Int. J. Appl. Math. Comput. Sci., 2009, Vol. 19, No. 2, 281–289
DOI: 10.2478/v10006-009-0024-4

ON DIRECTIONAL CHANGE AND ANTI-WINDUP COMPENSATION IN MULTIVARIABLE CONTROL SYSTEMS

DARIUSZ HORLA

Institute of Control and Information Engineering
Poznań University of Technology, ul. Piotrowo 3A, 60-965 Poznań, Poland
e-mail: dariusz.horla@put.poznan.pl

The paper presents a novel description of the interplay between the windup phenomenon and directional change in controls for multivariable systems (including plants with an uneven number of inputs and outputs), usually omitted in the literature. The paper also proposes a new classification of anti-windup compensators with respect to the method of generating the constrained control signal.

Keywords: windup phenomenon, multivariable systems, optimal control.

1. Introduction

Taking control limits, ubiquitous in the real world, into consideration is crucial for achieving high performance of designed control systems. There are two ways in which one can consider possible constraints during the synthesis of controllers. In the first approach, imposing constraints during the design procedure of the controller usually leads to difficulties with obtaining explicit forms of control laws, apart from computationally very simple cases. The other approach is to assume the system is fully linear and, subsequently, having designed the controller for the unconstrained system (by means of optimisation, using Diophantine equations, etc.), impose constraints, which would require additional changes in the control system due to the presence of constraints.

A situation when because of, e.g., constraints (or, in general, nonlinearities) internal controller states do not correspond to the actual signals present in the control systems is referred to in the literature as the windup phenomenon (Horla, 2004; Walgama and Sternby, 1993; Doná *et al.*, 2000). It is obvious that, due to not taking control signal constraints into account during the controller design stage, one can expect degraded performance because of the infeasibility of the computed control signals, as they can only be applied having been constrained first.

There are various methods of compensating the windup phenomenon in single-input single-output systems, but few result in satisfactory performance improvement in the case of multivariable systems (Horla, 2004; Öhr, 2003). In such a case, apart from the windup phenomenon itself, one can also observe directional change in the control vector due to different implementations of constraints, which could affect the direction of the unconstrained (i.e., computed) control vector.

The other nicety of multivariable systems is, in a general form, the problem with decoupling, with respect to the uneven number of control signals and output signals. In such a case, the control direction may correspond not only to principal input directions (Maciejowski, 1989; 2002) or the maximal directional gain of the transfer function matrix (Albertos and Sala, 2004), but also to the degree of decoupling.

The problem of directional change was discussed in (Walgama and Sternby, 1993), where a pioneering approach towards better understanding of connections in between anti-windup compensation and directional change was made. A review of multivariable anti-windup compensators was included in (Öhr, 2003) with some basic analysis of the topic.

Decoupling, anti-windup compensation and directional change are inherently connected and present in all industrial applications and are of prime importance in real-world environment. A great deal of papers has been written on anti-windup compensation in such areas as motor drive control (Hwi-Beon, 1998; Kuo-Kai and Cheng-Yuan, 2003), paper machine headbox control (Öhr, 2003), or the control of hydraulic/pneumatic drives

(Janiszowski, 2005; Hodel and Hall, 2001).

The present paper aims to answer the question of what the connections between directional change and the windup phenomena are. This has been shown for two strands in controller design subject to constraints, as mentioned before, and two ways of anti-windup compensation with respect to directional change in controls.

The paper has been motivated by the research carried in (Horla, 2004), where the initial problem was described and identified, and in (Horla, 2007a), where it was solved with the use of an *a priori* anti-windup compensator for the case of square systems. The current paper extends the understanding of anti-windup compensation to nonsquare systems with imposed constraints, comparing control performance with the optimisation-based approach, relating to MPC (Camacho and Bordons, 1999), which is widelyspread and applied in industry.

The novelty of the paper is the description of the interplay between the windup phenomenon and directional change in controls for multivariable systems, not present in the literature, as well as a proposition of a new classification of anti-windup compensators.

2. Anti-windup compensation

There are two general schemes of anti-windup compensation (AWC) connected with controller design (Horla, 2006). Let us suppose that a controller has been designed for a fully linear case, as to fit certain performance requirements. Introducing constraints would require (most often heuristic) modifications to the control law that usually feed back the difference between computed $\underline{v}_t$ and the constrained control vector $\underline{u}_t$, as in Fig. 1(a). This is referred to in the literature as *a posteriori* AWC (Horla, 2007a).

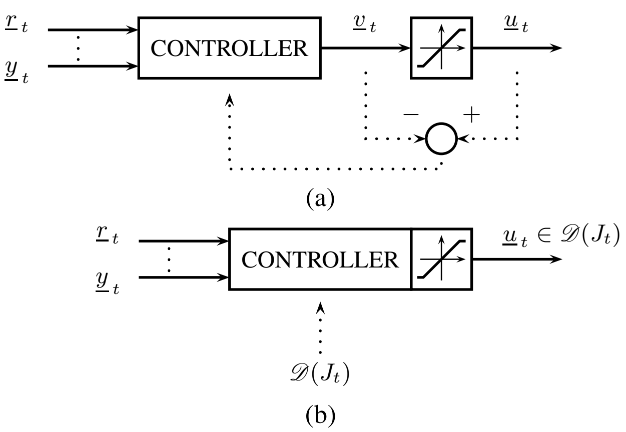


Fig. 1. AWC: (a) *a posteriori*, (b) *a priori*.

The second way of performing anti-windup compensation is to incorporate AWC implicitly into the controller, i.e., when the controller generates a feasible control vector only (belonging to the domain $\mathscr{D}$ of all control vectors for which the control performance index J_t is of finite value), which is addressed as *a priori* AWC (as in Fig. 1(b)). It can be implemented in various ways, one of which is constrained predictive control (Maciejowski, 2002; Camacho and Bordons, 1999). For a discussion of connections between AWC and model predictive control, see (Doná *et al.,* 2000; Horla, 2007b; 2006).

The next two sections describe selected representatives of two strands in anti-windup compensation, whose performance is to be further compared. It should be stressed that the main point of the paper is not to draw any conclusions concerning the stability of closed-loop systems, which can be assured by a proper choice of the (stabilising) control law, but to give a precise analysis of the directional change vs. anti-windup compensation interplay, which is potentially of practical importance.

