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STABILITY ANALYSIS AND H_∞ CONTROL OF DISCRETE T–S FUZZY HYPERBOLIC SYSTEMS

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This paper focuses on the problem of constraint control for a class of discrete-time nonlinear systems. Firstly, a new discrete T–S fuzzy hyperbolic model is proposed to represent a class of discrete-time nonlinear systems. By means of the parallel distributed compensation (PDC) method, a novel asymptotic stabilizing control law with the "soft" constraint property is designed. The main advantage is that the proposed control method may achieve a small control amplitude. Secondly, for an uncertain discrete T–S fuzzy hyperbolic system with external disturbances, by the proposed control method, the robust stability and H_∞ performance are developed by using a Lyapunov function, and some sufficient conditions are established through seeking feasible solutions of some linear matrix inequalities (LMIs) to obtain several positive diagonally dominant (PDD) matrices. Finally, the validity and feasibility of the proposed schemes are demonstrated by a numerical example and a Van de Vusse one, and some comparisons of the discrete T–S fuzzy hyperbolic model with the discrete T–S fuzzy linear one are also given to illustrate the advantage of our approach.

Keywords: discrete T–S fuzzy hyperbolic model, parallel distributed compensation (PDC), positive diagonally dominant (PDD) matrices, robust stability.

1. Introduction

The Takagi–Sugeno (T–S) fuzzy model (Takagi and Sugeno 1985) has been a popular choice in modeling and designing a systematic control for nonlinear systems containing uncertain information which cannot be described accurately by mathematical tools. The T–S fuzzy linear model adopts a linear dynamic model as the consequent part of a fuzzy rule, which makes it possible to apply the classical and mature linear systems theory to nonlinear systems. Thus, it becomes one of the more successful methods for studying nonlinear systems.

There have been many research results for it, such as stability analysis, guaranteed-cost and observer-based control designs (Tanaka and Sugeno, 1992; Jadbabaie *et al.*, 1998; Tanaka and Wang, 2001; Fuan and Chen, 2004; Chen and Liu, 2005; Feng, 2006; Kim *et al.*, 2008; Li *et al.*, 2009; Yan *et al.*, 2010; Zhang *et al.*, 2012; Zhao *et al.*, 2013; Tong *et al.*, 2011; 2012; 2014; Siavash and Alireza 2014). Especially, considering the uncertainties of the discrete T–S fuzzy linear model, numerous references have proposed different methods, such as the robust control strategy and the adaptive control approach (Cao and Frank, 2000; Cao *et al.*, 2000; Chen *et al.*, 2000; Tong *et al.*, 2009; 2010; Du, 2012; Qi *et al.*, 2012; Wang, 2014). A piecewise static-output-feedback controller and a piecewise Lyapunov function were designed to make the uncertain closed-loop fuzzy system stochastically stable with guaranteed performance (Qiu *et al.*, 2010). The works of Su *et al.* (2013; 2014), Qiu *et al.* (2009) and Li *et al.* (2011) discussed T–S fuzzy systems with time delay. Although there have been many successful applications for the discrete T–S fuzzy linear system, this model for approximation of nonlinear systems still has its structural limitations.

Considering the advantages of bilinear systems (Mohler, 1973; Elliott, 1999) and T–S fuzzy control, fuzzy control based on the T–S fuzzy bilinear model

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was firstly presented by Li and Tsai (2007), and stability conditions of the system were given via LMIs. Li and Tsai (2008) also presented robust fuzzy controllers for a class of discrete-time T–S fuzzy bilinear systems, in which the parallel distributed compensation method was utilized to design a fuzzy controller to ensure robust asymptotic stability of the closed-loop system and to guarantee an H_∞ norm-bound constraint on disturbance attenuation for all admissible uncertainties. Non-fragile guaranteed cost control was designed for the fuzzy bilinear system (Zhang and Li, 2010; Li and Zhang, 2012). Based on the piecewise quadratic Lyapunov function (PQLF), piecewise fuzzy observer-based controllers were designed for discrete T–S fuzzy bilinear systems with an unavailable state (Li *et al.*, 2013).

From the above discussions, it can be seen that the existing literature has faced extensive discussions on the T–S fuzzy model. However, notice that for practical applications any controller for dynamic systems should be designed such that it guarantees systems stability requiring permissible magnitudes of control inputs (Park *et al.*, 2004). In general, the approaches of constrained control include model predictive control (Bemporad *et al.*, 2003), control with saturation nonlinearity (Zhao and Gao, 2012) and probabilistic control (Datta *et al.*, 2012). Unfortunately, for most real-life problems, these methods often change the constraint control into very complex optimization problems. To tackle this issue, based on the fuzzy hyperbolic model (FHM) (Zhang and Quan, 2001; Zhang, 2009) and the T–S fuzzy one, Chen and Li (2012) established a new T–S model, namely, the T–S fuzzy hyperbolic model for complex continuous-time nonlinear systems. The consequent part of the proposed model is a hyperbolic dynamic model. The advantage of the model over its T–S fuzzy linear counterpart is that the control amplitude is much smaller than for the T–S fuzzy linear model.

Recently, the problems of non-fragile guaranteed cost constraint control for continuous-time T–S fuzzy hyperbolic models have been discussed further (Chen and Li, 2015). However, the control method has not been mentioned in discrete-time control systems. As we know, discrete-time systems have come to play a more important role than their continuous-time counterparts in the digital age, and discrete-time fuzzy-model-based control systems have drawn an increasing research interest. Motivated by the above concerns, we focus on constraint control of discrete-time nonlinear systems. Firstly, a novel discrete T–S fuzzy hyperbolic model for discrete-time nonlinear systems is proposed. Secondly, the PDC control is designed given the local control law $u_j(t) = H_j \tanh(Kx(t))$. By fuzzy blending, the overall fuzzy hyperbolic control law is obtained as $u(t) = \sum_{j=1}^{r} h_j(s(t))H_j \tanh(Kx(t))$, where the range of each component $tanh(k_j x_j(t))$, $j = 1, 2, \dots, r$, in vector $\tanh(Kx(t))$ belongs to $(-1, 1)$. This design approach can deal with the constraint problem via a soft constraint approach. Finally, the robust H_∞ constraint control problem for an uncertain discrete T–S fuzzy hyperbolic system with external disturbance is further investigated.

Section 2 presents a discrete T–S fuzzy hyperbolic model and analyzes the stability of the closed-loop discrete fuzzy system by utilizing the PDC method to design a fuzzy controller. In Section 3, for the problem of discrete nonlinear system with external disturbance, a robust fuzzy controller is designed and a robust H_∞ stability condition is given in terms of LMIs. Section 4 illustrates the effectiveness of the proposed schemes via some simulations. Some conclusions are included in Section 5.

