties of the estimators obtained by ordinary and weighted least squares methods. The basic model in (1) can be written in the form of the linear model $\mathbf{Y} = \mathbf{X}\boldsymbol{\alpha} + \boldsymbol{\varepsilon}$, where $\mathbf{X} = [\mathbf{x}, \mathbf{z}]$; $\mathbf{x}$ and $\mathbf{z}$ are control and noise factors vectors, respectively. In the above expression, $\boldsymbol{\varepsilon}$ is a random error vector with a covariance matrix denoted by $\mathbf{V}$. Table 2 shows the mean, variance, and estimators for the coefficient vector $\boldsymbol{\alpha}$ for the ordinary and weighted least squares methods. It should be pointed out that the OLS method is applicable when the covariance matrix can be presented as $\sigma_\varepsilon^{\mathbf{2}}\mathbf{I}$, where $\mathbf{I}$ denotes an identity matrix. When this condition does not hold, one should use the WLS method instead. A third method referred to as an incorrect method, henceforth denoted by IM, is also presented in this table. This case, which often happens in practice, is discussed by Draper and Smith (1998). They show that when the OLS method is used incorrectly in place of the WLS one, the variance of the estimator increases significantly. The incorrect use of the OLS method instead of the WLS one happens either when noise factors are present but cannot be controlled in the experiment or when noise factors are unknown to the experimenter. In general, when a random effects model is applicable and a fixed effects model is assumed for simplicity, then we should expect an increase in the variance of the coefficient vector. The impacts of this assumption will be shown through numerical examples later. When noise factors are random in an experiment, the covariance matrix is not a multiple of the identity matrix and the WLS method must be used so that more weight is given to the solutions having less dispersion.

Matrix $\mathbf{V}$ determines which estimation method is appropriate for use. As can be seen, all three estimation methods provide unbiased estimators for the coefficient vector, but as Draper and Smith (1998) state, the variance of the third case is significantly larger than that of the second case. The effects of a larger variance on RPD will be investigated numerically in the next section.

5. Numerical example

In this section, a numerical example is provided to investigate the effect of an incorrect selection of estimation methods on the optimum solution affected by an increase in the variance of the response model coefficients. The example is solved using the two aforementioned methods. In this example, it is assumed that actual observations required for the estimation process can be generated by a predefined response model. The Optimization Toolbox of MATLAB is used for generating 15000 vectors of coefficients assuming that the vector of coefficients follows a multivariate normal

Table 2. Three different estimation methods.

Method	**V**	$\hat{\boldsymbol{\alpha}}$	$E(\hat{\boldsymbol{\alpha}})$	$\mathrm{Cov}(\hat{\boldsymbol{\alpha}})$
OLS	$=\boldsymbol{\sigma}_\varepsilon^2\mathbf{I}$	$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$	$\boldsymbol{\alpha}$	$(\mathbf{X}'\mathbf{X})^{-1}\boldsymbol{\sigma}_\varepsilon^2$
WLS	$\neq\boldsymbol{\sigma}_\varepsilon^2\mathbf{I}$	$(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y}$	$\boldsymbol{\alpha}$	$(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}$
IM	$\neq\boldsymbol{\sigma}_\varepsilon^2\mathbf{I}$	$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$	$\boldsymbol{\alpha}$	$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$

distribution defined by $\mathcal{N}\left(\boldsymbol{\alpha}, \mathrm{Cov}(\hat{\boldsymbol{\alpha}})\right)$, where $\mathrm{Cov}(\hat{\boldsymbol{\alpha}})$ is given in Table 2. Note that we assume that the variance in the experiment related to estimation is similar to that of the process related to optimization.