One of the fields of potential applications where abiding certain directions as preferred directions, directions supplied by a higher-order layer of the control system or, simply, directions resulting from constraints is mobile robotics.

In (Galicki, 2005), one can find that under certain constraints, such as state constraints, further restrictions may arise and have to be taken into account. Such constraints limit the velocity vector to certain directions. One can therefore find the best path and then try to accomplish the task with minimum directional change.

A recent summary of up-to-date anti-windup compensators utilising optimisation techniques can be found in (Hu *et al.*, 2008) for the case of a linear system with input saturations.

3. *A posteriori* AWC

The most popular and easily implemented anti-windup compensators are based on the RST controller equation, which in the case of multivariable systems is of the form

$$\boldsymbol{R}(q^{-1})\underline{v}_t = -\boldsymbol{S}(q^{-1})\underline{y}_t + \boldsymbol{T}(q^{-1})\underline{r}_t\,, \qquad (1)$$

where controller polynomial matrices,

$$\begin{aligned}\boldsymbol{R}(q^{-1}) &= \boldsymbol{I} + \boldsymbol{R}_1 q^{-1} + \cdots + \boldsymbol{R}_{n\boldsymbol{R}} q^{-n\boldsymbol{R}},\\ \boldsymbol{S}(q^{-1}) &= \boldsymbol{S}_0 + \boldsymbol{S}_1 q^{-1} + \cdots + \boldsymbol{S}_{n\boldsymbol{S}} q^{-n\boldsymbol{S}},\\ \boldsymbol{T}(q^{-1}) &= \boldsymbol{T}_0 + \boldsymbol{T}_1 q^{-1} + \cdots + \boldsymbol{T}_{n\boldsymbol{T}} q^{-n\boldsymbol{T}},\end{aligned}$$

are of sizes $\boldsymbol{R}_i \in \mathbb{R}^{m\times m}$ ($i = 1, \ldots, n\mathbb{R}$), $\boldsymbol{S}_i \in \mathbb{R}^{m\times p}$ ($i = 0, \ldots, n\mathbf{S}$), $\boldsymbol{T}_i \in \mathbb{R}^{m\times p}$ ($i = 0, \ldots, n\mathbf{T}$), designed for the unconstrained case, $\underline{y}_t \in \mathbb{R}^p$ is the output vector, $\underline{v}_t \in \mathbb{R}^m$ is the computed control vector, and $\underline{r}_t \in \mathbb{R}^p$ is the reference vector.

When nonlinearities, such as control limits, are taken into consideration, the computed control vector $\underline{v}_t$ is different from the constrained (i.e., applied) control vector $\underline{u}_t$.

In such a case, one can modify the control law according to many AWC schemes (Horla, 2004; Öhr, 2003)—one of most popular is the Conditioning Technique (CT) AWC, which is a version of the generalised CT (Horla, 2007b) with no additional parameters to tune.

The restoration of the consistency of the states (anti-windup compensation) is performed by modifying the reference vector, i.e., computing the so-called feasible reference vector, in general given as a function f,

$$\underline{r}_t^r = \begin{cases} f[\underline{v}_t, \underline{u}_t, \underline{r}_t] & \text{if} \quad \exists_{1 \le i \le m}\, u_{i,t} \ne v_{i,t}, \\ \underline{r}_t & \text{if} \quad \underline{v}_t = \underline{u}_t. \end{cases} \tag{2}$$

The control and reference vectors are computed as

$$\begin{aligned} \underline{v}_t &= (\boldsymbol{I} - \boldsymbol{R}(q^{-1}))\underline{u}_t - \boldsymbol{S}(q^{-1})\underline{y}_t \\ &\quad + (\boldsymbol{T}(q^{-1}) - \boldsymbol{T}_0)\underline{r}_t^r + \boldsymbol{T}_0 \underline{r}_t, \end{aligned} \tag{3}$$

$$\underline{r}_t^r = \underline{r}_t + \boldsymbol{T}_0^{-1}(\underline{u}_t - \underline{v}_t), \tag{4}$$

and, subsequently, the computed control vector is subject to constraints, e.g., cut-off saturation. The matrix $\boldsymbol{T}_0$ is assumed to be nonsingular.

The controller is fed back with the constrained control vector and informed about the constraints violation by means of a modified reference vector. If $m \ne p$, then Eqn. (4) takes the form

$$\underline{r}_t^r = \underline{r}_t + \boldsymbol{T}_0^{\dagger}(\underline{u}_t - \underline{v}_t), \tag{5}$$

where † denotes a Moore-Penrose pseudo-inverse. A block diagram of CT-AWC for the general case is shown in Fig. 2.

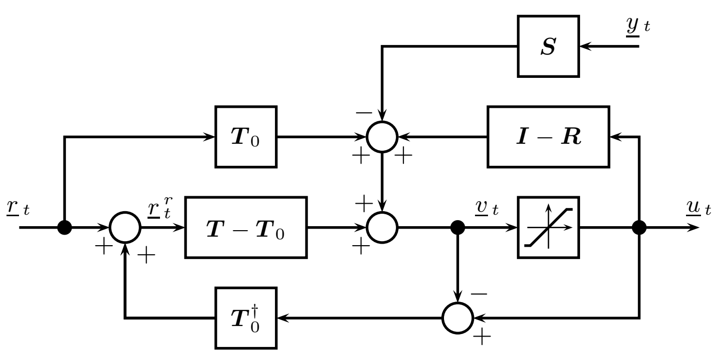


Fig. 2. Block diagram of CT-AWC with the area of nonlinearity (here: cut-off saturation).