Notation. The notation used throughout this paper is fairly standard, $A > 0$ ($A \geq 0, A \geq 0, A \leq 0$, respectively) means that the matrix A is positive definite (positive semi-definite, negative definite, negative semi-definite, respectively). The identity matrix, which is of appropriate dimensions, will be denoted by I. The superscript "T" stands for the matrix transpose, R^n denotes the n-dimensional Euclidean space. The symbol "*" in a square matrix stands for the transposed elements in the symmetric positions. The shorthand $\text{diag}\{k_1, k_2, \dots, k_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $k_1, k_2, \dots, k_n$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Modeling and stability analysis of a discrete T–S fuzzy hyperbolic model

2.1. Modeling of a discrete T–S fuzzy hyperbolic model. The continuous T–S fuzzy hyperbolic model was firstly presented to represent continuous-time nonlinear systems (Chen and Li, 2012). In this subsection, a discrete T–S fuzzy hyperbolic model will be proposed to represent discrete-time nonlinear systems. This novel fuzzy model is still described by fuzzy "IF-THEN" rules, which express local dynamics in a hyperbolic tangent model. Finally, the overall fuzzy system is obtained by fuzzy, smooth "blending" of the local hyperbolic tangent model. The i-th rule of the discrete T–S fuzzy hyperbolic model is described below:

Plant rule i: If $s_1(t)$ is F_{i1} and … and $s_g(t)$ is F_{ig}, then

$$x(t+1) = A_i \tanh(Kx(t)) + B_i u(t), \quad i \in S = \{1, 2, \dots, r\}, \tag{1}$$

where r is the number of fuzzy rules and F_{ij} is the fuzzy set, $x(t) \in \mathbb{R}^n$ stands for the state vector, and $u(t) \in \mathbb{R}$ signifies the control input, $s(t) = [s_1(t), s_2(t), \dots, s_g(t)] \in \mathbb{R}^s$ are the known premise variables. It is assumed that the premise variables do not depend on the control input $u(t)$ or disturbances $\omega(t)$ in this paper. $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^n$. $\tanh(Kx(t)) = [\tanh(k_1x_1(t)), \dots, \tanh(k_nx_n(t))]$ with

$$\tanh(k_1x_1(t)) = \frac{e^{k_ix_i} - e^{-k_ix_i}}{e^{k_ix_i} + e^{-k_ix_i}}$$

and k_i is a specified constant.

By using the fuzzy inference method with a singleton fuzzifier, product inference and a center average defuzzifier, the overall discrete T–S fuzzy hyperbolic system can be rewritten as

$$x(t+1) = \sum_{i=1}^{r} h_i(s(t))(A_i \tanh(Kx(t)) + B_iu(t)), \quad (2)$$

where

$$h_i(s(t)) = \frac{\mu_i(s(t))}{\sum_{i=1}^{r} \mu_i(s(t))}$$

and

$$\mu_i(s(t)) = \prod_{j=1}^{g} F_{ij}(s_j(t)), \quad i \in S, \quad F_{ij}(s_j(t))$$

is a membership degree of $s_j(t)$ in F_{ij}. In this paper, $\mu_i(s(t))$ are assumed such that $\mu_i(s(t)) \geq 0, i \in S$, and $\sum_{i=1}^{r} \mu_i(s(t)) > 0$ for all t. From the definition of $h_i(s((t))$, we can see that $h_i(s((t)) \geq 0, \quad i \in S$, and $\sum_{i=1}^{r} h_i(s(t)) = 1$. We write $h_i(s(t))$ as h_i for a brief description.

Before presenting the main results of this paper, we introduce some lemmas, which will be used in the sequel.

Lemma 1. (Margaliot and Langholz, 2003) *If a square matrix P is positive diagonally dominant (PDD), then for all $x \neq 0$ the following result holds:*

$$\tanh^T(x(t))P\tanh(x(t)) \leq x^T(t)Px(t).$$

Lemma 2. (Zhang and Li, 2010) *Given any matrices M, N, and a symmetric matrix $P > 0$ with appropriate dimensions, for any real scalar $\varepsilon > 0$, the following inequality holds:*

$$M^TPN + N^TPM \leq \varepsilon M^TPM + \varepsilon^{-1}N^TPN.$$

2.2. Fuzzy controller design and stability analysis.

Based on the parallel distributed compensation (PDC) method (Tanaka and Wang, 2001), the j-th fuzzy controller of the discrete T–S fuzzy hyperbolic system (2) is designed as follows:
Control rule j: If $s_1(t)$ is F_{j1} and ... and $s_g(t)$ is F_{jg}, then

$$u_j(t) = -H_j \tanh(Kx(t)), \quad j \in S = \{1, 2, \dots, r\}, \quad (3)$$

where $H_j \in \mathbb{R}^{1\times n}$ is the controller gain matrix to be determined, $K = \mathrm{diag}(k_1, k_2, \dots, k_n), k_i$ is a positive constant, which has been obtained by system identification.

By using the fuzzy inference method, the overall fuzzy control law is represented by

$$u(t) = -\sum_{j=1}^{r} h_j(s(t))H_j \tanh(Kx(t)). \quad (4)$$

Remark 1. In (4), each component in vector $\tanh(Kx(t))$ is bounded whose range of is $(-1, 1)$, so it is obvious that the fuzzy hyperbolic controller (4) is also bounded. It can be seen intuitively that when the variation range of the x value is very big, the controller (4) has a constraint control property of compressibility, and can achieve a small control amplitude when H_j is a limited value. This advantage will be illustrated by simulation results.

Substituting (4) into (2), the overall closed-loop system can be rewritten as

$$x(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j(A_i - B_iH_j)\tanh(Kx(t)). \quad (5)$$

2.2.1. Main results.

Theorem 1. *Assume that there exist matrixes $P > 0, Z = Z^T$ and some constant matrixes M_i, M_j, such that the following LMIs are satisfied:*

$$\begin{bmatrix} -Y & * \\ A_iY - B_iM_i & -K^{-1}YK^{-T} \end{bmatrix} < 0, \quad 1 \leq i \leq r, \quad (6)$$

$$\begin{bmatrix} -Y & * \\ \frac{A_iY + A_jY - B_iM_j - B_jM_i}{2} & -K^{-1}YK^{-T} \end{bmatrix} < 0, \quad 1 \leq i < j \leq r \quad (7)$$

$$z_{ij} \geq 0, \quad \forall i \neq j, \quad (8)$$

$$y_{ij} + z_{ij} \geq 0, \quad \forall i \neq j, \quad (9)$$

$$y_{ii} - \sum_{i\neq j}(y_{ij} + 2z_{ij}) \geq 0, \quad \forall i. \quad (10)$$

where $Y = P^{-1}, H_i = M_iY^{-1}, H_j = M_jY^{-1}, i, j = 1, 2, \dots, r$. Then the closed-loop system (5) is globally asymptotically stable, *Proof.* Choose the following Lyapunov function candidate for the system (5):

$$V(t) = x^T(t)K^T PKx(t),$$

where K is defined in (5), $P > 0$.

