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CONTROLLABILITY AND RECONSTRUCTABILITY OF A SYSTEM DESCRIBED BY THE N–D ROESSER MODEL

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The controllability and reconstructability (global) of the system described by a digital N-D Roesser model are defined. Then, necessary and sufficient conditions for system controllability and reconstructability are given. The conditions constitute a generalization of the corresponding conditions for 1-D systems.

Keywords: N-D systems, Roesser model, controllability, reconstructability

1. Introduction

Multidimensional systems have found many applications in different fields: digital data filtering, image processing (Roesser, 1975), modelling of partial differential equations (Kaczorek, 1985; Marszalek, 1984), etc. The most popular models for multidimensional systems were proposed by Attasi (1973), Fornasini and Marchesini (1978), and Roesser (1975). A generalization of these models was presented in (Kurek, 1985).

In this paper we deal with N-dimensional (N-D) linear digital systems described by the Roesser model (N-DRM), cf. (Roesser, 1975). We will consider the controllability and reconstructability of the system. In the paper (Roesser, 1975) the observability and controllability of the 2-DRM were considered. Unfortunately, these notions were only local and therefore they are not very important (Kung *et al.*, 1977). More important and more interesting issues are global properties of the system.

The paper is organized as follows. In Section 2 the model is presented. In Section 3 the system's controllability is defined. Then necessary and sufficient conditions for the global controllability of the system are proven. In Section 4 the notion of reconstructability is proposed. Next, necessary and sufficient conditions for the global reconstructability of the system are given and a numerical example is presented. Finally, concluding remarks are given.

2. Model

The N-DRM is described by the following equation:

$$\begin{bmatrix} x_1(k_1+1, k_2, \dots, k_N) \\ \vdots \\ x_N(k_1, k_2, \dots, k_N+1) \end{bmatrix} = Ax(k) + Bu(k),$$

$$y(k) = Cx(k), \tag{1}$$

where $x \in \mathbb{R}^n$ is a local (l) state vector, $x_i \in \mathbb{R}^{n_i}$, $i = 1, 2, \dots, N$, and $n_1 + \dots + n_N = n$, $u \in \mathbb{R}^m$ is an input vector, $y \in \mathbb{R}^p$ is an output vector and

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ \vdots \\ B_N \end{bmatrix},$$

$$C = \begin{bmatrix} C_1 & \dots & C_N \end{bmatrix}$$

are real matrices of appropriate dimensions.

The (standard) boundary condition set (BCS) for the model is defined as follows (Roesser, 1975):

$$\mathrm{BCS}(0, \dots, 0): \quad \big\{x_i(k_1, \dots, k_{i-1}, 0, k_{i+1}, \dots, k_N) = x_{i0}$$
$$\text{for } k_1, \dots, k_{i-1}, k_{i+1}, \dots, k_N = 0, 1, \dots$$
$$\text{and } i = 1, \dots, N\big\}. \tag{2}$$

For abbreviation, we will use the following notation: $(k_1,\dots,k_N) \ge (h_1,\dots,h_N)$ *if and only if* $k_i \ge h_i$ for $i = 1,2,\dots,N$, and $(k_1,\dots,k_N) \ngeq (h_1,\dots,h_N)$ *if and only if* for some i we have $k_i < h_i$. Moreover, we denote by I_q the $q \times q$ identity matrix and then $I(z_1,\dots,z_N) =$ block diag $(z_1 I_{n_1},\dots,z_N I_{n_N})$.

3. Controllability

Following (Bisiacco, 1985; Kurek, 1990), we define the controllability of the system (1) as follows.

Definition 1. The N-DRM (1) is (globally) controllable *if and only if* for any $x(0,\dots,0) = x_0 \in \mathbb{R}^n$ and BCS $(0,\dots,0)$ equal to zero there exist $(k_1,\dots,k_N) > 0$ and an input sequence $\{u(i_1\dots,i_N),(i_1,\dots,i_N) \ge 0\}$ such that $x(t_1,\dots,t_N) = 0$ for $(t_1,\dots,t_N) \nless (k_1,\dots,k_N)$.

Remark 1. Clearly, for a (globally) controllable N-DRM a proper input sequence can be calculated such that $x(i_1,\dots,i_N) = 0$ for $(t_1,\dots,t_N) \nleq (k_1,\dots,k_N)$.