CT example *(Horla, 2007b; 2004; Öhr, 2003)* Let us assume that a two-input two-output system ($m = p = 2$) is driven by an RST controller decoupling it. Let

$$\begin{aligned} \boldsymbol{T}(q^{-1}) &= \boldsymbol{S}(q^{-1}) \\ &= \begin{bmatrix} -0.4 & -0.2 \\ 1.9 & -3.0 \end{bmatrix} + \begin{bmatrix} -0.3 & 0.2 \\ 0.0 & -0.1 \end{bmatrix} q^{-1}. \end{aligned}$$

Having assumed a cut-off nonlinearity at the controller output, zero initial conditions for $t < 1$, and $\underline{r}_t = [1,\, 0]^T$ for $t = 1$, with symmetrical cut-off control amplitude constraints $\alpha = \pm 1$, the computed control vector $\underline{v}_1 = [-0.4,\, 1.9]^T$ saturates at $t = 1$, thus $\underline{u}_1 = [-0.4,\, 1.0]^T$ (the control direction is changed).

According to (4),

$$\begin{aligned} \underline{r}_1^r &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.4 & -0.2 \\ 1.9 & -3.0 \end{bmatrix}^{-1} \left(\begin{bmatrix} -0.4 \\ 1.9 \end{bmatrix} - \begin{bmatrix} -0.4 \\ 1.0 \end{bmatrix} \right) \\ &= \begin{bmatrix} 0.9241 \\ -0.3481 \end{bmatrix} = \begin{bmatrix} r_{1,1}^r \\ r_{2,1}^r \end{bmatrix}. \end{aligned}$$

One can see that the output that was intended to be left unmodified has been modified from zero to a nonzero value. The latter is a result of directional change, motivated by anti-windup compensation. The two issues, as will be shown in further sections of the paper, are therefore connected.

4. *A priori* AWC

As has been previously stated, the second way of performing anti-windup compensation is to incorporate AWC implicitly into the controller. In order to use all the advantages of such an approach (e.g., the optimality of the solution, no need to design decoupling stages, etc.), let the optimal constrained control vector follow from

$$\underline{u}_t^{\star} : J_t(\underline{u}_t^{\star}) = \inf_{\underline{u}_t \in \mathscr{D}(J_t)} \{J_t(\underline{u}_t)\},$$

where its j-th component has been symmetrically constrained,

$$|u_{j,t}| \le \alpha_j,$$

and the constrained control vector is

$$\underline{u}_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}. \tag{6}$$

Assuming that the optimal control signal results from one-step optimisation, which deprives predictive control of all its advantages (i.e., prediction feature), this enables further comparison with *a posteriori* AWC performance. Since the constrained control vector contains one control action for each plant input only (6), a one-step predictive controller is considered. The performance index has been chosen as a sum of squared tracking errors resulting from a reference model output $\underline{r}_{M,t+1}$ (which can be chosen as an original reference vector),

$$J_t = \sum_{k=1}^{p} (r_{M,k,t+1} - y_{k,t+1})^2, \tag{7}$$

or, equivalently, using a squared L_2 norm,

$$J_t = \|\underline{r}_{M,t+1} - \underline{\hat{y}}_{t+1} - \underline{\hat{\hat{y}}}_{t+1}\|_2^2, \tag{8}$$

where one-step prediction of system output comprises as in (8) forced and free-response output vectors $\underline{\hat{y}}_{t+1}$ and $\underline{\hat{\hat{y}}}_{t+1}$, respectively (Horla, 2006).

The performance index can be rewritten in a quadratic form (Boyd and Vandenberghe, 2004),

$$\begin{aligned} J_t = &\left(\boldsymbol{G}\underline{u}_t + \underline{\hat{\hat{y}}}_{t+1} - \underline{r}_{M,t+1}\right)^T \\ &\cdot \left(\boldsymbol{G}\underline{u}_t + \underline{\hat{\hat{y}}}_{t+1} - \underline{r}_{M,t+1}\right), \end{aligned} \tag{9}$$

where $\boldsymbol{G}$ is an impulse-response matrix, and the optimisation can be performed with the use of a linear matrix inequality (LMI) statement of the problem with the last two LMIs responsible for control constraints, as below (Boyd and Vandenberghe, 2004; Boyd *et al.*, 1994):

$$\begin{aligned} &\min\ \gamma \\ &\begin{bmatrix} I & \star \\ \underline{u}_t^T(\boldsymbol{G}^T\boldsymbol{G})^{1/2} & \begin{pmatrix} \gamma - (\underline{r}_{M,t+1} - \underline{\hat{\hat{y}}}_{t+1})^T \times \\ \times(\underline{r}_{M,t+1} - \underline{\hat{\hat{y}}}_{t+1}) + \\ +2(\underline{r}_{M,t+1} - \underline{\hat{\hat{y}}}_{t+1})^T\boldsymbol{G}\underline{u}_t \end{pmatrix} \end{bmatrix} \geq 0, \\ &\operatorname{diag}\{\alpha_1 - u_{1,t}, \ldots, \alpha_m - u_{m,t}\} \geq 0, \\ &\operatorname{diag}\{\alpha_1 + u_{1,t}, \ldots, \alpha_m + u_{m,t}\} \geq 0, \end{aligned} \tag{10}$$

where '$\star$' denotes a symmetrical entry and the scalars $\alpha_1, \ldots, \alpha_m$ correspond to symmetrical constraints imposed on the components of the constrained control vector.

5. Constraining the control vector

Depending on the method of imposing constraints on the control vector, one can observe directional change, illustrated in Fig. 3(a) in the case of cut-off saturation (for future reference: SAT1), which is not present when saturation is performed according to imposed constraints (dashed lines) with constant direction, Fig. 3(b) (for future reference: SAT2).

6. Description of the windup phenomenon in multivariable systems

The problem of the windup phenomenon in multivariable systems with its connection to directional change in controls is rarely addressed in the literature. However, the following concepts can be found in (Walgama and Sternby, 1993):

Solving the windup phenomenon problem does not mean that the constrained control vector is of the same direction as computed control vector.

