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GROUNDING AND EXTRACTING MODAL RESPONSES IN COGNITIVE AGENTS: 'AND' QUERY AND STATES OF INCOMPLETE KNOWLEDGE

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In this study an original way of modeling language grounding and generation for a simple set of language responses is presented. It is assumed that the language is used by a cognitive agent and consists of a few modal belief and possibility formulas that are used by this agent to communicate its opinions on the current state of an object. The cognitive agent is asked a simple AND query and the language is tailored to this situation. The agent's knowledge bases are characterized by certain incompleteness of information on the current state of objects. The language of the available responses is originally grounded in the agent's previous empirical experience. According to the assumptions of the cognitive linguistics and the phenomenology of knowledge, this experience is the basic source of meaning represented by the available formulas (responses). In the study the idea of an epistemic satisfaction relation is introduced that describes states of the agent's knowledge in which particular formulas are satisfied in the epistemic sense. Additionally, a formal description of the semantic power of formulas is presented. The concepts of the empirical satisfaction relation and the semantic power of formulas are used to define a model of particular language behavior that preserves the assumption of language grounding. Two examples of possible implementations are given. These implementations are basic ones and refer to statistical characteristics of the stored empirical experience of the cognitive agent.

Keywords: cognitive agent, semantic communication, language grounding

1. Introduction

Cognitive linguistics claims that symbols of semantic languages are always correlated with their meaning embodied in communicating agents (Fauconnier, 1997; Lakoff and Johnson, 1999). This phenomenon is known as symbol grounding (Harnad, 1990). A symbol of a language is treated as grounded if it is bound to some content stored in cognitive structures constituting the body of a cognitive agent. From the cognitive agent's point of view, each ungrounded symbol is meaningless and useless as an external representation of intentions, beliefs, desires, attitudes, etc. At the same time the phenomenology of knowledge assumes that the ultimate source of any meaning accessible to cognitive agents is strictly determined by their perceptions (Husserl, 1913; 1921). Perceptions are assumed to be the basic arguments for cognitive processes that constitute and create a more advanced higher level meaning.

The above assumptions of the necessity of symbol grounding and the ultimate role of perceptions are applicable in situations in which an artificial cognitive agent is asked to describe the current state of an object, provided that at the moment of being asked this agent is not able to observe the object in a direct way. In such situations the agentis replies can only represent its views on this object derived from previous experiences and therefore need to involve modal operators pointing at related kinds of knowledge vagueness.

Unfortunately, if the necessity of symbol grounding is accepted as a fundamental requirement for the semantic communication of cognitive agents, the related models of language behavior become surprisingly complex. There are two basic reasons for this complexity. Firstly, each model of the process of constructing replies needs to specify the way in which modal operators are related to their meaning given at the level of cognitive structures. In particular, a precise relation has to be defined between sets of perceptions represented in cognitive systems and particular sentences of the semantic language of replies. In some relatively simple cases this relation appears to be quite complex. Secondly, usually more than one sentence of the semantic language is well grounded in sets of stored perceptions and can be used as a reply. Therefore at the stage of constructing replies the cognitive agent needs to refer to additional criteria for the choice of the sentence which is the most adequate for the existing state of knowledge. Very often these criteria are not simple and involve concepts from linguistics and the mathematical theory of communication.

Below an original approach to defining language behaviors of cognitive agents is presented provided that the language has some semantics for communicating agents and is used in the situation of some incompleteness of their knowledge. The language is a set of modal formulas, each of which can represent a potential reply to the AND question built in the following way: *Does the object o exhibit the property P and the property Q?* An additional assumption is that while being asked this question the cognitive agent is not able to observe the current state of the properties P and Q in the referred object o. Therefore, to construct its reply it refers to previous experiences stored at the level of cognitive structures. This relatively simple case of information query results in a rather complex model that integrates a few interesting concepts from cognitive linguistics, the mathematical theory of communication and formal semantics. It is necessary to stress that the way in which the semantics for replies is understood and defined in this paper is different from the semantics known from the BDI approach (Cohen and Levesque, 1990; Halpern and Moses, 1992; Lindern *et al.*, 1998).