Along the trajectories of the system (5), the corresponding time difference of $V(t)$ is given by

$$\begin{aligned}
&\Delta V \\
&= V(t+1) - V(t) \\
&= x^T(t+1)K^T PKx(t+1) - x^T(t)K^T PKx(t) \\
&= \Big\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i - B_i H_j)\tanh(Kx)\Big\}^T K^T PK \\
&\quad \times \Big\{\sum_{n=1}^{r}\sum_{l=1}^{r} h_n h_l (A_n - B_n H_l)\tanh(Kx)\Big\} \\
&\quad - x^T K^T PKx \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_i h_j h_n h_l \tanh^T(Kx)(A_i - B_i H_j)^T \\
&\quad \times K^T PK(A_n - B_n H_l)\tanh(Kx) - \tanh^T(Kx) \\
&\quad \times P\tanh(Kx) \\
&= \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_i h_j h_n h_l \tanh^T(Kx)(G_{ij} + G_{ji})^T \\
&\quad \times K^T PK(G_{nl} + G_{\ln})\tanh(Kx) \\
&\quad - \tanh^T(Kx) \\
&\quad \times P\tanh(Kx) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \{\tanh^T(Kx)\Big(\frac{G_{ij}+G_{ji}}{2}\Big)^T K^T PK \\
&\quad \times \Big(\frac{G_{ij}+G_{ji}}{2}\Big)\tanh(Kx) \\
&\quad - \tanh^T(Kx)P\tanh(Kx) \\
&= \sum_{i=1}^{r} h_i^2 \tanh^T(Kx)(G_{ii}^T K^T PKG_{ii})\tanh(Kx) \\
&\quad + 2\sum_{i=1}^{r}\sum_{i<j} h_i h_j \tanh^T(Kx)\Big(\frac{G_{ij}+G_{ji}}{2}\Big)^T K^T PK \\
&\quad \times \Big(\frac{G_{ij}+G_{ji}}{2}\Big)\tanh(Kx) - \tanh^T(Kx)P\tanh(Kx),
\end{aligned}$$

where $G_{ij} = A_i - B_i H_j, H_{ij} = G_{ij} + G_{ji}$.

If

$$G_{ii}^T K^T PKG_{ii} - P < 0, \tag{11}$$

$$\Big(\frac{G_{ij}+G_{ji}}{2}\Big)^T K^T PK\Big(\frac{G_{ij}+G_{ji}}{2}\Big) - P < 0, \tag{12}$$

we can obtain $V(t+1) - V(t) < 0$, which implies that the closed-loop system (5) is asymptotically stable at the equilibrium point $x = 0$. Pre-multiplying and post-multiplying (11) and (12) by Y, and applying the Schur complement (Li *et al.*, 2009), we obtain (6) and (7). Moreover, since

$$\begin{aligned}
y_{ii} &\ge \sum_{j\ne i}(y_{ij} + 2z_{ij}) = \sum_{j\ne i}(|y_{ij} + z_{ij}| + |-z_{ij}|) \\
&\ge \sum_{j\ne i}|y_{ij}|,
\end{aligned}$$

the matrix Y is positive diagonally dominant. This completes the proof. ■

3. Robust H_∞ control of the discrete T–S fuzzy hyperbolic model

3.1. Problem formulation and preliminaries. In this section, we will deal with the robust stability and H_∞ control problem of discrete T–S fuzzy hyperbolic systems with external disturbance. The i-th rule of the uncertain discrete T–S fuzzy hyperbolic system is designed as follows:

If $s_1(t)$ is F_{i1} and … and $s_g(t)$ is F_{ig}, then

$$x(t+1) = A_i\tanh(Kx(t)) + B_i u(t) + N_i\omega(t) \qquad i \in S = \{1, 2, \ldots, r\}, \tag{13}$$

where $\omega(t) \in \mathbb{R}^m$ stands for the external disturbance inputs which are assumed to belong to $L_\infty[0,\infty)$, $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^n, N_i \in \mathbb{R}^n$.

By using the fuzzy inference method, the overall uncertain T–S fuzzy hyperbolic system is represented by

$$\begin{aligned}
x(t+1) = \sum_{i=1}^{r} h_i(s(t))[&A_i\tanh(Kx(t)) \\
&+ B_i u(t) + N_i\omega(t)]
\end{aligned} \tag{14}$$

Next, a fuzzy state-feedback controller with a small amplitude will be designed to robustly asymptotically stabilize the discrete T–S fuzzy hyperbolic system (14) and to make this fuzzy system satisfy the H_∞ performance index

$$\begin{aligned}
J = \sum_{t=0}^{\infty}\tanh^T(x(t))\tanh(x(t)) &< x(0)^T Px(0) \\
&+ \gamma^2\sum_{t=0}^{\infty}\omega^T(t)\omega(t),
\end{aligned} \tag{15}$$

where $x(0)$ is the initial value of the state vector, γ represents a prescribed disturbance attenuation constant, $P > 0$ is positive diagonally dominant (PDD) .

Then, substituting (4) into (14), the overall closed-loop system can be rewritten as

$$x(t+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [(A_i - B_i H_j)\tanh(Kx(t) + N_i\omega(t)]. \tag{16}$$

3.2. Main results.

Theorem 2. *Given some scalars ε, ς and $\gamma > 0$, assume that there exist some matrixes $P > 0, Z = Z^T$ and constant matrices M_i, M_j, such that*

$$\Phi_{ij} < 0, \quad 1 \le i, j \le r, \tag{17}$$

$$z_{ij} \ge 0, \quad \forall i \ne j, \tag{18}$$

$$y_{ij} + z_{ij} \ge 0, \quad \forall i \ne j, \tag{19}$$

$$y_{ii} - \sum_{i \ne j}(y_{ij} + 2z_{ij}) \ge 0, \quad \forall i, \tag{20}$$

where

$$\Phi_{ij} = \begin{bmatrix} -Y+I & 0 & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * & * \\ \Xi_1 & 0 & -K^{-1}YK^{-T} & * & * & * & * \\ A_iY - B_iM_j & 0 & 0 & -\varepsilon^{-1}K^{-1}YK^{-T} & * & * & * \\ Y & 0 & 0 & 0 & -I & * & * \\ 0 & N_i & 0 & 0 & 0 & \Xi_2 & * \\ 0 & N_j & 0 & 0 & 0 & 0 & -\frac{\varsigma}{2}K^{-1}YK^{-T} \end{bmatrix},$$

$$\Xi_1 = \frac{A_iY - B_iM_j + A_jY - B_jM_i}{2},$$

$$\Xi_2 = -\Big(\frac{\varsigma}{2} + \varepsilon^{-1}\Big)^{-1} K^{-1}YK^{-T},$$

where $H_i = M_iY^{-1}, H_j = M_jY^{-1}, Y = P^{-1}, i, j = 1, 2, \dots, r$. Then the uncertain discrete closed-loop system (16) is robust asymptotically stable and satisfies H_∞ performance index for all $\omega(t) \in L_2[0, \infty)$.

Proof. Choose the following Lyapunov function candidate for the system (16):

$$V(t) = x^T(t)K^TPKx(t), \tag{21}$$

where K is defined in (14), $P > 0$.