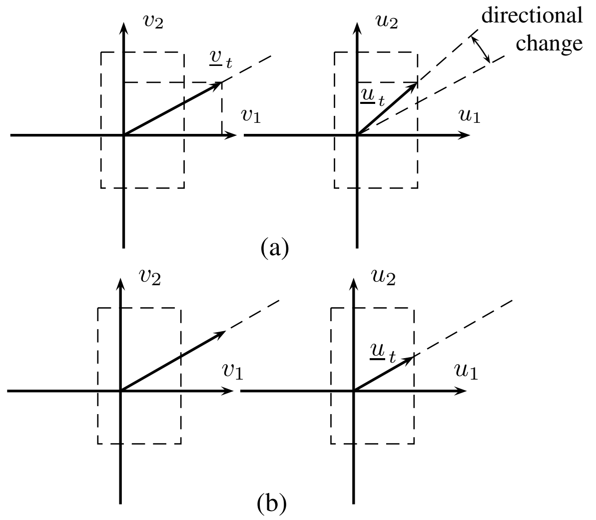


Fig. 3. Saturation: (a) direction-changing, (b) direction-preserving (left: control vector before saturation, right: after saturation).

On the other hand, avoiding directional changes in control enables one to avoid windup phenomenon.

In further sections, it will be shown where the latter description holds, in what cases it is invalid and how it can be extended for the cases of nonsquare systems.

7. Directional change phenomenon—An example

Let us suppose that a two-input two-output system is not coupled (Horla, 2004; Öhr, 2003) and both loops are driven by separate controllers (with no cross-coupling). The system output $\underline{y}_t$ is to track the reference vector comprising two sine waves, which corresponds to drawing a circular shape in the (y_1, y_2) plane .

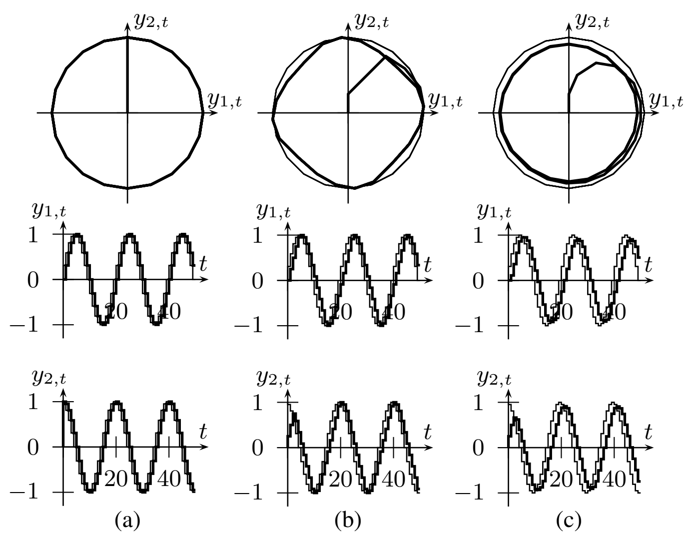


Fig. 4. (a) unconstrained system, (b) cut-off saturation, (c) direction-preserving saturation.

As can be seen in Fig. 4(a), the unconstrained system performs best, whereas in the case of SAT1 imposed on both elements of the control vector (Fig. 4(b)), tracking performance is poor. This is because of directional change in controls, changing proportions between the components. In the application to, e.g., shape-cutting, the performance of the system from Fig. 4(c) (SAT2) is superior. Nevertheless, it should be borne in mind that the system is always perfectly decoupled—such a situation does not usually take place for systems with $m = p$ and hardly ever for $m \neq p$.

8. Simulation studies

The simulation results presented in this section, as well as conclusions, arise from logical inference concerning the performance indices and overall performance of the systems visible in the figures. The examined plants that are presented below are chosen for three configurations of m and p.

The following multivariable CARMA plant model will be of interest:

$$\boldsymbol{A}(q^{-1})\underline{y}_{\,t} = \boldsymbol{B}(q^{-1})\underline{u}_{\,t-d}\,, \tag{11}$$

with left co-prime polynomial matrices $\boldsymbol{A}(q^{-1})$ ($\boldsymbol{A}(0) = \boldsymbol{I}$), $\boldsymbol{B}(q^{-1})$ and delay $d = 1$. The plants considered are assumed to be cross-coupled:

- P1 ($m = 2, p = 2$)

$$\begin{aligned}\boldsymbol{A}(q^{-1}) &= \boldsymbol{I} + \begin{bmatrix} 0.8 & -0.1 \\ 0.4 & -1.0 \end{bmatrix} q^{-1} \\ &\quad + \begin{bmatrix} -0.49 & -0.10 \\ 0.10 & 0.25 \end{bmatrix} q^{-2}, \\ \boldsymbol{B}(q^{-1}) &= \begin{bmatrix} 1.0 & 0.3 \\ 0.5 & 0.8 \end{bmatrix}.\end{aligned}$$

- P2 ($m = 3, p = 2$)

$$\begin{aligned}\boldsymbol{A}(q^{-1}) &= \boldsymbol{I} + \begin{bmatrix} 0.8 & -0.1 \\ 0.4 & -1.0 \end{bmatrix} q^{-1} \\ &\quad + \begin{bmatrix} -0.49 & -0.10 \\ 0.10 & 0.25 \end{bmatrix} q^{-2}, \\ \boldsymbol{B}(q^{-1}) &= \begin{bmatrix} 1.0 & 0.2 & 0.3 \\ 0.5 & 0.3 & 0.8 \end{bmatrix}.\end{aligned}$$

- P3 ($m = 2, p = 3$)

$$\begin{aligned}\boldsymbol{A}(q^{-1}) &= \boldsymbol{I} + \begin{bmatrix} -0.7 & 0.0 & 0.1 \\ -0.1 & -0.8 & 0.2 \\ 0.1 & 0.0 & -0.8 \end{bmatrix} q^{-1} \\ &\quad + \begin{bmatrix} -0.1 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} q^{-2},\end{aligned}$$

$$\boldsymbol{B}(q^{-1}) = \begin{bmatrix} 1.0 & 0.1 \\ 0.2 & 1.0 \\ 0.5 & -0.1 \end{bmatrix}.$$

The above-mentioned plants have been chosen to represent three basic input vs. output configurations. By such a choice, the cases where directional change must take place ($m > p$) and may take place (other cases) are described. This is due to the assumption that if there is a plant with $m < p$, then it is impossible to design a controller/compensator to decouple it in dynamic situations, because of the insufficiency of control inputs. In order to present the interplay between directional change and anti-windup compensation in a simple manner, second-order systems have been taken into consideration. The other plants may be approximated by second-order dynamics to a certain extent.