Remark 2. For every initial state

$$x_{0j} = \begin{bmatrix} 0 & \dots & 1_j & \dots & 0 \end{bmatrix}^T, \quad j = 1,\dots,n$$

there is an N-set $(k_{1j},\dots,k_{Nj})$ such that $x(i_1,\dots,i_N) = 0$ for $(t_1,\dots,t_N) \nleq (k_{1j},\dots,k_{Nj})$. Therefore, because of the system linearity and the superposition principle, there is always a common N-set $(k_{1c},\dots,k_{Nc})$ such that for any $x_0 \in \mathbb{R}^n$ we have $x(t_1,\dots,t_N) = 0$ for $(t_1,\dots,t_N) \nless (k_{1c},\dots,k_{Nc})$ for a (globally) controllable N-DRM.

Let us compare the above definition with the property of l-controllability given by Roesser (1975).

Definition 2. (Roesser, 1975) The N-DRM (1) is (locally) controllable *if and only if* for any $x(0,\dots,0) = x_0 \in \mathbb{R}_n$ and BCS $(0,\dots,0)$ equal to zero there exist $(k_1,\dots,k_N) > 0$ and an appropriate input sequence $\{u(i_1,\dots,i_N),\ (i_1,\dots,i_N) \ge 0\}$ such that $x(k_1,\dots,k_N) = 0$.

The difference between both notions is quite easy to see. Whereas the latter is really a local property, the former is rather a global one. Indeed, since the N-DRM is linear and causal, it is obvious that (global) controllability implies that for any BCS $(0,\dots,0)$ there exist $(k_{1c},\dots,k_{Nc}) > 0$ and an input sequence $\{u(i_1,\dots,i_N),\ (i_1,\dots,i_N) \ge 0\}$ such that BCS $(k_{1c},\dots,k_{Nc})$ is equal to zero, i.e. the system is controllable in Kalman's sense of controllability. However, the difference between the proposed notion of controllability and that Kalman's is caused by the fact that the BCS cannot be considered a global state of the N-DRM, whereas Kalman's concept of controllability refers to the system state. As a matter of fact, Definition 1 gives the notion of the N-DRM controllability with respect to the BCS. However, it is clear that an N-DRM which is controllable according to Definition 1 is controllable with respect to any other BCS.

Theorem 1. *The N-DRM (1) is controllable* if and only if

$$\operatorname{rank}\big[I - AI(z_1,\dots,z_N)B\big] = n \tag{3}$$

for $z_1,\dots,z_N \in \mathbb{C}$.

Proof. For the N-DRM (1) with the BCS $(0,\dots,0)$ equal to zero except for $x(0,\dots,0) = x_0$ we obtain, after the N-D Z transformation,

$$\begin{aligned} I(z_1,\dots,z_N)&\big[X(z_1,\dots,z_N) - x_0\big] \\ &= AX(z_1,\dots,z_N) + BU(z_1,\dots,z_N), \end{aligned} \tag{4}$$

where

$$\begin{aligned} X(z_1,\dots,z_N) &= Z\big\{x(k_1,\dots,k_N)\big\} \\ &= \sum_{i_1=0}^{\infty}\cdots\sum_{i_N=0}^{\infty} x(i_1,\dots,i_N) z_1^{-i_1}\dots z_N^{-i_N}. \end{aligned} \tag{5}$$

Since $x_1(0,k_2,\dots,k_N) = 0$ for $(k_2,\dots,k_N) \nleq 0$, we can write

$$X_1(z_1,\dots,z_N) = x_{10} + z_1^{-1}\tilde{X}_1(z_1,\dots,z_N), \tag{6}$$

where

$$\begin{aligned} &\tilde{X}_1(z_1,\dots,z_N) \\ &\quad = \sum_{i_1=0}^{\infty}\cdots\sum_{i_N=0}^{\infty} x_1(i_1+1,i_2,\dots,i_N) z_1^{-i_1}\dots z_N^{-i_N}. \end{aligned}$$

Similarly, this occurs for $X_i(z_1,\dots,z_N)$, $i = 2,3,\dots,N$. Thus

$$\begin{aligned} &X(z_1,\dots,z_N) \\ &\quad = I(z_1^{-1},\dots,z_N^{-1})\tilde{X}(z_1,\dots,z_N) + x_0, \end{aligned} \tag{7}$$

where

$$X(z_1,\dots,z_N) = \begin{bmatrix} X_1(z_1,\dots,z_N) \\ \vdots \\ X_N(z_1,\dots,z_N) \end{bmatrix},$$

$$\tilde{X}(z_1,\dots,z_N) = \begin{bmatrix} \tilde{X}_1(z_1,\dots,z_N) \\ \vdots \\ \tilde{X}_N(z_1,\dots,z_N) \end{bmatrix},$$

$$x_0 = \begin{bmatrix} x_{10} \\ \vdots \\ x_{N0} \end{bmatrix}.$$

Then, since $I(z_1^{-1}, \dots, z_N^{-1}) I(z_1, \dots, z_N) = I$, from (4) and (7) we get

$$\begin{aligned} &\left[I - AI(z_1^{-1}, \dots, z_N^{-1}) \;\; -B \right] \\ &\qquad \times \begin{bmatrix} \tilde{X}(z_1, \dots, z_N) \\ U(z_1, \dots, z_N) \end{bmatrix} = Ax_0. \end{aligned} \tag{8}$$