Along the trajectories of the system (16), the corresponding time difference of $V(t)$ is given by

$$\begin{aligned}
&\Delta V \\
&= V(t+1) - V(t) \\
&= x^T(t+1)K^TPKx(t+1) - x^T(t)K^TPKx(t) \\
&= \Big\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j[(A_i - B_iH_j)\tanh(Kx) + N_i\omega(t)]\Big\}^T \\
&\quad \times K^TPK\Big\{\sum_{n=1}^{r}\sum_{l=1}^{r} h_nh_l[(A_n - B_nH_l)\tanh(Kx) \\
&\quad + N_n\omega(t)]\Big\} - x^TK^TPKx \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l[\tanh^T(Kx)(A_i - B_iH_j)^T \\
&\quad + \omega(t)^T {N_i}^T]K^TPK[(A_n - B_nH_l)\tanh(Kx) \\
&\quad + N_n\omega(t)] - \tanh^T(Kx)P\tanh(Kx) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l\Big\{\tanh^T(Kx)(A_i - B_iH_j)^T \\
&\quad \times K^TPK(A_n - B_nH_l)\tanh(Kx) \\
&\quad + \tanh^T(Kx) \\
&\quad \times (A_i - B_iH_j)^TK^TPKN_n\omega(t) + (N_i\omega(t))^T \\
&\quad \times K^TPK(A_n - B_nH_l)\tanh(Kx) + (N_i\omega(t))^T \\
&\quad \times K^TPKN_n\omega(t)\Big\} - \tanh^T(Kx)P\tanh(Kx) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l\Big\{\tanh^T(Kx)(A_i - B_iH_j)^T \\
&\quad \times K^TPK(A_n - B_nH_l)\tanh(Kx)\Big\} \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l\tanh^T(Kx)(A_i - B_iH_j)^T \\
&\quad \times K^TPKN_n\omega(t) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l(N_n\omega(t))^TK^TPK(A_i \\
&\quad - B_iH_j)\tanh(Kx) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l(N_i\omega(t))^TK^TPKN_n\omega(t) \\
&\quad - \tanh^T(Kx)P\tanh(Kx) \\
&= \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l\tanh^T(Kx)(G_{ij} + G_{ji})^T \\
&\quad \times K^TPK(G_{nl} + G_{ln})\tanh(Kx) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l[\varepsilon\tanh^T(Kx)(A_i
\end{aligned}$$

$$
\begin{aligned}
&- B_iH_j)^T K^T PK(A_i - B_iH_j)\tanh(Kx) \\
&+ \varepsilon^{-1}(N_n\omega(t))^T K^T PKN_n\omega(t)] \\
&+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r} h_ih_jh_nh_l(N_i\omega(t))^T K^T PKN_n\omega(t) \\
&-\tanh^T(Kx)P\tanh(Kx) \\
\le &\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\Big\{\tanh^T(Kx)\Big(\frac{G_{ij}+G_{ji}}{2}\Big)^T K^T PK \\
&\times\Big(\frac{G_{ij}+G_{ji}}{2}\Big)\tanh(Kx) + \varepsilon\tanh^T(Kx) \\
&\times(A_i - B_iH_j)^T K^T PK(A_i - B_iH_j)\tanh(Kx) \\
&+\varepsilon^{-1}(N_i\omega(t))^T K^T PKN_i\omega(t) + \frac{\varsigma}{2}(N_i\omega(t))^T \\
&\times K^T PKN_i\omega(t) + \frac{\varsigma^{-1}}{2}(N_j\omega(t))^T K^T PKN_j\omega(t) \\
&-\tanh^T(Kx)P\tanh(Kx) + \tanh^T(Kx)\tanh(Kx) \\
&- \gamma^2\omega^T(t)\omega(t) - \tanh^T(Kx)\tanh(Kx) \\
&+ \gamma^2\omega^T(t)\omega(t) \\
= &\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\eta^T\Omega\eta - \tanh^T(Kx)\tanh(Kx) \\
&+\gamma^2\omega^T(t)\omega(t),
\end{aligned}
$$

where

$$
\begin{aligned}
\eta^T &= \Big[\tanh^T(Kx)\omega^T(t)\Big], \\
G_{ij} &= A_i - B_iH_j,\ H_{ij} = G_{ij} + G_{ji}, \\
\Omega &= \begin{bmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{bmatrix}, \\
\Omega_{11} &= \Big(\frac{G_{ij}+G_{ji}}{2}\Big)^T K^T PK\Big(\frac{G_{ij}+G_{ji}}{2}\Big) \\
&\quad + \varepsilon(A_i - B_iH_j)^T K^T PK(A_i - B_iH_j) - P + I, \\
\Omega_{22} &= \Big(\frac{\varsigma}{2} + \varepsilon^{-1}\Big)N_i{}^T K^T PKN_i + \frac{\varsigma^{-1}}{2}N_j{}^T K^T PKN_j \\
&\quad - \gamma^2 I.
\end{aligned}
$$

If $\Omega < 0$, we can obtain

$$
\begin{aligned}
V(t+1) - V(t) < &-\tanh^T(Kx)\tanh(Kx) \\
&+ \gamma^2\omega^T(t)\omega(t). \qquad (22)
\end{aligned}
$$

Based on the accumulated result of (22) from $t = 0$ to $t = \infty$, we have the following inequality:

$$
\begin{aligned}
V(x(\infty)) - V(x(0)) < &-\sum_{t=0}^{\infty}\tanh^T(Kx)\tanh(Kx) \\
&+ \gamma^2\sum_{t=0}^{\infty}\omega^T(t)\omega(t).
\end{aligned}
$$

That is to say,

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\tanh^T(Kx)\tanh(Kx) \\
&\qquad < V(x(0)) + \gamma^2\sum_{t=0}^{\infty}\omega^T(t)\omega(t). \qquad (23)
\end{aligned}
$$

Thus, the H_∞ performance index is satisfied.

Let $Y = P^{-1}$ and $M_i = H_iY, M_j = H_jY$. Pre- and post-multiplying both the sides of Ω by $\mathrm{diag}\{Y, I\}$, using the Schur complements (Li *et al.*, 2009), we will obtain the LMI (17). Finally, since

$$
\begin{aligned}
y_{ii} &\ge \sum_{j\neq i}(y_{ij} + 2z_{ij}) \\
&= \sum_{j\neq i}(|y_{ij} + z_{ij}| + |-z_{ij}|) \ge \sum_{j\neq i}|y_{ij}|,
\end{aligned}
$$

the matrix Y is positive diagonally dominant. This completes the proof. ■

4. Simulation examples

In this section, a discretization of the Van de Vusse system and a mathematical constructive example will be presented to illustrate the effectiveness of the proposed method. Some comparisons with the results in recent publications are given to clarify the superiority of our approach.

Example 1. Consider the dynamics of an isothermal continuous stirred-tank reactor (CSTR) for the Van de Vusse example (Li *et al.*, 2008) of the following form:

$$
\begin{aligned}
\dot{x}_1 &= F_1(x_1, x_2, u) \\
&= -k_1x_1 - k_3x_1^2 + u(C_{A0} - x_1), \qquad (24) \\
\dot{x}_2 &= F_2(x_1, x_2, u) \\
&= k_1x_1 - k_2x_2 + u(-x_2) \qquad (25) \\
y &= x_2, \qquad (26)
\end{aligned}
$$

where the state x_1 [mol/L] represents the concentration of the reactant inside the reactor, the state x_2[mol/L] is the concentration of the product in the output stream of the CSTR, the output $y = x_2$ determines the grade of the final product, the input-feed stream to the CSTR consists of a reactant with concentration C_{A0} and the controlled input is the dilution rate $u = F/V$ [h^{-1}], F the input flow rate to the reactor [L/h] and V is the constant volume of the CSTR in liters.

In the following, in the system (23)–(25), the parameters are chosen as $k_1 = 5h^{-1}, k_2 = 1h^{-1}, k_3 = 1$ [L/(mol h)], $C_{A0} = 5$ [mol/L], and $V = 1$ [L].