In the case of *a posteriori* AWC, the plant is controlled by a multivariable pole-placement controller with a characteristic polynomial matrix,

$$\boldsymbol{A}_M(q^{-1}) = (1 - 0.5q^{-1})\boldsymbol{I}^{p\times p}.$$

The controller is given in an RST structure with polynomial matrices $\boldsymbol{R}(q^{-1})$ ($n\boldsymbol{R} = n\boldsymbol{B} + d - 1$) and $\boldsymbol{S}(q^{-1})$ ($n\boldsymbol{S} = n\boldsymbol{A} - 1$) resulting from the Diophantine equation, which in the case of P1 takes the form

$$\begin{aligned}\boldsymbol{A}(q^{-1})\boldsymbol{R}(q^{-1}) &+ q^{-1}\boldsymbol{B}(q^{-1})\boldsymbol{S}(q^{-1}) \\ &= \boldsymbol{A}_M(q^{-1})\boldsymbol{A}_o(q^{-1}),\end{aligned} \tag{12}$$

with an observer polynomial matrix $\boldsymbol{A}_o(q^{-1}) = (1 - 0.5q^{-1})\boldsymbol{I}^{p\times p}$ for all plants, $\boldsymbol{T}(q^{-1}) = \boldsymbol{K}\boldsymbol{A}_o(q^{-1})$, $\boldsymbol{A}_M(1) = \boldsymbol{B}(1)\boldsymbol{K}$. From the Diophantine equation, one obtains eight equations with eight variables for P1, for P2—eight equations with 12 variables, and for P3—18 equations with 12 variables. In the latter cases, the set of equations is solved in a least-squares sense.

Having imposed control limits on the RST controller, in order to restore good performance quality, as designed for the linear case via the Diophantine equation, it is required to implement *a posteriori* AWC techniques.

As has already been stated, in order to enable a comparison of two different anti-windup compensation schemes together with their connection with directional change phenomenon, *a priori* AWC has been designed in such a way that it has been deprived of all its advantages—the prediction horizon has been chosen as one step, and the reference model vector corresponds to closed-loop tracking with dynamics described by $\boldsymbol{A}_M(q^{-1})$. In such a way, one can compare the performance of two pole-placement controllers with different AWCs.

The evaluation of control performance connected with anti-windup compensation performance requires the following performance indices to be introduced:

$$J_1 = \frac{1}{N} \sum_{i=1}^{p} \sum_{t=1}^{N} |r_{i,t} - y_{i,t}|, \tag{13}$$

$$J_2 = \frac{1}{N} \sum_{i=1}^{p} \sum_{t=1}^{N} (r_{i,t} - y_{i,t})^2, \tag{14}$$

$$\overline{\varphi}_1 = \frac{1}{N} \sum_{t=1}^{N} |\varphi(\underline{v}_t) - \varphi(\underline{u}_t)| \quad [^\circ], \tag{15}$$

$$\overline{\varphi}_2 = \frac{1}{N} \sum_{t=1}^{N} (\varphi(\underline{v}_t) - \varphi(\underline{u}_t))^2, \tag{16}$$

where (13) corresponds to a mean absolute tracking error of p outputs, (15) is a mean absolute direction change between the computed and the constrained control vector, and φ denotes an angular measure in $\mathbb{R}^m$. The indices (14) and (16) lack a physical interpretation. In the case of $m = 3$, φ corresponds to the absolute angle measure, due to difficulties with choosing its appropriate direction.

The purpose of introducing (15) and (16) is that the two performance indices are computed with different weights with respect to the angle deflection, and larger deflections are penalised more severely in the case of (16), similarly as in the case of L_1 and L_2 norms.

Simulations labelled as SAT1 and SAT2 have been performed for the system with cut-off saturation and direction-preserving saturation, respectively, and simulations labelled as CT- with the same saturation algorithms as above but with CT-AWC. The label 'a-pr AWC' refers to optimal *a priori* AWC. In the following figures, φ_t denotes directional change defined as the angle deflection between computed and constrained control vectors. The constraint levels have been chosen so that all control signals saturate enabling both windup and directional change phenomena to take place and triggering asymptotic tracking properties of the closed-loop system.

As can be seen from Table 1(a) and Fig. 5, cut-off saturation causes a directional change, which is visible during reference vector changes. The *a priori* AWC angle deflection has been computed as a difference between angles of the control vector for the unconstrained case and the control vector for the constrained one. In order to restore high control performance, it is necessary to alter the decoupling of the plant. As can be seen, the greater the mean direction change, the lesser the performance index, see (14). Thus it can be said that directional change supports anti-windup compensation. In the case of the SAT2 algorithm, both closed-loop systems behave worse than with time-varying direction of control vectors. No change in direction makes severe coupling to be visible in output vector plots. The *a priori* AWC performance shows clearly that for the presented system with $m = p$, greater directional change corresponds to better anti-windup phenomenon compensation.

Table 1. Performance indices for (a) P1, (b) P2, (c) P3.

	SAT1	SAT2	CT-SAT1	CT-SAT2	a-pr AWC
J_1	0.5366	0.7095	0.4465	0.6443	0.4009
J_2	0.8725	1.2755	0.8235	1.2108	0.8486
$\overline{\varphi}_1$	0.4644	0.0000	0.7689	0.0000	0.7171
$\overline{\varphi}_2$	6.7954	0.0000	11.0431	0.0000	11.8327

(a)

	SAT1	SAT2	CT-SAT1	CT-SAT2	a-pr AWC
J_1	0.4748	0.7585	0.3796	0.6287	0.3535
J_2	0.7675	1.5131	0.6951	1.3720	0.7373
$\overline{\varphi}_1$	0.4022	0.0000	0.5970	0.0000	75.6537
$\overline{\varphi}_2$	4.8936	0.0000	6.9177	0.0000	6217.70

(b)

	SAT1	SAT2	CT-SAT1	CT-SAT2	a-pr AWC
J_1	1.3601	1.5326	1.4187	1.6291	1.3293
J_2	1.9862	2.0637	2.0299	2.1535	1.8375
$\overline{\varphi}_1$	3.6796	0.0000	5.9229	0.0000	2.5244
$\overline{\varphi}_2$	84.5076	0.0000	159.1085	0.0000	37.0812