In a typical ‘the smaller the better’ scenario, the goal is to minimize the mean while holding the variance at a constant level. However, in robust parameter design, the control variables should be selected so that a desirable mean response with a low variance is obtained. The response surface model that Myers and Montgomery (2002) used is defined as

$$\begin{aligned} y(\mathbf{x},\mathbf{z}) = {} & 30.37 - 2.92x_1 - 4.13x_2 + 2.87x_1x_2 \\ & - 2.6x_1^2 + 2.18x_2^2 + 2.73z_1 - 2.33z_2 \\ & + 2.33z_3 - 0.27x_1z_1 + 0.89x_1z_2 + 2.58x_1z_3 \\ & + 2.01x_2z_1 - 1.43x_2z_2 + 1.56x_2z_3. \end{aligned} \tag{5}$$

It can be shown that

$$\begin{aligned} \mathrm{E}\left[y(\mathbf{x},\mathbf{z})\right] = {} & 30.37 - 2.92x_1 - 4.13x_2 - 2.87x_1x_2 \\ & - 2.6x_1^2 + 2.18x_2^2 \end{aligned} \tag{6}$$

and

$$\begin{aligned} & \mathrm{V}ar\left[y(\mathbf{x},\mathbf{z})\right] \\ & = \left(\begin{pmatrix} 2.73 \\ -2.33 \\ 2.33 \end{pmatrix} + \begin{pmatrix} -.27 & 0.89 & 2.58 \\ 2.01 & -1.43 & 1.56 \end{pmatrix}' \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right)' \\ & \times \Sigma_z \left(\begin{pmatrix} 2.73 \\ -2.33 \\ 2.33 \end{pmatrix} + \begin{pmatrix} -.27 & 0.89 & 2.58 \\ 2.01 & -1.43 & 1.56 \end{pmatrix}' \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right) + \sigma_\varepsilon^2. \end{aligned} \tag{7}$$

We arbitrarily keep $\boldsymbol{\sigma}_\varepsilon^2$ at a constant value of one. We consider the model in Eqn. (5) along with a modified 23-run central composite design (CCD) given in Table 3 to generate observations needed to estimate the model coefficients. In the original problem, Myers and Montgomery (2002) considered z_is as fixed factors in the experiment, whereas in the example discussed here z_is are assumed to be uncontrollable factors. Two cases are considered in this study. In the first one, it is assumed that there is no correlation between the noise factors, and in the second case a high correlation of 0.9 in magnitude is assumed between the noise factors. The reason for selecting a high correlation on the one hand is its effect on yielding relatively higher values in the variance response. On the other hand, it allows us to investigate the impact of correlation between noise factors on the estimation methods in a much easier way. The correlation matrices corresponding to the first and second cases are denoted by $\boldsymbol{\Sigma}_z^I$ and $\boldsymbol{\Sigma}_z^{II}$, respectively, and are defined as

$$\Sigma_z^I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\Sigma_z^{II} = \begin{bmatrix} 1 & -0.9 & 0.9 \\ -0.9 & 1 & -0.9 \\ 0.9 & -0.9 & 1 \end{bmatrix}.$$

To determine the optimum solutions, we can apply the methods presented in Table 1. The results are discussed below.

Method I. This method uses the formulation proposed by Copeland and Nelson (1996) to optimize the problem. Based on the design in Table 3 and the correlation matrices defined as $\boldsymbol{\Sigma}_z^I$ and $\boldsymbol{\Sigma}_z^{II}$, a set of observations for the response is generated and then the vector of coefficients using the WLS method and the IM is estimated. Since we now have the estimates for the coefficients of the response in (5), the equations for the mean and standard deviation of the response, i.e., Eqn. (6) and the square root of Eqn. (7), can be constructed. Using Method I, the settings for the control factors leading to the optimum values of the mean and standard deviation are determined. This step is repeated 15000 times and the average values for the mean and standard deviation are presented in Figs. 1 and 2, and Figs. 3 and 4, respectively. Figures 1 and 3 correspond to the case when $\boldsymbol{\Sigma}_z^I$ is used, and Figs. 2 and 4 correspond to the case when $\boldsymbol{\Sigma}_z^{II}$ is considered as the correlation matrix. In each figure, the

Table 3. Modified central composite design with 23 runs.