In Section 2 of the paper a model of a class of simple cognitive agents is given. The model includes basic structures for representing empirical perceptions. The overall collection of perceptions constitutes the content from which the so-called grounding experience is extracted for the language of possible replies. This language of replies is presented in Section 3. Its formulas are extensions built from modal operators of belief and possibility. The extended formulas are built from conjunctions, alternatives and exclusive alternatives, each of them built from two different negated or non-negated literals. In Section 4 the concept of the semantic power of formulas is discussed. In particular, it is explained why some formulas of the language of possible replies are treated by the cognitive agent as more informative than others. In further sections the concept of the semantic power of formulas is also used to define particular decision procedures for choosing the most relevant replies. In consequence, Section 4 consists of a formalization of a very important common-sense idea of the strength of statements with precise means of the mathematical theory of communication proposed by Shannon. Section 5 presents the idea of grounding. In particular, some introductory notes on the role of grounding are given and two strategies for determining the grounding experience for particular reply formulas are defined. The first strategy does not take into account the similarity between the situation in which the cognitive agent determines its reply to the information query and these past situations in which it collected the related grounding experience. This strategy is called context independent. The second strategy is based on the similarity of the above-mentioned situations and is called context dependent. In Section 6 the concept of an epistemic satisfaction relation is introduced. The epistemic satisfaction relation is similar to the classic definition for the truth in the sense that it specifies circumstances in which an external formula is perceived as corresponding to the existing state of knowledge at best. The definition for the epistemic satisfaction of the modal formula of belief and possibility involves both the idea of grounding and the idea of the semantic power of formulas. Section 6 defines the actual language behavior of the cognitive agent given in Section 2 provided that the agent is asked the above-mentioned question and its knowledge of the current situation is incomplete. In Section 7 extended computational examples are presented. Final remarks are given in Section 8.

2. The External World and Internal Knowledge Structures

It is assumed that a cognitive agent is provided with its own and internally realized system of concepts. This system makes it possible for the agent to autonomously construct private conceptualizations of the states of an external world W. This external world W is a dynamic system built from atom objects $\Omega = \{o_1, o_2, \dots, o_N\}$. Each atom object $x \in \Omega$ can be described by means of a property $P \in \Delta = \{P_1, P_2, \dots, P_K\}$. In particular, the cognitive agent can perceive or "think" of an object $x \in \Omega$ as having or not having a particular property $P \in \Delta$.

The world W is a dynamic system that changes over time. External events recognized by the agent are ordered along the line of time points $T = \{t_0, t_1, t_2, \dots\}$. The weak temporal order $\leq^{\mathrm{TM}} \subseteq T \times T$ and the strong temporal order $<^{\mathrm{TM}} \subseteq T \times T$ are defined over the set T. For each $i, j \in \{0, 1, 2, \dots\}$ the relation $t_i \leq^{\mathrm{TM}} t_j$ holds if and only if $i \leq j$ and the relation $t_i <^{\mathrm{TM}} t_j$ holds if and only if $i < j$.

It is assumed that the cognitive agent is able to construct internal models of the world. However, these models can be built only within the cognitive constraints inherently bound to each cognitive agent and realized as its system of internally available concepts. The elements of the model refer to the basic aspects of the world state. In particular, they make it possible to create on the level of internal representations particular models of the states of properties in objects of the world. The overall universe of all possible models is given by the following definition:

Definition 1. (*Universe of modal states*) The universe $\textit{Universe}_{\mathrm{ModalStates}}$ of modal states accessible to the cognitive agent is the following set of relational systems:

$$\textit{Universe}_{\mathrm{ModalStates}} = \big\{ s : s = \langle \Omega, Z_1^+, Z_2^+, \dots, Z_K^+ \rangle \text{ and } Z_i^+ \subseteq \Omega \big\}.$$

The following interpretations are assumed:

For each $x \in \Omega$, the relation $x \in Z_i^+$ represents the statement "The object x has the property P_i."

For each $x \in \Omega$, the relation $x \notin Z_i^+$ represents the statement "The object x does not have the property P_i."

The internal model of the world is rarely a complete enumeration of all properties in all objects of the world. The actual models usually grasp parts of the world and the remaining aspects are not known. Therefore the universe of modal states has mainly a theoretical meaning, and actual and mentally accessible models of the world are rather complete models of parts of this world (Johnson-Laird, 1983).