For the study of a discrete T–S fuzzy hyperbolic system, based on the work of (Li *et al.*, 2013), we can obtain the following discrete Van de Vusse model:

$$x_1(t+1) = x_1(t) + T(-k_1x_1(t) - k_3x_1^2(t)) + T(u(t)(C_{A0} - x_1(t))), \tag{27}$$

$$x_2(t+1) = x_2(t) + T(k_1x_1(t) - k_2x_2(t)) + T(u(t)(-x_2(t))), \tag{28}$$

$$y(t) = x_2(t), \tag{29}$$

where $T = 0.05$ ms is the sampling time. Then, some equilibrium points of (26)–(28) are tabulated in Table 1. Under these equilibrium points $[\ x_e \quad u_e\ ]$, which are also chosen as the desired operating points $[\ x_d \quad u_d\ ]$, we can use the T–S fuzzy-model-based modeling method of Hsiao *et al.* (2010) to construct all system matrices when we represent the system (26)–(28) by using the T–S fuzzy linear model (Tanaka and Wang, 2001).

Based on the modeling method by Hsiao *et al.* (2010), we can obtain

$$\begin{aligned} f_1(x) &= T(-k_1x_1(t) - k_3x_1^2(t)),\\ g_1(x) &= T(u(t)(C_{A0} - x_1(t))),\\ f_2(x) &= T(k_1x_1(t) - k_2x_2(t)),\\ g_2(x) &= T(u(t)(-x_2(t))). \end{aligned}$$

Applying the results of Hsiao *et al.* (2010) to the system (26)–(28) at operating points, the system matrices $A_i, B_i, i = 1, 2, 3, 4$ can be obtained. These matrices are the same as those in the T–S fuzzy linear model when we represent the system (26)–(28) by utilizing T–S fuzzy hyperbolic model. Accordingly, we can obtain the following discrete fuzzy hyperbolic control laws:

R^1: If x_1 is about 0.6835, then

$$\begin{aligned} x_\delta(t+1) &= A_1 \tanh(Kx_\delta(t)) + B_1u_\delta(t),\\ u_\delta(t) &= H_1 \tanh(Kx_\delta(t)). \end{aligned}$$

R^2: If x_1 is about 1.1343, then

$$\begin{aligned} x_\delta(t+1) &= A_2 \tanh(Kx_\delta(t)) + B_2u_\delta(t),\\ u_\delta(t) &= H_2 \tanh(Kx_\delta(t)). \end{aligned}$$

Table 1. Data for equilibrium points.

x_{e1}	x_{e2}	u_e
0.6835	1.7987	0.9
1.1343	2.0256	1.8
1.2949	2.0233	2.2
2.0711	1.7259	5

R^3: If x_3 is about 1.2949, then

$$\begin{aligned} x_\delta(t+1) &= A_3 \tanh(Kx_\delta(t)) + B_3u_\delta(t),\\ u_\delta(t) &= H_3 \tanh(Kx_\delta(t)). \end{aligned}$$

R^4: If x_4 is about 2.0711, then

$$\begin{aligned} x_\delta(t+1) &= A_4 \tanh(Kx_\delta(t)) + B_4u_\delta(t),\\ u_\delta(t) &= H_4 \tanh(Kx_\delta(t)), \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0.2509 & -1.0151 \\ 0.2649 & 0.9443 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0673 & -1.0963 \\ 0.2841 & 0.9208 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.2296 & -1.1564 \\ 0.2999 & 0.9180 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -1.5134 & -1.5052 \\ 0.3730 & 0.8025 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2158 \\ -0.0899 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1933 \\ -0.1013 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.1853 \\ -0.1012 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.1464 \\ -0.0863 \end{bmatrix},$$

$$x_\delta(t) = x(t) - x_d, \quad u_\delta(t) = u(t) - u_d.$$

By solving the LMIs (6)–(10), the positive diagonally dominant matrix can be calculated as

$$P = \begin{bmatrix} 0.6741 & 0 \\ 0 & 0.6114 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0 & 0.0100 \\ 0.0100 & 0 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.9716 & -8.4223 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 2.2301 & -9.2796 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} -2.5465 & -10.7967 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} -11.0728 & -14.7438 \end{bmatrix},$$

and the following controller gain matrices are obtained:

$$H_1 = \begin{bmatrix} 0.6550 & -5.1495 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 1.5034 & -5.6737 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} -1.7116 & -6.6013 \end{bmatrix},$$

$$H_4 = \left[\begin{array}{cc} -7.4644 & -9.0146 \end{array} \right].$$

Based on the stable fuzzy controller design approach for a discrete T–S fuzzy linear system (Tanaka and Wang, 2001), we can figure out the positive symmetric matrix

$$P = \left[\begin{array}{cc} 0.0512 & 0.0034 \\ 0.0034 & 0.0509 \end{array} \right]$$

and controller gain matrices:

$$F_1 = \left[\begin{array}{cc} 0.5169 & -5.5961 \end{array} \right],$$

$$F_2 = \left[\begin{array}{cc} -0.2418 & -6.2432 \end{array} \right],$$

$$F_3 = \left[\begin{array}{cc} -1.7310 & -6.8761 \end{array} \right],$$

$$F_4 = \left[\begin{array}{cc} -6.8668 & -9.3380 \end{array} \right].$$

The membership function of state x_1 is shown in Fig. 1, and

$$\tanh(Kx) = \left[\begin{array}{cc} \tanh(0.5x_1) & \tanh(0.7x_2) \end{array} \right]^T.$$

Thus, the whole discrete fuzzy hyperbolic control law is

$$u = (h_1H_1 + h_2H_2 + h_3H_3 + h_4H_4) \times \tanh(Kx_\delta)u_d, \quad (30)$$

where h_1, h_2 and h_3 satisfy $h_1 + h_2 + h_3 = 1$.

To illustrate the advantage of the proposed control, here, the system response curves under the conditions of different initial values are studied. Figures 2–5 denote respectively the simulation results of applying the discrete fuzzy hyperbolic controller (29) and the discrete fuzzy linear controller (Tanaka and Wang, 2001) to the discrete Van de Vusse model (26)–(28), with the operating point $x_d^T = [\ 1.2949 \quad 2.0233\ ]$ and $u_d = 2.2000$ under the initial conditions $x(0) = [\ 1 \quad 0.55\ ]^T$ and $x(0) = [\ 15 \quad -2.5\ ]^T$.

From these simulations, we can find that the state of the discrete nonlinear system (26)–(28) under the discrete fuzzy hyperbolic controller (29) can converge to the operating point faster than that under the discrete fuzzy linear controller (Tanaka and Wang, 2004). Furthermore, the amplitude of the fuzzy hyperbolic controller is also smaller than that of the fuzzy linear controller.

♦

Example 2. An uncertain discrete T–S fuzzy hyperbolic model is given as

R^i: If x_i is $L_i, i = 1, 2, 3$, then

$$x(t + 1) = A_i \tanh(Kx(t)) + B_iu(t) + N_i\omega(t),$$
$$u = -H_i \tanh(Kx(t)),$$

where

$$A_1 = \left[\begin{array}{cc} 0.09 & -0.19 \\ 0.07 & -0.24 \end{array} \right],$$

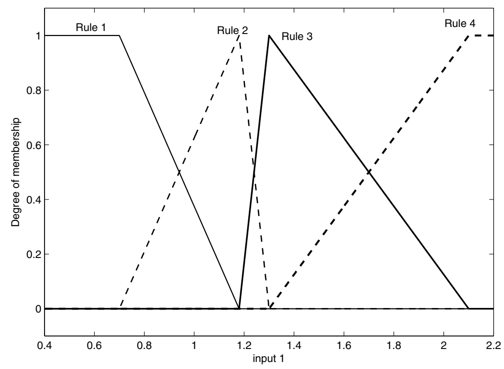

Fig. 1. Membership functions of x_1.