Design points	x_1	x_2	z_1	z_2	z_3
1	-1	-1	-1	-1	1
2	-1	1	-1	-1	-1
3	-1	-1	1	-1	-1
4	1	1	1	-1	-1
5	-1	-1	-1	1	-1
6	1	1	-1	1	-1
7	1	-1	1	1	-1
8	-1	1	1	1	-1
9	-1	-1	-1	-1	1
10	1	1	-1	-1	1
11	1	-1	1	-1	1
12	-1	1	1	-1	1
13	1	-1	-1	1	1
14	-1	1	-1	1	1
15	-1	-1	1	1	1
16	1	1	1	1	1
17	0	0	0	0	-2
18	0	0	0	0	2
19	0	0	0	-2	0
20	0	0	0	2	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0

Table 4. Ideal points and their locations.

	x_1	x_2	$\hat{\mu}_y$	$\hat{\sigma}_y$
$\sigma^{\min}$	0.003775	-1.4785	41.214	1.0052
$\mu^{\min}$	0.060831	0.90721	28.408	11.65

mean values obtained from the WLS and IM estimation methods and the exact values obtained from the true mean response equation are presented. Figures 5 and 6 show the standard deviation of the mean and standard deviation, respectively. Figures 5 and 6 are related to the estimation methods described in Figs. 1 and 2, and Figs. 3 and 4, respectively. For comparison purposes, the values of the mean and standard deviation when the exact values of the coefficients are used in Eqns. (6) and (7) are also shown in Figs. 1–4. Negative and positive signs in the figures are related to uncorrelated and correlated, namely $\mathbf{\Sigma}_z^I$ and $\mathbf{\Sigma}_z^{II}$, noise matrices, respectively.

Figures 1 and 2 show that the WLS method produces results which are closer to the actual values than the IM. Figures 3 and 4 show that for both correlated and uncorrelated cases, the results yielded by the two estimation methods are close to the values obtained

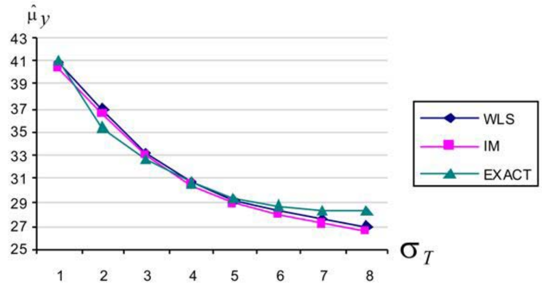

Fig. 1. Average values for the mean response with no correlation.

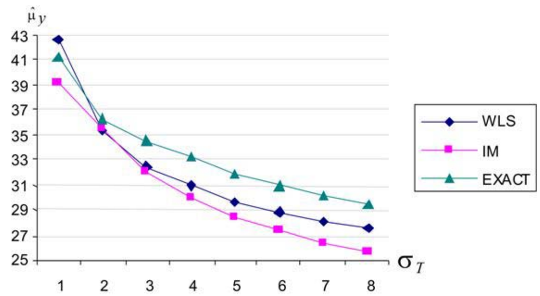

Fig. 2. Average values for the mean response with correlation.

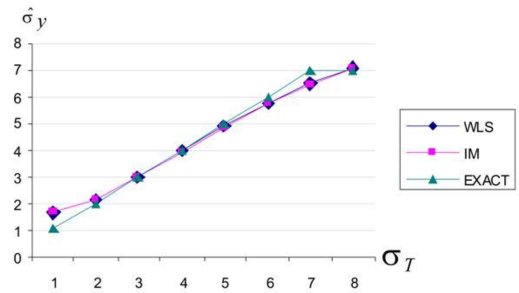

Fig. 3. Average values for the standard deviation response with no correlation.

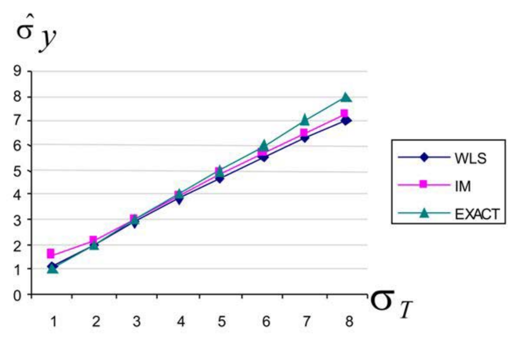

Fig. 4. Average values for the standard deviation response with correlation.

from the actual standard deviation response. Figures 5 and 6 show the standard deviation of the mean and standard deviation values estimated by the two methods for both correlated and uncorrelated cases. In these two figures, the results obtained by the estimation method