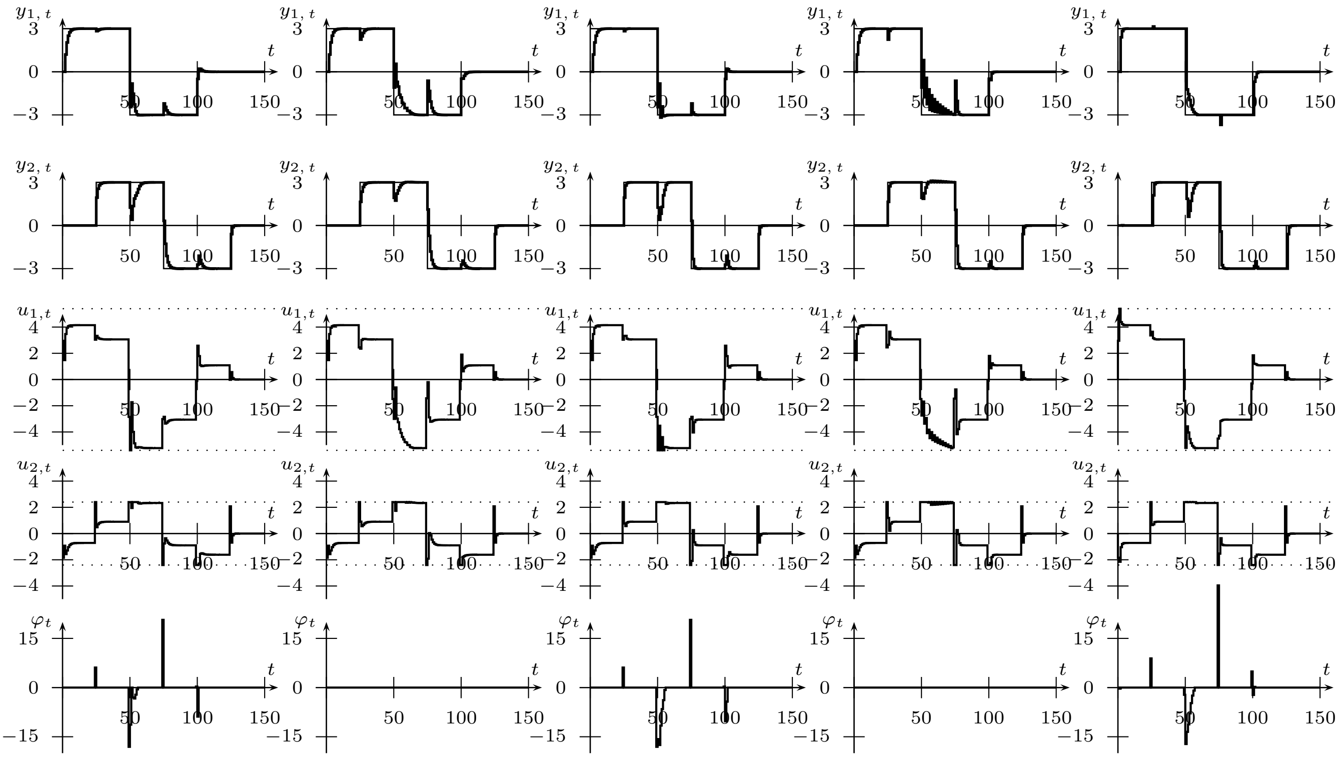
(c)

The introduction of CT-AWC to both saturation methods enables one to achieve improvement due to taking possible saturations into consideration. The requirement of no directional change is a more demanding control regime.

For the case of $m > p$ (as in Table 1(b) and Fig. 6), similar conclusions hold; however, in the case of a greater number of control signals, the requirement to preserve the control direction causes performance indices to increase more, regardless of the method of anti-windup compensation. Once again, *a priori* compensation, i.e., frequent changes in control directions, corresponds to almost perfect decoupling and the best tracking (and anti-windup compensation) performance.

The situation is different with a deficient number of control inputs in comparison with plant outputs, i.e., $m < p$ (as in Table 1(c) and Fig. 7). In such a case, when certain modes of the plant are uncontrollable, and if coupled with other, they may cause a control performance degradation. For the plant P3, the output $y_{3,t}$ is coupled with remaining two outputs and, because of the described deficiency, it is impossible to keep it near the zero value, with respect to $r_{3,t} = r_{M,3,t} = 0$.

For the system satisfying $m < p$, the direction-preservation requirement causes performance degradation (as before) and gives rise to the windup phenomenon, but CT-AWC does not work well due to persistent coupling present in the control system, i.e., in such a case, *a posteriori* anti-windup compensation should not be implemented. The *a priori* AWC foresees coupling effects, and with moderate directional change it assures better control performance and anti-windup compensation. In such an



(a) (b) (c) (d) (e)

Fig. 5. P1, $\alpha_1 = 5.4$, $\alpha_2 = 2.4$, (a) no AWC, SAT1, (b) no AWC, SAT2, (c) CT-AWC, SAT1, (d) CT-AWC, SAT2, (e) *a priori* AWC.

application, it is not advisable to allow severe directional changes in controls, which is, nevertheless, still needed to obtain a decoupling effect. It would then be advisable to design a saturation algorithm taking the AWC vs. directional change interplay into consideration, which can be a subject of further research.

9. Conclusions

With respect to the simulation results presented, one can state that in the case of a superior number of control inputs over outputs, provided that the plant is controllable, one can achieve better anti-windup compensation, resulting from changing directions of control vectors. On the contrary, one should rather preserve the information given in the computed control vector for the opposite case, with respect to connections of slight (occasional) directional changes and coupling effects.

Thus a novel decription can be formulated, which covers the cases not depicted in (Walgama and Sternby, 1993) and extends their results to nonsquare systems with input constraints filling the gap in the literature:

Solving the windup phenomenon problem does not ***have to*** *mean that the constrained control vector is of the same direction as the computed control vector* ***if cross-coupling is present in the control system, or if $m < p$ and the plant cannot be perfectly decoupled at all times.***

On the other hand, avoiding directional change in control enables one to avoid the windup phenomenon ***if and only if the plant is perfectly decoupled or is not coupled at all****.* (Which, due to the constraints, is hardly ever met, and usually impossible for $m < p$.)