Note that there always exist nonsingular matrices $T_1 \in \mathbb{R}^{n \times n}$ and $T_2 \in \mathbb{R}^{(n+m)\times(n+m)}$ such that

$$T_1 A = \begin{bmatrix} 0 \\ A_1 \end{bmatrix}$$

and

$$\begin{aligned} &T_1 \left[I - AI(z_1^{-1}, \dots, z_N^{-1}) \;\; -B \right] T_2 \\ &\quad = \begin{bmatrix} I_q & 0 \\ Z_1(z_1^{-1}, \dots, z_N^{-1}) & Z_2(z_1^{-1}, \dots, z_N^{-1}) \end{bmatrix}, \end{aligned} \tag{9}$$

where A_1 has full row rank, i.e. $\operatorname{rank} A_1 = \operatorname{rank} A = n - q$.

Then from (8) and (9) we obtain

$$\begin{aligned} &\begin{bmatrix} I_q & 0 \\ Z_1(z_1^{-1}, \dots, z_N^{-1}) & Z_2(z_1^{-1}, \dots, z_N^{-1}) \end{bmatrix} \\ &\qquad \times \begin{bmatrix} V_1(z_1, \dots, z_N) \\ V_2(z_1, \dots, z_N) \end{bmatrix} = \begin{bmatrix} 0 \\ A_1 \end{bmatrix} x_0, \end{aligned}$$

where

$$\begin{bmatrix} V_1(z_1, \dots, z_N) \\ V_2(z_1, \dots, z_N) \end{bmatrix} = T_2^{-1} \begin{bmatrix} \tilde{X}(z_1, \dots, z_N) \\ U(z_1, \dots, z_N) \end{bmatrix},$$

$\dim V_1 = q$ and $\dim V_2 = n + m - q$.

From this it follows easily that $V_1(z_1, \dots, z_N) \equiv 0$. Next, from Definition 1 and Eqns. (5) and (7) we see that an N-DRM is controllable *if and only if* for any x_0 there exist $\tilde{X}(z_1, \dots, z_N)$ and $U(z_1, \dots, z_N)$ which are polynomials in $z_1^{-1}, \dots, z_{N-1}^{-1}$ and z_N^{-1}. Thus, an N-DRM is controllable *if and only if* there exists $V_2(z_1, \dots, z_N)$, a polynomial in $z_1^{-1}, \dots, z_{N-1}^{-1}$ and z_N^{-1}, such that

$$Z_2(z_1^{-1}, \dots, z_N^{-1}) V_2(z_1, \dots, z_N) = A_1 x_0.$$

Since the matrix A_1 has a full row rank, from Theorem 2 in (Youla, 1979) it follows that there exists an appropriate polynomial matrix $V_2(z_1, \dots, z_N)$, i.e. $\tilde{X}(z_1, \dots, z_N)$ and $U(z_1, \dots, z_N)$, *if and only if*

$$\operatorname{rank} Z_2(z_1^{-1}, \dots, z_N^{-1}) = n - q$$

for $(z_1^{-1}), \dots, (z_N^{-1}) \in \mathbb{C}$. Hence, based on (9), we get (3) after the change of $(z_1^{-1}, \dots, z_N^{-1})$ into $(z_1, \dots, z_N)$, respectively. ■

Remark 3. The controllability of the N-DRM is generic for $m \geq N$ and nongeneric for $m = 1, 2, \dots, N-1$. This results from (3) and (Kurek, 1990), since there exists an uncontrollable N-DRM with $m = 1, 2, \dots, N-1$ and a controllable one with $m = N$.

Remark 4. A necessary condition for the controllability of an N-DRM is the controllability of the pairs $(A_{11}, B_1), \dots, (A_{NN}, B_N)$. Indeed, for $z_2 = z_3 = \dots = z_N = 0$ we obtain

$$\operatorname{rank} \begin{bmatrix} I - A_{11} z_1 & 0 & \dots & 0 & B_1 \\ -A_{21} z_1 & I & & 0 & B_2 \\ \vdots & & \ddots & & \vdots \\ -A_{N1} z_1 & 0 & & I & B_N \end{bmatrix} = n$$

for $z_1 \in \mathbb{C}$. However, it simply implies that the pair (A_{11}, B_1) is controllable. The rest of the proof is similar and therefore it is omitted. ■

4. Reconstructability

Following (Kurek, 1987), we define the reconstructability of the system (1) as follows.