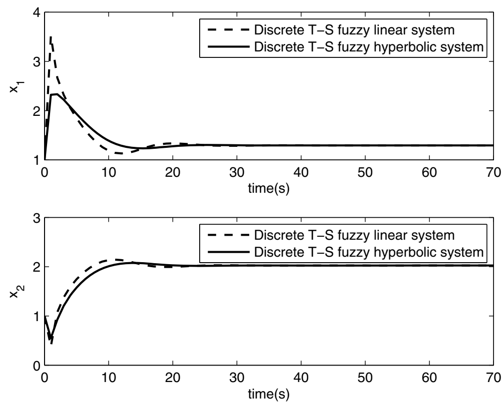

Fig. 2. State response curves and comparisons under the initial condition $x(0) = [1 \ \ 0.55]$. (DTSFLS: T–S fuzzy linear system, DTSFHS: T–S fuzzy hyperbolic system).

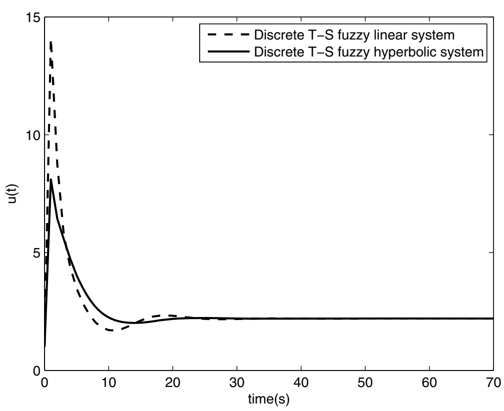

Fig. 3. Control curves and comparisons under the initial condition $x(0) = [1 \ \ 0.55]$.

$$A_2 = \begin{bmatrix} -0.07 & 0.28 \\ -0.18 & 0.15 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.17 & -0.05 \\ 0.6 & -0.06 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.02 \\ 0.04 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.03 \\ 0.05 \end{bmatrix}, \quad N_3 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}.$$

Let γ=2, ε=0.25, ς=4. By solving the LMI (17) in Theorem 2, the positive diagonally dominant matrix and controller gain matrices are obtained as

$$P = \begin{bmatrix} 94.4987 & -0.0622 \\ -0.0622 & 391.1528 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0 & 0.001 \\ 0.001 & 0 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.0487 & -0.0169 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.0100 & 0.0115 \end{bmatrix},$$
$$M_3 = \begin{bmatrix} 0.1632 & -0.0045 \end{bmatrix},$$

and the controller gain matrices are

$$H_1 = \begin{bmatrix} 4.6030 & -6.6193 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} 0.9440 & 4.4871 \end{bmatrix},$$
$$H_3 = \begin{bmatrix} 15.4247 & -1.7716 \end{bmatrix}.$$

Let $\tanh(Kx) = \begin{bmatrix} \tanh(0.1x_1) & \tanh(0.2x_2) \end{bmatrix}$, and choose membership functions and external disturbance as follows:

$$\mu L_1 = \frac{1}{15(1 + 5\exp^2(-x_1))},$$

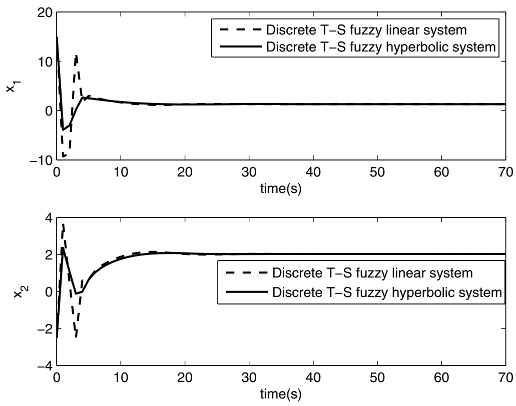

Fig. 4. State response curves and comparisons under the initial condition $x(0) = [15 \quad -2.5]$.

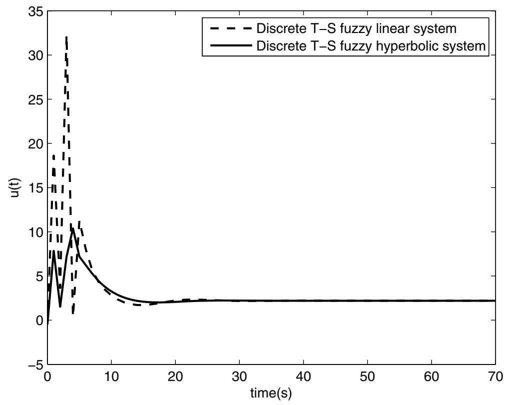

Fig. 5. Control curves and comparisons under the initial condition $x(0) = [15 \quad -2.5]$.

$$\mu L_2 = \frac{1}{15(1 + 2\exp^2(-x_1))},$$

$$\mu L_3 = 1 - \mu L_1 - \mu L_2.$$

and $\omega(t) = 2\sin(10t)\exp(-0.5t)$. In order to get a better comparative result, here, let the two systems have the same $A_i, B_i, N_i, i = 1, 2, 3$, initial conditions, membership functions and external disturbances. Two different the initial values are used to illustrate the advantage of the proposed control; the initial conditions are respectively

$$x(0) = \begin{bmatrix} 4 & -1 \end{bmatrix}^T$$

and

$$x(0) = \begin{bmatrix} -3 & 10 \end{bmatrix}^T.$$

For different initial conditions, the simulation results of the comparisons between the discrete T–S fuzzy hyperbolic system and the discrete T–S fuzzy linear system are shown respectively in Figs. 6–9.

The simulation results show that the uncertain discrete closed-loop T–S fuzzy hyperbolic system (16) is robust asymptotically stable and satisfies the H_∞ performance index (15). Moreover, the uncertain discrete T–S fuzzy hyperbolic system has a smaller control amplitude than the uncertain discrete T–S fuzzy linear system. ♦

Remark 2. For different initial conditions, many comparisons are done. From Figs. 3–9, it is clearly seen that the controller design approach to the fuzzy hyperbolic controller based on a T–S fuzzy hyperbolic model needs a much smaller control amplitude than a T–S fuzzy linear model. This illustrated effectively the advantages of the proposed method.

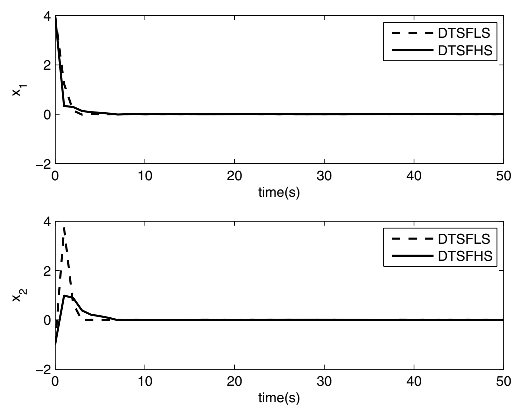

Fig. 6. State response curves and comparisons under the initial condition $x(0) = [4 \ \ -1]$.