Such a description of the interplay definitely changes the way one should look at the windup phenomenon and its connection with the directional change problem.

References

Albertos, P. and Sala, A. (2004). *Multivariable Control Systems*, Springer-Verlag, London.

Boyd, S., Ghaoui, L. E., Feron, E. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory, 3rd Edn.*, SIAM, Philadelphia, PA.

Boyd, S. and Vandenberghe, L. (2004). *Convex Optimization*, Cambridge University Press, Cambridge.

Camacho, E. and Bordons, C. (1999). *Model Predictive Control*, Springer-Verlag, London.

Doná, J. D., Goodwin, G. and Seron, M. (2000). Anti-windup and model predictive control: Reflections and connections, *European Journal of Control* **6**(5): 455–465.

Galicki, M. (2005). Control-based solution to inverse kinematics for mobile manipulators using penalty functions, *Journal of Intelligent and Robotic Systems* **42**(3): 213–238.

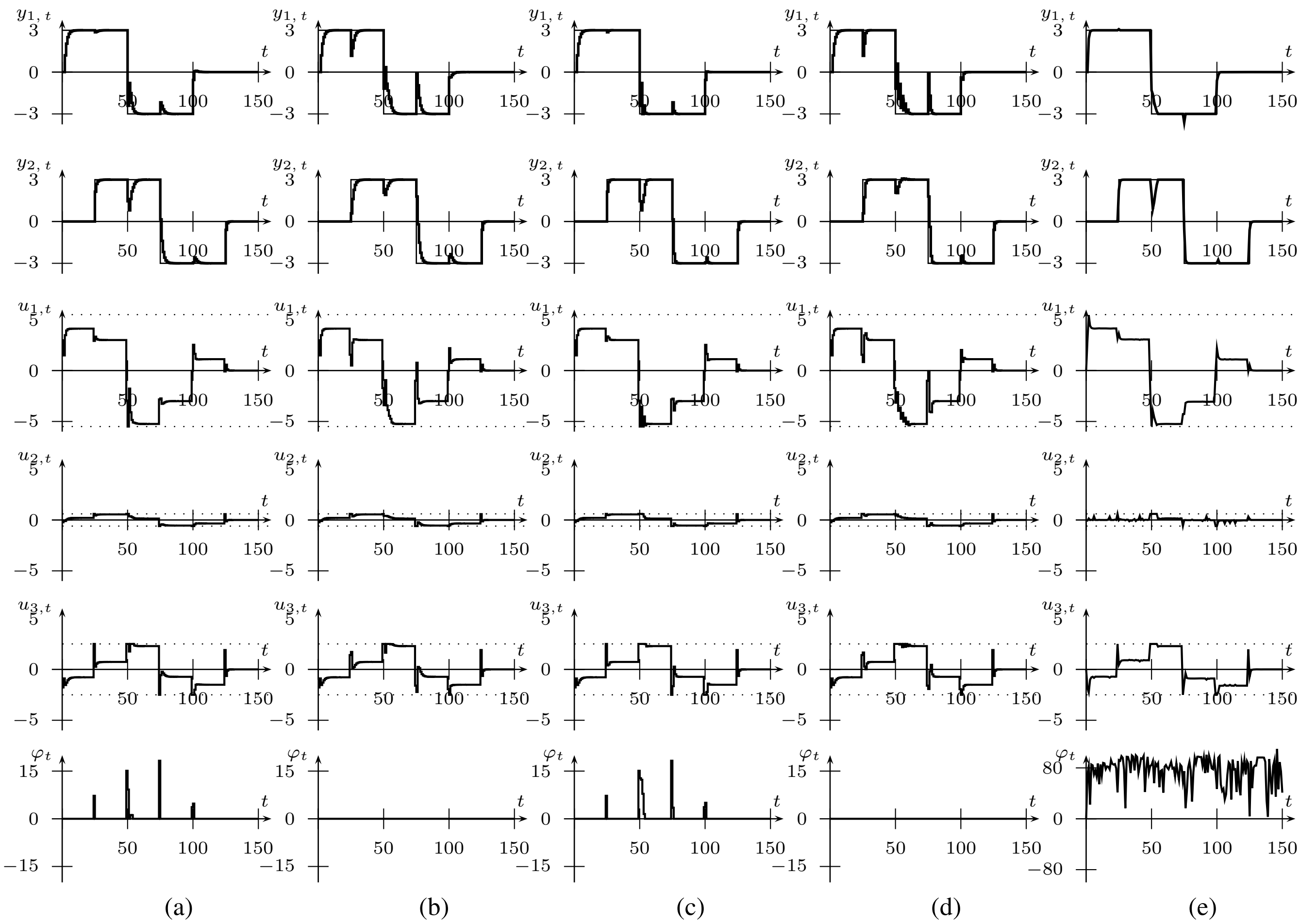


Fig. 6. P2, $\alpha_1 = 5.5$, $\alpha_2 = 0.6$, $\alpha_3 = 2.5$, (a) no AWC, SAT1, (b) no AWC, SAT2, (c) CT-AWC, SAT1, (d) CT-AWC, SAT2, (e) *a priori* AWC.

Hodel, A. and Hall, C. (2001). Variable structure PID control to prevent integrator windup, *IEEE Transactions on Industrial Electronics* **42**(2): 442–451.

Horla, D. (2004). Directional change and anti-windup compensation for multivariable systems, *Studies in Automation and Information Technology* **28/29**: 53–68.

Horla, D. (2006). LMI-based multivariable adaptive predictive controller with anti-windup compensator, *Proceedings of the 12th IEEE International Conference on Methods and Models in Automation and Robotics, Międzyzdroje, Poland*, pp. 459–462.

Horla, D. (2007a). Directional change and windup phenomenon, *Proceedings of the 4th IFAC International Conference on Informatics in Control Automation and Robotics, Angers, France*, (on CD-ROM).