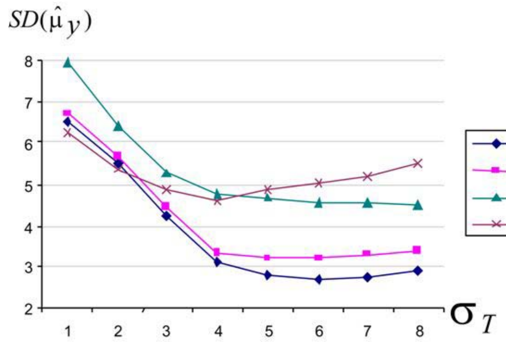

Fig. 5. Average values for the standard deviation of the mean response.

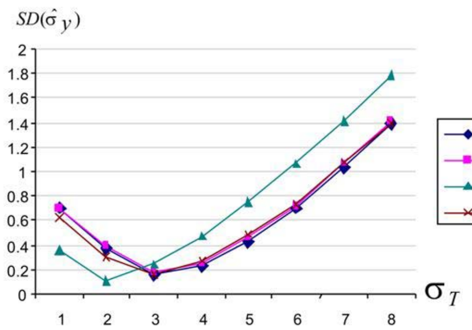

Fig. 6. Average values for the standard deviation of the standard deviation response.

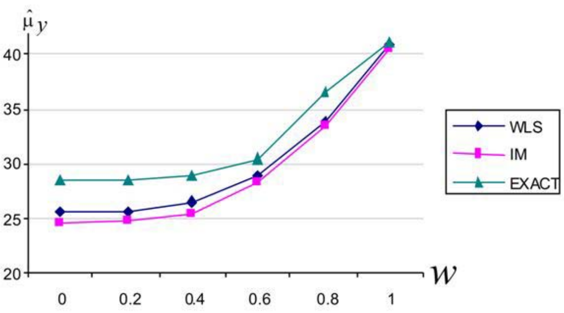

Fig. 7. Average values for the mean response with no correlation and $p = 1$.

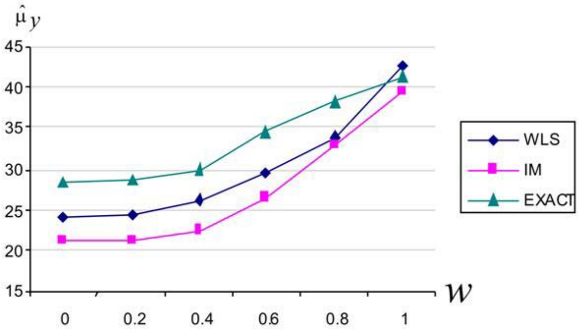

Fig. 8. Average values for the mean response with correlation and $p = 1$.

when correlation exists are denoted by "$a+$" superscripts, and the results obtained when no correlation exists are shown by "$a-$" superscripts. From the figures we see that, as expected, the standard deviation related to the uncorrelated case is relatively smaller than for the correlated case.

Method II. This method uses the L_p metric discussed in the previous section. The L_p metric requires the ideal values for the mean and the standard deviation. In our example, the ideal value for the mean response is 28.408, i.e., $\mu^{\min} = 28.408$. In Eqn. (7) we arbitrarily set the value of σ_ε^2 equal to one. Hence, the ideal value for the square root of the variance response becomes 1.0052, i.e., $\sigma^{\min} = 1.0052$. Table 4 shows the values of the controllable factors which lead to ideal points for the mean and standard deviation responses. Note that we perform optimization in the feasible region 4×4, that is, x_1 and x_2 take values between -2 and 2.