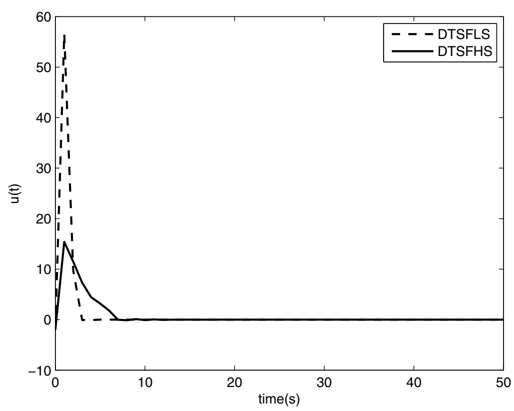

Fig. 7. Control curves and comparisons under the initial condition $x(0) = [4 \ \ -1]$.

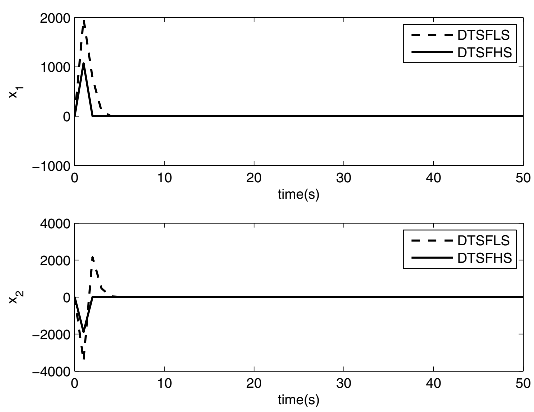

Fig. 8. State response curves and comparisons under the initial condition $x(0) = [-3 \ \ 10]$.

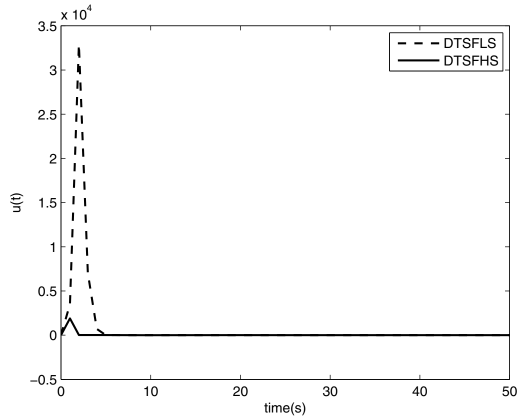

Fig. 9. Control curves and comparisons under the initial condition $x(0) = [-3 \ \ 10]$.

5. Conclusion

In this paper, a discrete T–S fuzzy hyperbolic model for a class of discrete nonlinear systems was proposed. The parallel distributed compensation method was utilized to design a fuzzy hyperbolic controller. Sufficient conditions for the asymptotic stability of the closed-loop system were formulated by LMIs. In addition, for the discrete T–S fuzzy hyperbolic system with external disturbance, the global robust stability and H_∞ performance were developed by designing a robust H_∞ constraint controller. Finally, we presented some simulation examples to illustrate the validity and feasibility of the proposed schemes. From these simulation results, the control input requirements of the discrete T–S fuzzy hyperbolic system are much lower than for the discrete T–S fuzzy linear system, while the state stabilization time of the two systems is almost the same. In a future work, based on fuzzy piecewise Lyapunov functions, the proposed hyperbolic controller can be employed to control discrete-time nonlinear systems with time delays.

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References

Bemporad, A., Borrelli, F. and Morari, M. (2003). Min-max control of constrained uncertain discrete-time linear system, *IEEE Transactions on Automatic Control* **48**(9): 1600–1606.

Cao, S.G., Rees, N.W., Feng, G. and Liu, W. (2000). H_∞ control of nonlinear discrete-time systems based on dynamical fuzzy models, *International Journal of System Science* **31**(31): 229–241.

Cao, Y.Y. and Frank, P.M. (2000). Robust H_∞ disturbance attenuation for a class of uncertain discrete-time fuzzy systems, *IEEE Transactions on Fuzzy Systems* **8**(4): 406–415.

Chen, B. and Liu, X.P. (2005). Delay-dependent robust H_∞ control for TCS fuzzy systems with time delay, *IEEE Transactions on Fuzzy Systems* **13**(4): 544–556.

Chen, B.-S., Tseng, C.-S. and Uang, H.-J. (2000). Mixed H_2/H_∞ fuzzy output feedback control design for nonlinear dynamic systems: An LMI approach, *IEEE Transactions on Fuzzy Systems* **8**(3): 249–265.

Chen, M.L. and Li, J.M. (2012). Modeling and control of T–S fuzzy hyperbolic model for a class of nonlinear systems, *International Conference on Modelling, Identification and Control, Wuhan, China*, pp. 57–62.

Chen, M.L. and Li, J.M. (2015). Non-fragile guaranteed cost control for Takagi–Sugeno fuzzy hyperbolic systems, *International Journal of System Science* **46**(9): 1614-1627.

Datta, R., Bittermann, M.S., Deb, K. and Ciftcioglu, O. (2012). Probabilistic constraint handling in the framework of joint evolutionary-classical optimization with engineering application, *IEEE Congress on Evolutionary Computation, Brisbane, Australia*, pp. 1–8.

Du, D.S. (2012). Reliable H_∞ control for Takagi–Sugeno fuzzy systems with intermittent measurements, *Nonlinear Analysis: Hybrid Systems* **6**(4): 930-941.

Elliott, D.L. (1999). Bilinear systems, *Encyclopedia of Electrical Engineering*, Wiley, New York, NY.

Feng, G. (2006). A survey on analysis and design of model-based fuzzy control systems, *IEEE Transactions on Fuzzy Systems* **14**(5): 676-697.

Guan, X. and Chen, C. (2004). Delay-dependent guaranteed cost control for T–S fuzzy system with time delays, *IEEE Transactions on Fuzzy Systems* **12**(2): 236-249.

Hsiao, M.Y., Liu, C.H., Tsai, S.H., Chen, P.S. and Chen, T.T. (2010). A Takagi–Sugeno fuzzy-model-based modeling method, *IEEE International Conference on Fuzzy Systems, Barcelona, Spain*, pp. 1–6.

Jadbabaie, A., Jamshidi, M. and Titli, A. (1998). Guaranteed-cost design of continuous-time Takagi–Sugeno fuzzy controllers via linear matrix inequalities, *IEEE World Congress on Computational Intelligence, Anchorage, AK, USA*, Vol. 1, pp. 268–273.

Kim, S.H., Lee, C.H. and Park, P.G. (2008). Relaxed delay-dependent stabilization conditions for discrete-time fuzzy systems with time delays, *IEEE 10th International Conference on Control, Automation, Robotics and Vision, Hanoi, Vietnam*, pp. 999–1004.

Li, J.M., Li, J., and Du, C.X. (2009). *Linear Control System Theory and Methods*, Xidian University Press, Xian, pp. 10–13.

Li, J.R., Li, J.M. and Xia, Z.L. (2011). Delay-dependent generalized H_2 control for discrete T–S fuzzy large-scale stochastic systems with mixed delays, *International Journal of Applied Mathematics and Computer Science* **21**(4): 583–603, DOI: 10.2478/v10006-011-0046-6.