It is assumed below that the entire knowledge of the agent is derived from the set of the so-called base profiles. The concept of the base profile has already been used in an effective way to model other aspects of knowledge processing in multiagent systems (Katarzyniak and Nguyen, 2000). Base profiles are internal models of empirical observations of the external world carried out by the cognitive agent during its interactions with its environment. Base profiles possess the status of empirically verified pieces of knowledge and from the phenomenological point of view they constitute the ultimate source of any meaning accessible to the agent during the processes of language interpretation. The following formal definition of base profiles is given:

Definition 2. (*Base profile*) The base profile of the state of the world related to the time point $t \in T$ that has been autonomously created by the cognitive processes of the agent and encapsulated in its body is given as the following relational system:

$$\textit{BaseProfile}(t) = \big\langle \Omega, P_1^+(t), P_1^-(t), P_2^+(t), P_2^-(t), \ldots, P_K^+(t), P_K^-(t)\big\rangle,$$

where

(a) t denotes the time point, to which the profile is related,

(b) For each $i = 1, 2, \ldots, K$, the relation $P_i^+(t) \subseteq \Omega$ holds. For each object $x \in \Omega$ it satisfies the relation $x \in P_i^+(t)$ if and only if at the time point t the agent perceived x as exhibiting the property P_i.

(c) For each $i = 1, 2, \ldots, K$, the relation $P_i^-(t) \subseteq \Omega$ holds. For each object $x \in \Omega$ it satisfies the relation $x \in P_i^-(t)$ if and only if at the time point t the agent perceived x as not exhibiting the property P_i.

The relational system $\textit{BaseProfile}(t)$ will also be called the t-related base profile.

Let the universe of all possible base profiles be denoted by the symbol $\textit{Universe}_{\text{Profiles}}$. The following common-sense constraints are accepted for the base profile:

$$P_i^+(t) \cap P_i^-(t) = \emptyset, \quad P_i^+(t) \cup P_i^-(t) \subseteq \Omega.$$

The related interpretation is that while observing an object the cognitive agent is not able to perceive it simultaneously as exhibiting and not exhibiting a particular property P.

The sum $P_i^+(t) \cup P_i^-(t)$ does not need to be equal to the set Ω. This means that in the case of each property the area of the agent's incompetence as regards the t-related state of objects can be given:

Definition 3. (*Incompetence of the agent*) The area of the cognitive agent's incompetence as regards the property P at the time point t is given by the following set:

$$P^{\pm}(t) = \Omega / \big(P^+(t) \cup P^-(t)\big).$$

The set $P^{\pm}(t)$ will also be called the t-related P-incompetence.

Having defined the above concept, a simple definition for the agent's state of knowledge can be given:

Definition 4. (*Internal knowledge state*) The agent's state of knowledge at the time point t is defined by the following temporal data base of base profiles:

$$\textit{KnowledgeState}(t) = \big\{\textit{BaseProfile}(l) : l \in T \text{ and } l \leq^{\text{TM}} t\big\}.$$

The role of $\textit{KnowledgeState}(t)$ is basic. As has already been stressed, cognitive linguistics and the phenomenology of knowledge assume that any language statement that is generated at a particular time point t needs to be grounded in $\textit{KnowledgeState}(t)$. In other words, the meaning of any statement cannot be separated from what is stored in $\textit{KnowledgeState}(t)$ because this meaning is derived in a direct or indirect way from representations of perceptions stored in $\textit{KnowledgeState}(t)$.

The next idea formally introduced in the paper is strictly related to the knowledge incompleteness area and the base profiles. At each particular time point $t \in T$ the t-related P-incompetence $P^{\pm}(t)$, $P \in \Delta$, can be substituted by mental models of particular states. These states are not observed by the agent in the external world. They are rather mental images constructed by the agent processes within its cognitive system of concepts. From the formal point of view the process of reducing the scope of knowledge incompleteness by the creation of images of properties' states is equivalent to decreasing the cardinalities of all non-empty sets $P^{\pm}(t)$, $P \in \Delta$. This formal and technical procedure has a well-known natural counterpart, namely, the process of creating possible worlds (Brentano, 1924; Husserl, 1913; 1921).