The results for the L_p metric method for the case of $p = 1$ are presented in Figs. 7–12. Figures 7 and 8 show the results for the mean equation as a function of w determined by the DM in the L_p equation. The weight value should be selected in advance by the DM. It determines the importance of the standard deviation response. For instance, if the standard deviation response is more important compared with the mean response, then a relatively larger value should be assigned to it. The case of w equal to one refers to a situation when the DM does not want to allow a smaller value for $\hat{\mu}_y$ at the expense of a larger value for $\hat{\sigma}_y$. From Figs. 7 and 8, one can infer that for both cases of correlated and uncorrelated noise matrices, incorrect use of an estimation method yields a relatively larger bias compared with the weighted least squares method. As we expect, the bias for the correlated case is larger than that for the uncorrelated one. Figures 9 and 10 show the standard deviation response as a function of w for uncorrelated and correlated noise matrices, respectively. These figures indicate that as the value of w increases, the standard deviation for the response decreases. In addition, an inappropriate estimation method leads to a larger bias in the standard deviation response. Figures 11 and 12 show the standard deviations for the mean and standard deviation responses as a function of w, respectively. These two figures indicate that an incorrect estimation method induces a larger variation in the estimates and, consequently, a less robust process.

In comparison with the CN method, the L_p method seems to lead to better solutions. For instance, when the weight in the WLS method is set equal to 0.6, we obtain the value of 4.48 and 29.032 from Figs. 9 and 7, respectively. In Fig. 3, the same standard deviation can be obtained by the CN method when σ_T is set approximately equal to 4.5. In Fig. 1, when σ_T is set equal to 4.5, the value of 29.98 can be obtained for the mean response.

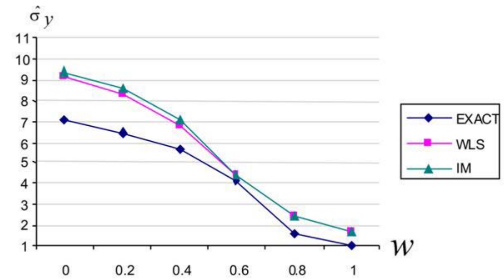

Fig. 9. Average values for the standard deviation response with no correlation and $p = 1$.

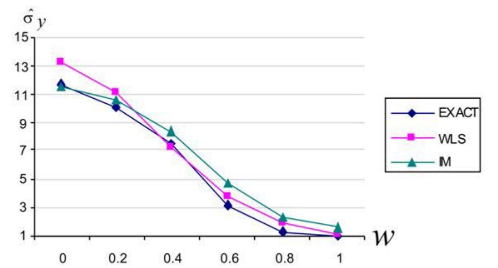

Fig. 10. Average values for the standard deviation response with correlation and $p = 1$.

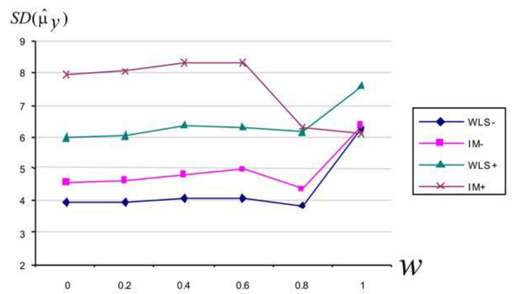

Fig. 11. Average values for the standard deviation of the mean response and $p = 1$.

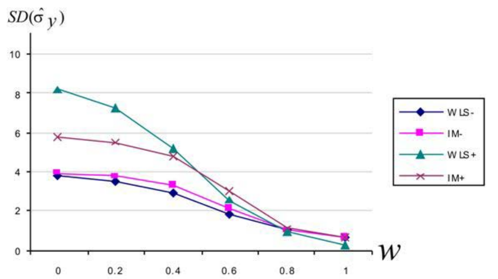

Fig. 12. Average values for the standard deviation of the standard deviation response and $p = 1$.

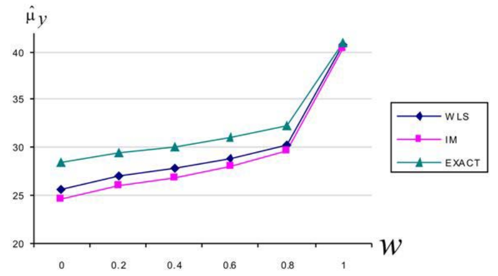

Fig. 13. Average values for the mean response with no correlation and $p = 2$.

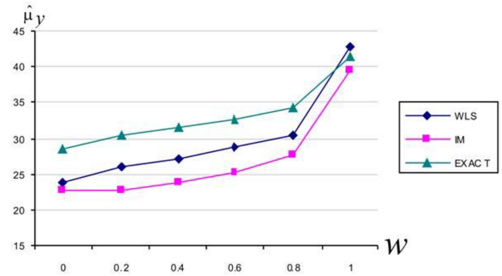

Fig. 14. Average values for the mean response with correlation and $p = 2$.