Definition 3. The N-DRM (1) is (causally) reconstructable *if and only if* any local state $x(k) \in \mathbb{R}^n$ can be determined from the knowledge of the past output and input sequences of the system

$$\begin{aligned} \big\{ y(i_1, \dots, i_N), u(i_1, \dots, i_N), (i_1, \dots, i_N) \\ \leq (k_1, \dots, k_N) \big\}. \end{aligned}$$

Theorem 2. *The N-DRM (1) is controllable* if and only if

$$\operatorname{rank} \begin{bmatrix} I - AI(z_1, \dots, z_N) \\ C \end{bmatrix} = n \tag{10}$$

for $z_1, \dots, z_N \in \mathbb{C}$.

Proof. (Sufficiency) Since the system is linear, we can assume that $u(i_1,\dots,i_N)=0$. Then, using backward shift operators z_i, $z_j^h f(i_1,\dots,i_N)=f(i_1,\dots,i_j-h,i_N)$, we can rewrite (1) in the form

$$\begin{bmatrix} I-AI(z_1,\dots,z_N) \\ C \end{bmatrix} x(k)=\begin{bmatrix} 0 \\ y(k) \end{bmatrix}. \tag{11}$$

Then, if the condition of the theorem is satisfied, there exist polynomial matrices $H_1(z_1,\dots,z_N)$, $\dim H_1=n\times n$, and $H_2(z_1,\dots,z_N)$, $\dim H_2=n\times p$ such that (Youla, 1979)

$$\begin{bmatrix} H_1(z_1,\dots,z_2) & H_2(z_1,\dots,z_2)\end{bmatrix} \times \begin{bmatrix} I-AI(z_1,\dots,z_N) \\ C \end{bmatrix} = I.$$

Thus we have

$$H_2(z_1,\dots,z_N)y(k)=x(k).$$

(Necessity) If the condition is not satisfied, there exist $(z_{10},\dots,z_{N0})\neq 0$ and $x_0\neq 0$ such that

$$\begin{bmatrix} I-AI(z_{10},\dots,z_{N0}) \\ C \end{bmatrix} x_0=0.$$

From this, for non-zero initial conditions

$$\mathrm{BCS}\,(0,\dots,0):$$

$$\big\{x_i(j_1,\dots,j_{i-1},0,j_{i+1},\dots,j_N) = z_{10}^{k_1-i_1}\dots z_{j-1,0}^{k_{j-1}-i_{j-1}} z_{j0}^{k_j-i_j} z_{j+1,0}^{k_{j+1}-i_{j+1}} \dots z_{N0}^{k_N-i_N}x_{01}, \quad \text{for} \quad 0\le i\le k\big\}$$

we obtain

$$x(i_1,i_2,\dots,i_N)=I(z_1^{k_1-i_1}z_2^{k_2-i_2}\cdots z_N^{k_N-i_N})x_0$$

and $y(i)=0$ for $0\le i\le k$. Thus the actual state $x(k)=x_0$ is indistinguishable from the local state $x(k)=0$. ∎

It is easy to notice that the unreconstructable system is also unobservable, i.e. its non-zero initial state is indistinguishable from the zero initial state. However, the reconstructability of the system does not mean that the system is observable (consider, e.g., the system (1) with $A=0$).

5. Illustrative Example

Now we present an example illustrating the controllability and reconstructability of the system described by the 3-DRM.

Example 1. Consider the system described by the 3-D Roesser model with $n_1=2$, $n_2=1$, $n_3=1$ and

$$A=\left[\begin{array}{cc|c|c} 0.1 & 1 & 0 & 0.1 \\ 0.2 & 0.5 & 1 & 0 \\ \hline -0.2 & -1 & 0 & 0.5 \\ \hline -0.5 & 0.1 & 1 & -1 \end{array}\right],$$

$$B=\left[\begin{array}{cc} 0 & 0.1 \\ 1 & -0.2 \\ \hline -0.5 & 0 \\ \hline 0 & 1 \end{array}\right],$$

$$C=\left[\begin{array}{cc|c|c} 2 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 2 \end{array}\right].$$