Possible worlds are understood as complete mental models of a t-related state of the world W that are members of $\mathit{Universe}_{\mathrm{ModalStates}}$ and have the content coherent with the content of the encapsulated t-related base profile:

Definition 5. (*Possible worlds*) The set of possible worlds accessible to the cognitive agent at the time point t and understood by this agent as alternative mental models of the current state of the world is formally given as follows:

$$\begin{aligned}&\mathit{PossibleWorlds}(t)\\&= \big\{\langle \Omega, P_1^+(t) \cup M_1, P_2^+(t) \cup M_2, \ldots, P_K^+(t) \cup M_K \rangle :\\&\qquad \text{for each } i = 1, 2, \ldots, K, \ \ M_i \subseteq \Omega(P_i^+(t)\big\}.\end{aligned}$$

The following is true for possible worlds:

1. Each relational system $s \in \mathit{PossibleWorlds}(t)$ belongs to *Universe*. This means that each possible world is also a case of modal states accessible to the agent in general.

2. From the formal point of view each possible world $s \in \mathit{PossibleWorlds}(t)$ is equivalent to a base profile in which no knowledge incompleteness is present. Namely, the possible world $s = \langle \Omega,\, P_1^+(t) \cup M_1,\, P_2^+(t) \cup M_2, \ldots, P_K^+(t) \cup M_K \rangle$ is semantically equivalent to a base profile with the following structure and content:

$$\begin{aligned}&\big\langle \Omega, P_1^+(t) \cup M_1, \Omega/(P_1^+(t) \cup M_1),\\&\qquad P_2^+(t) \cup M_2, \Omega/(P_2^+(t) \cup M_2),\\&\qquad\quad \ldots, P_K^+(t) \cup M_K, \Omega/(P_K^+(t) \cup M_K)\big\rangle.\end{aligned}$$

Let $P, Q \in \Delta$, $t \in T$ and $x \in \Omega$ be given. The following x-related classification of possible worlds can be defined:

$Z^{++}(P,Q,t,x)$ consists of all $s \in \mathit{PossibleWorlds}(t)$, in which $x \in P^+(t)$ and $x \in Q^+(t)$.

$Z^{+-}(P,Q,t,x)$ consists of all $s \in \mathit{PossibleWorlds}(t)$, in which $x \in P^+(t)$ and $x \in Q^-(t)$.

$Z^{-+}(P,Q,t,x)$ consists of all $s \in \mathit{PossibleWorlds}(t)$, in which $x \in P^-(t)$ and $x \in Q^+(t)$.

$Z^{--}(P,Q,t,x)$ consists of all $s \in \mathit{PossibleWorlds}(t)$, in which $x \in P^-(t)$ and $x \in P^-(t)$.

Obviously, the following is true:

1. The sets $Z^{++}(P,Q,t,x)$, $Z^{+-}(P,Q,t,x)$, $Z^{-+}(P,Q,t,x)$, $Z^{--}(P,Q,t,x)$ are mutually disjoint.
2. We have $\Omega = Z^{++}(P,Q,t,x) \cup Z^{+-}(P,Q,t,x) \cup Z^{-+}(P,Q,t,x) \cup Z^{--}(P,Q,t,x)$.

Possible worlds and modal states are used in defining the strength of grounding formulas of the language of possible replies. These definitions are given below.

3. The External Language of Communication

The external language of communication studied in this paper is tailored to a situation in which the cognitive agent has to answer the question “Does an object o exhibit properties P and Q?.” An additional assumption is that the agent is not able to verify the current state of these properties in the object o. This means that the formulas of the language need to correspond to the states of knowledge incompleteness as regards the distribution of the properties P and Q in the object o. What follows is that from the formal point of view the language of replies is not a subset of the prepositional or first-order language but requires the use of modal operators corresponding to relevant kinds of information vagueness, e.g., beliefs and possibilities.

The external language of communication considered in this paper consists of logic-like formulas built with two kinds of components. The first component of each formula of the language of replies is called the core component. The core components and their intentional (common-sense) semantics are given in Table 1. The set of core components will be denoted by $L_\Phi = \{\varphi_i : i = 1, \ldots 6\}$.

The second component of each formula is either the modal operator of belief *Bel* or the modal operator of possibility *Pos*. The core components are arguments for these

Table 1. Core components of modal replies.