Li, J.R., Li, J.M. and Xia, Z.L. (2013a). Observer-based fuzzy control design for discrete-time T-S fuzzy bilinear systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **21**(3): 435–454.

Li, J.R., Li, J.M. and Xia, Z.L. (2013b). Delay-dependent generalized H_2 fuzzy static-output-feedback control for discrete T–S fuzzy bilinear stochastic systems with mixed delays, *Journal of Intelligent and Fuzzy Systems Applications in Engineering and Technology* **25**(4): 863–880.

Li, J.M. and Zhang, G. (2012). Non-fragile guaranteed cost control of T–S fuzzy time-varying state and control delays systems with local bilinear models, *IEEE Transactions on Systems, Man and Cybernetics* **9**(2): 43–62.

Li, T.H.S. and Tsai, S.H. (2007). T–S fuzzy bilinear model and fuzzy controller design for a class of nonlinear systems, *IEEE Transactions on Fuzzy Systems* **15**(3): 494–506.

Li, T.H.S. and Tsai, S.H. (2008). Robust H_∞ fuzzy control for a class of uncertain discrete fuzzy bilinear systems, *IEEE Transactions on Systems, Man and Cybernetics B: Cybernetics* **38**(2): 510–527.

Li, T.H.S., Tsai, S.H., Lee, J.Z., Hsiao, M.Y. and Chao, C.H. (2008). Robust H_∞ fuzzy control for a class of uncertain discrete fuzzy bilinear systems, *IEEE Transactions on Systems, Man and Cybernetics B: Cybernetics* **38**(2): 510–527.

Margaliot, M. and Langholz, G. (2003). A new approach to fuzzy modeling and control of discrete-time systems, *IEEE Transactions on Fuzzy Systems* **11**(4): 486–494.

Mohler, R.R. (1973). *Bilinear Control Processes*, Academic Press, New York, NY.

Park, Y., Tahk, M.J. and Bang, H. (2004). Design and analysis of optimal controller for fuzzy systems with input constraint *IEEE Transactions on Fuzzy System* **12**(6): 766–779.

Qi, R.Y., Tao, G., Jiang, B. and Tan, C. (2012). Adaptive control schemes for discrete-time T–S fuzzy systems with unknown parameters and actuator failures, *IEEE Transactions on Fuzzy Systems* **20**(3): 471–486.

Qiu, J.B., Feng G. and Yang J. (2009). A new design of delay-dependent robust H_∞ filtering for discrete-time T–S fuzzy systems with time-varying delay, *IEEE Transactions on Fuzzy Systems* **17**(5): 1044-1058.

Qiu, J.B., Feng G. and Gao H.J. (2010). Fuzzy-model-based piecewise H_∞ static-output-feedback controller design for networked nonlinear systems, *IEEE Transactions on Fuzzy Systems* **18**(5): 919-934.

Siavash, F.D. and Alireza, F. (2014). Non-monotonic Lyapunov functions for stability analysis and stabilization of discrete time Takagi–Sugeno fuzzy systems, *International Journal of Innovative Computing, Information and Control* **10**(4): 1567–1586.

Su, X.J., Shi P.G., Wu L.G. and Song Y.-D. (2013). A novel control design on discrete-time Takagi–Sugeno fuzzy systems with time-varying delays, *IEEE Transactions on Fuzzy Systems* **21**(4): 655–671.

Su, X. J., Shi P., Wu L. and Basin M.V. (2014). Reliable filtering with strict dissipativity for T–S fuzzy time-delay systems, *IEEE Transactions on Cybernetics* **44**(12): 2470–2483, DOI: 10.1109/TCYB.2014.2308983.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems, Man and Cybernetics* **15**(1): 116–132.

Tanaka, K. and Sugeno, M. (1992). Stability analysis and design of fuzzy control systems, *Fuzzy Sets and Systems* **45**(2): 135–156.

Tanaka, K. and Wang, H.O. (2001). *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, NY.

Tong, S.C., He, X.L. and Zhang, H.C. (2009). A combined backstepping and small-gain approach to robust adaptive fuzzy output feedback control, *IEEE Transactions on Fuzzy Systems* **17**(5): 1059–1069.

Tong, S.C., Huo, B.Y. and Li, Y.M. (2014). Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures, *IEEE Transactions on Fuzzy Systems* **22**(1): 1–15.

Tong, S.C., Liu, C.L. and Li, Y.M. (2010). Fuzzy-adaptive decentralized output-feedback control for large-scale nonlinear systems with dynamical uncertainties, *IEEE Transactions on Fuzzy Systems* **18**(5): 845–861.

Tong, S.C. and Li, Y.M. (2012). Adaptive fuzzy output feedback tracking backstepping control of strict-feedback nonlinear systems with unknown dead zones, *IEEE Transactions on Fuzzy Systems* **20**(1): 168–180.

Tong, S., Yang, G. and Zhang, W. (2011). Observer-based fault-tolerant control against sensor failures for fuzzy systems with time delays, *International Journal of Applied Mathematics and Computer Science* **21**(4): 617–627, DOI: 10.2478/v10006-011-0048-4.

Wang, J. (2014). Adaptive fuzzy control of direct-current motor dead-zone systems, *International Journal of Innovative Computing, Information and Control* **10**(4): 1391–1399.

Yan, H.C., Zhang, H., Shi, H.B. and Meng, M.Q.-H. (2010). H_∞ fuzzy filtering for discrete-time fuzzy stochastic systems with time-varying delay, *IEEE 29th Chinese Control Conference, Beijing, China*, pp. 59993–59998.

Zhang, G. and Li, J.M. (2010). Non-fragile guaranteed cost control of discrete-time fuzzy bilinear system, *Journal of Systems Engineering and Electronics* **21**(4): 629–634.

Zhang, H.G. (2009). *Fuzzy Hyperbolic Model: Modeling Control and Applications*, Science Press, Beijing, pp. 121–131.

Zhang, H.G. and Quan, Y.B. (2001). Modeling, identification and control of a class of nonlinear system, *IEEE Transactions on Fuzzy Systems* **9**(2): 349-354.

Zhang, H., Shi, Y., and Mehr, A.S. (2012). On filtering for discrete-time Takagi–Sugeno fuzzy systems, *IEEE Transactions on Fuzzy Systems* **20**(2): 396–401.

Zhao, Y., and Gao, H.J. (2012). Fuzzy-model-based control of an overhead crane with input delay and actuator saturation *Transactions on Fuzzy Systems* **20**(1): 181–186.

Zhao, T., Xiao, J., Li, Y. and Li, Y.X. (2013). A fuzzy Lyapunov function approach to stabilization of interval type-2 T–S fuzzy systems, *IEEE 25th Chinese Control and Decision Conference, Xian, China*, pp. 2234-2238.

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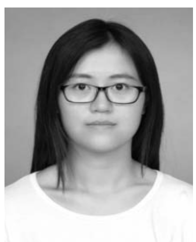

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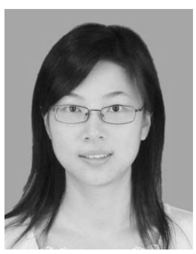

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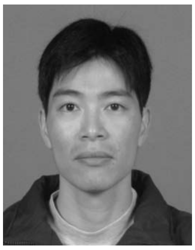

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