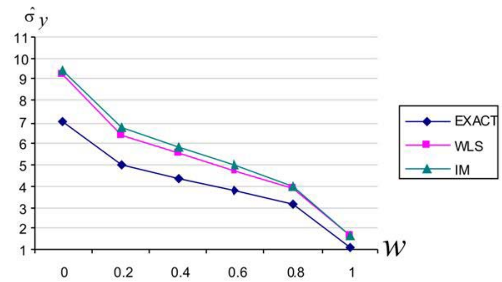

Fig. 15. Average values for the standard deviation response with no correlation and $p = 2$.

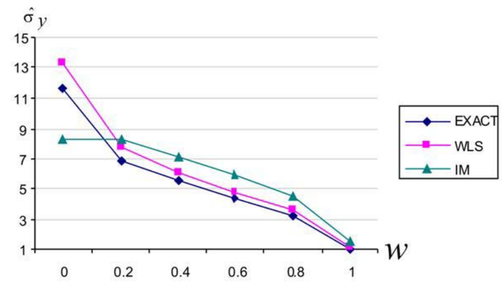

Fig. 16. Average values for the standard deviation response with correlation and $p = 2$.

Thus, the value of the mean response obtained by the CN method is greater (worse) than that of the L_p method. The results for the case of p equal to two are presented in Figs. 13–18.

From Figs. 7–18 it can be inferred that, as the p value increases, the slope of the curves also increases. It indicates that the DM makes a more conservative decision in the trade off between the standard deviation and mean responses. Similar analyses in which the details are not given were conducted for $p = \infty$ and the same results were obtained. The results indicate that the L_p is a flexible method by which researchers are able to express their

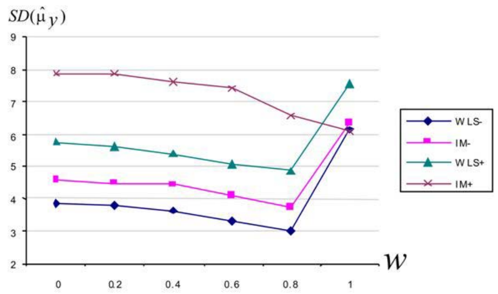

Fig. 17. Average values for the standard deviation of the mean response and p=2.

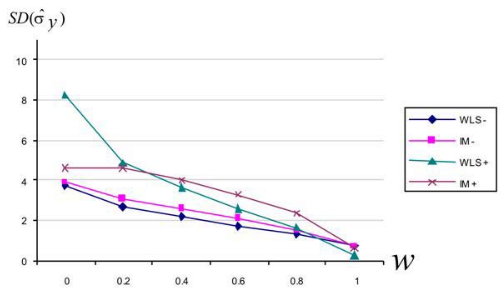

Fig. 18. Average values for the standard deviation of the standard deviation response and $p = 2$.

preferences for responses through w and p. In addition, it is less sensitive to the estimation errors compared with the CN method. However, if an incorrect estimation method is used for estimating the coefficients of the response model, one should expect a larger variance for the estimated coefficients.

6. Conclusion

In this paper, two different optimization criteria, namely, CN and L_p, for 'the smaller the better' problems were considered, and the impact of incorrect estimation method on the estimates of the coefficients of the mean and standard deviation responses in the context of robust parameter design study was investigated. Numerical results indicate that the use of the incorrect estimation method leads to a relatively significant bias in the mean and standard deviation responses. Hence, researchers should be more cautious about the presence of noise factors when estimating coefficients of the response function. By comparing optimization criteria, we found out that the solutions obtained by the first method, i.e., the CN method, are relatively less robust. The second optimization method, or the L_p criterion, heavily depends on the weights reflecting the importance of the responses from the viewpoint of the decision maker. An advantage of this method is its ability to yield efficient solutions regardless of the values of p and w.

Acknowledgment

Dr. Noorossana's research was supported partially by a grant from the Iran National Science Foundation.

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Received: 28 July 2007
Revised: 22 April 2008