	Core formula	Intentional (common-sense) meaning
φ_1	$P(o) \wedge Q(o)$	The object o exhibits the property P and exhibits the property Q.
φ_2	$P(o) \wedge \neg Q(o)$	The object o exhibits the property P and does not exhibit the property Q.
φ_3	$\neg P(o) \wedge Q(o)$	The object o does not exhibit the property P and exhibits the property Q.
φ_4	$\neg P(o) \wedge \neg Q(o)$	The object o does not exhibit the property P and does not exhibit the property Q.
φ_5	$P(o) \underline{\vee} Q(o)$	The object o exhibits either the property P or exhibits the property Q.
φ_6	$P(o) \vee Q(o)$	The object o exhibits the property P or exhibits the property Q.

operators. The following rules of extending core components with modal operators are assumed:

Formulas $\varphi_i,\ i = 1,\dots,4$, built only with logic connectives of conjunction $\wedge$ can be extended with the modal operator of possibility *Pos*. This extension belongs to the language of possible replies. The intentional meaning of each modal extension $\mathit{Pos}(\varphi_i)$ is given as "It is possible that φ_i." For instance, the extended modal formula $\mathit{Pos}(\neg P(o) \wedge Q(o))$ is understood as "It is possible that the object o does not exhibit the property P and exhibits the property Q."

Formulas $\varphi_i,\ i = 1, 2\dots,6$, built with logic connectives of conjunction $\wedge$, classic alternative $\vee$ or exclusive alternative $\underline{\vee}$ can be extended with the modal operator of belief *Bel*. This extension belongs to the language of possible replies. The intentional meaning of each modal extension $\mathit{Bel}(\varphi_i)$ is given as "I believe that φ_i." For instance, the extended belief formula $\mathit{Bel}(P(o) \vee Q(o))$ is understood as "I believe that the object o exhibits the property P or exhibits the property Q."

It is important to remember that the above formulas of the semantic language are treated as external and interpreted statements spoken out by the cognitive agent in order to give an answer to the question "Does the object o exhibit the property P and the property Q?". It also has to be stressed that this language does not allow us to extend the core components φ_5 and φ_6 with the modal operator of possibility. Obviously, these assumptions can be rejected in different approaches to modeling the language behavior.

4. The Semantic Power of the Language

The semantic power of logic formulas is an important concept underlying further procedures for the choice of the most relevant replies. The semantic power of a logic formula has already been used in order to analyze semantics for logic alternative (Ajdukiewicz, 1956). Unfortunately, this concept has not been formally defined, in particular for the case of semantic languages of communication and the language behavior of cognitive agents.

Let the following description of the semantic power of formulas in L_Φ be given: At first, the semantic content function named *cont* is introduced. This function assigns to each formula $\varphi \in L_\Phi$ a set of all complete mental models $s \in \mathit{Universe}_{\mathrm{ModalStates}}$ that are "mentally" accessible to the agent and are models satisfying the formula φ in the sense of the Tarskian definition of truth (Hunter, 1971; Tarski, 1935). The signature of this function is

$$\mathit{cont} : L_\Phi \to 2^{\mathit{Universe}_{\mathrm{ModalStates}}}$$

and its values are given as follows:

$$c_1 = \mathit{cont}(\varphi_1) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_1\},$$

$$c_2 = \mathit{cont}(\varphi_2) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_2\},$$

$$c_3 = \mathit{cont}(\varphi_3) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_3\},$$

$$c_4 = \mathit{cont}(\varphi_4) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_4\},$$

$$c_5 = \mathit{cont}(\varphi_5) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_5\} = \mathit{cont}(\varphi_2) \cup \mathit{cont}(\varphi_3),$$

$$c_6 = \mathit{cont}(\varphi_6) = \{s \in \mathit{Universe}_{\mathrm{ModalStates}} : s \models_{\mathrm{TARSKIAN}} \varphi_6\} = \mathit{cont}(\varphi_1) \cup \mathit{cont}(\varphi_2) \cup \mathit{cont}(\varphi_3),$$

where $s \models_{\mathrm{TARSKIAN}} \varphi$ denotes the Tarskian satisfaction relation (Hunter, 1971; Tarski, 1935). The function *cont* assigns to each formula its embodied meaning. Symbol $c_1, c_2, \dots, c_6$ are introduced to simplify the notation.