Then we obtain

$$P_1(z)=\left[\begin{array}{c|c} I-AI(z_1,z_2,z_3) & B\end{array}\right]$$

$$=\left[\begin{array}{cc|c|c|cc} 1-0.1z_1 & -z_1 & 0 & -0.1z_3 & 0 & 0.1 \\ -0.2z_1 & 1-0.5z_1 & -z_2 & 0 & 1 & -0.2 \\ \hline 0.2z_1 & z_1 & 1 & -0.5z_3 & -0.5 & 0 \\ \hline 0.5z_1 & -0.1z_1 & -z_2 & 1+z_3 & 0 & 1 \end{array}\right]$$

and

$$P_2(z)=\left[\begin{array}{c} I-AI(z_1,z_2,z_3) \\ \hline C \end{array}\right]$$

$$=\left[\begin{array}{cc|c|c} 1-0.1z_1 & -z_1 & 0 & -0.1z_3 \\ -0.2z_1 & 1-0.5z_1 & -z_2 & 0 \\ \hline 0.2z_1 & z_1 & 1 & -0.5z_3 \\ \hline 0.5z_1 & -0.1z_1 & -z_2 & 1+z_3 \\ \hline 2 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 2 \end{array}\right].$$

By elementary row and column operations on $P_1(z)$, we get

$$
\left[\begin{array}{cc|c|c|cc}
1-0.1z_1 & -z_1 & 0 & -0.1z_3 & 0 & 0.1 \\
-0.2z_1 & 1-0.5z_1 & -z_2 & 0 & 1 & -0.2 \\
\hline
0.2z_1 & z_1 & 1 & -0.5z_3 & -0.5 & 0 \\
\hline
0.5z_1 & -0.1z_1 & -z_2 & 1+z_3 & 0 & 1
\end{array}\right]
$$

$$
\rightarrow \left[\begin{array}{cc|c|c|cc}
a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\
0 & 0 & 0 & 0.6 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -0.5 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right],
$$

where

$$
a_{11} = 0.3333 - 1.3333z_3 + 0.3z_1z_3,
$$

$$
a_{12} = 0.1666 - 0.0833z_1 + 0.3333z_3 + 1.8133z_1z_3,
$$

$$
a_{13} = 0.3333 - 0.1666z_2 + 0.6666z_3 - 0.5333z_2z_3.
$$

Then, solving the three equations $a_{11} = 0$, $a_{12} = 0$ and $a_{13} = 0$, we find $z_{10} = -0.7868$, $z_{20} = 1.6965$ and $z_{30} = 0.2124$ such that $\operatorname{rank} P_1(z_0) = 3 < n = 4$. Thus the system is uncontrollable.

Next, by elementary column and row operations on $P_2(z)$, we obtain

$$
\left[\begin{array}{cc|c|c}
1-0.1z_1 & -z_1 & 0 & -0.1z_3 \\
-0.2z_1 & 1-0.5z_1 & -z_2 & 0 \\
\hline
0.2z_1 & z_1 & 1 & -0.5z_3 \\
\hline
0.5z_1 & -0.1z_1 & -z_2 & 1+z_3 \\
\hline
2 & 0 & 1 & -1 \\
0 & 1 & 1 & 0 \\
-1 & 1 & 0 & 2
\end{array}\right]
$$

$$
\rightarrow \left[\begin{array}{cc|c|c}
-0.3262+0.1z_3 & 0 & 0 & 0 \\
2.2727+3z_2 & 0 & 0 & 0 \\
\hline
-8+18.7z_2 & 0 & 0 & 0 \\
\hline
-6.4492 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0
\end{array}\right].
$$

However, this implies $\operatorname{rank} P_2(z) = 4$. Thus the system is reconstructable.

6. Concluding Remarks

The conditions for the controllability and reconstructability of the N-DRM were presented. They are dual, analogously to the 1-D case. Note that slightly modified conditions given in Remark 2, namely the reachability of the pairs $(A_{11}, B_1), \dots, (A_{NN}, B_N)$, imply the so-called real reachability of the N-DRM, which guarantees the controllability of the system in Kalman's sense with respect to its standard BCS (2) (Kurek, 1987). The conditions are weaker than those given in Theorem 1. Unfortunately, real reachability does not guarantee the controllability of the N-DRM if the BCS has a different form than the standard one (2), e.g., in the following case (Fornasini and Marchesini, 1978):

$$
\{x(k_1, \dots, k_N) = x_{k_1,\dots,k_N}, \quad k_1 + \cdots + k_N = 0\}.
$$

Finally, controllability is the necessary and sufficient condition for the stabilizability of the N-DRM by state feedback (Bisiacco, 1985). Moreover, it implies the local and modal controllability (Kung *et al.*, 1977; Roesser, 1975) of the system. For these reasons controllability seems to be one of the basic properties of N-DRMs.

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