Horla, D. (2007b). Optimised conditioning technique for a priori anti-windup compensation, *Proceedings of the 16th International Conference on Systems Science, Wrocław, Poland*, pp. 132–139.

Hu, T., Teel, A. and Zaccarian, L. (2008). Anti-windup synthesis for linear control systems with input saturation: Achieving regional, nonlinear performance, *Automatica* **44**(2): 512–519.

Hwi-Beon, S. (1998). New anti-windup PI controller for variable-speed motor drives, *IEEE Transactions on Industrial Electronics* **45**(3): 445–450.

Janiszowski, K. (2005). Modification of digital PID controller for application in servo-drives, *International Conference Hydraulics and Pneumatics, Vratna, Slovakia*, pp. 614–624.

Kuo-Kai, S. and Cheng-Yuan, C. (2003). Anti-windup controller design for piezo-electric ceramic linear ultrasonic motor drive, *Proceedings of the 29th Annual Conference of the IEEE Industrial Electronics Society, Roandre, VA, USA*, Vol. 1, pp. 341–346.

Maciejowski, J. (1989). *Multivariable Feedback Design*, Addison-Wesley, Cambridge.

Maciejowski, J. (2002). *Predictive Control with Constraints*, Pearson Education Limited.

Öhr, J. (2003). *Anti-windup and Control of Systems with Multiple Input Saturations: Tools, Solutions and Case Studies*, Ph.D. thesis, Uppsala University, Uppsala.

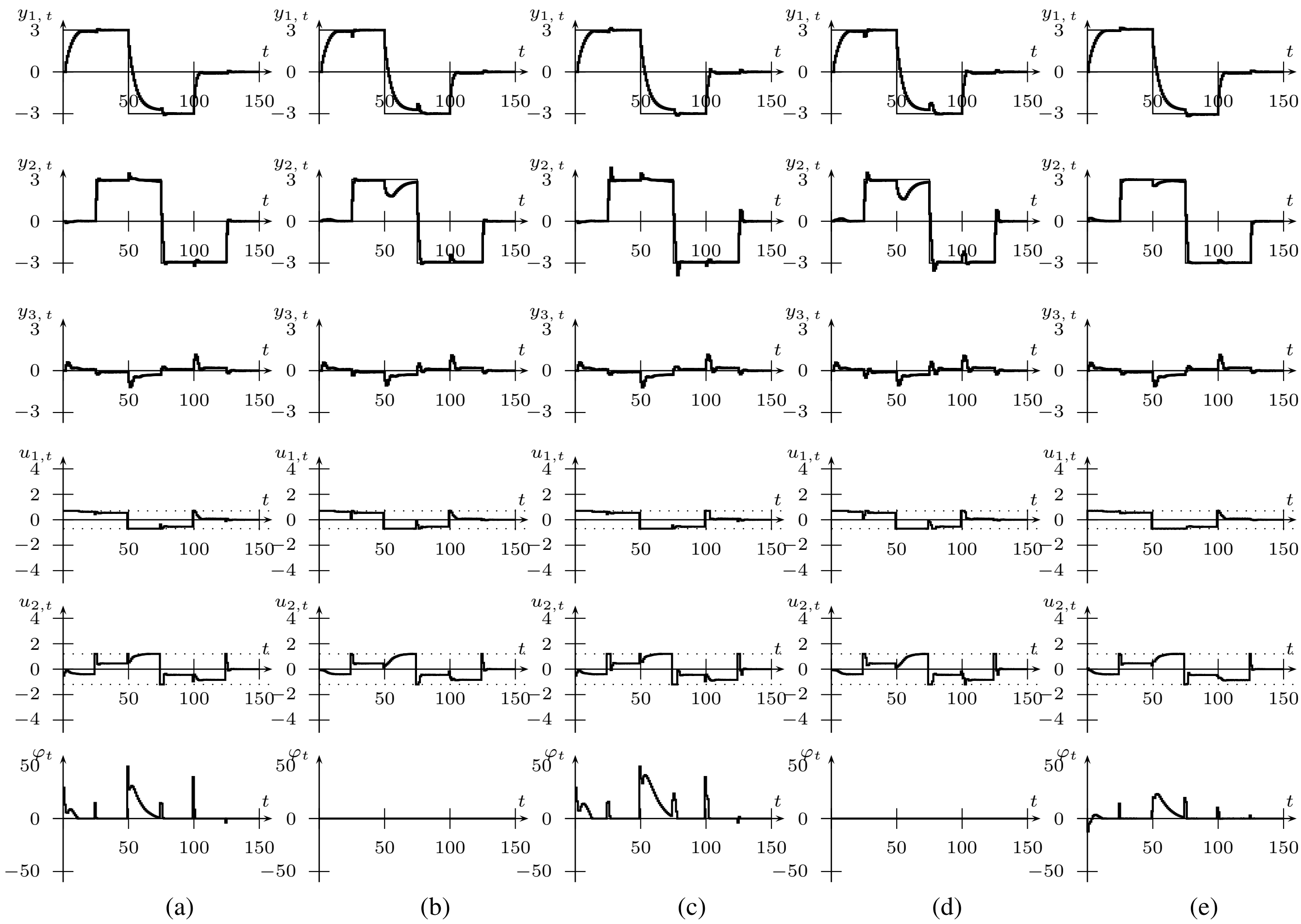


Fig. 7. P3, $\alpha_1 = 0.7$, $\alpha_2 = 1.2$, (a) no AWC, SAT1, (b) no AWC, SAT2, (c) CT-AWC, SAT1, (d) CT-AWC, SAT2, (e) *a priori* AWC

Walgama, K. and Sternby, J. (1993). Conditioning technique for multiinput multioutput processes with input saturation, *IEEE Proceedings—D* **140**(4): 231–241.

Dariusz Horla is with the Institute of Control and Information Engineering of the Poznań University of Technology. He received his M.Sc. degree in control engineering in 2002 and his Ph.D. degree in 2005 from the Faculty of Electrical Engineering of the Poznań University of Technology. His research interests include anti-windup compensation, optimal control, linear matrix inequalities, and adaptive control.

Received: 8 June 2008
Revised: 22 September 2008
Re-revised: 28 January 2009