Property 1. For $i, j \in \{1, 2, 3, 4\}$, if $i \neq j$, then $c_i \cap c_j = \emptyset$.

Property 2. $c_1 \cup c_2 \cup c_3 \cup c_4 = \mathit{Universe}_{\mathrm{ModalStates}}$.

Secondly, the following binary relation $\succeq^{\inf} \subseteq L_\Phi \times L_\Phi$ can be defined:

Definition 6. (*Semantic strength of formulas*) For each pair of formulas $\varphi_1, \varphi_2 \in L_\Phi$ such that $\varphi_1 \neq \varphi_2$, the relation

$$\varphi_1 \succeq^{\inf} \varphi_2$$

holds if and only if $\mathit{cont}(\varphi_1) \subseteq \mathit{cont}(\varphi_2)$. The symbol $\varphi_1 \succeq^{\inf} \varphi_2$ denotes the statement "φ_1 is semantically richer than φ_2."

The relation $\succeq^{\inf}$ defines a binary metastructure over the set L_Φ that reflects the differences of the semantic power of particular core components of the language considered. The relation $\succeq^{\inf}$ has the following properties:

Property 3. The relation $\succeq^{\inf}$ corresponds to the mathematical definition of information and entropy given by Shannon (1948). This correspondence is given in the following way:

According to the set-based definition of probability, each modal state $s \in \mathit{Universe}_{\mathrm{ModalStates}}$ can be assigned the following value of its probability:

$$P(s) = 1/\operatorname{card}(\mathit{Universe}_{\mathrm{ModalStates}}) = p_e,$$

where the symbol $\mathrm{card}(X)$ denotes the cardinality of the set X. This probability can be used to define the information I carried out by particular formulas of L_Φ:

$$\begin{aligned} I_1 &= I(\varphi_1) = I\big(P(o) \wedge Q(o)\big) \\ &= -\sum_{s \in c_1} P(s) \log P(s) \\ &= -p_e \log\big(p_e \cdot \mathrm{card}(c_1)\big) \cdot \mathrm{card}(c_1), \end{aligned}$$

$$\begin{aligned} I_2 &= I(\varphi_2) = I\big(P(o) \wedge \neg Q(o)\big) \\ &= -\sum_{s \in c_2} P(s) log P(s) \\ &= -p_e \log\big(p_e \cdot \mathrm{card}(c_2)\big) \cdot \mathrm{card}(c_2), \end{aligned}$$

$$\begin{aligned} I_3 &= I(\varphi_3) = I\big(\neg P(o) \wedge Q(o)\big) \\ &= -\sum_{s \in c_3} P(s) log P(s) \\ &= -p_e \log\big(p_e \cdot \mathrm{card}(c_3)\big) \cdot \mathrm{card}(c_3), \end{aligned}$$

$$\begin{aligned} I_4 &= I(\varphi_4) = I\big(\neg P(o) \wedge \neg Q(o)\big) \\ &= -\sum_{s \in c_4} P(s) log P(s) \\ &= -p_e \log\big(p_e \cdot \mathrm{card}(c_4)\big) \cdot \mathrm{card}(c_4), \end{aligned}$$

$$\begin{aligned} I_5 &= I(\varphi_5) = I\big(P(o) \vee Q(o)\big) \\ &= I_2 + I_3 = (-1) \cdot \mathrm{card}(c_5) \cdot p_e \log p_e \\ &= (-p_e \log p_e) \cdot \big(\mathrm{card}(c_2) + \mathrm{card}(c_3)\big), \end{aligned}$$

$$\begin{aligned} I_6 &= I(\varphi_6) = I\big(P(o) \vee Q(o)\big) \\ &= I_1 + I_2 + I_3 = (-p_e \log p_e) \cdot \mathrm{card}(c_6) \cdot p_e \log p_e \\ &= (-p_e \log p_e) \cdot \big(\mathrm{card}(c_1) + \mathrm{card}(c_2) + \mathrm{card}(c_3)\big). \end{aligned}$$

It is easy to notice that for each pair of core elements $\varphi, \phi \in L_\Phi$, the relation $\varphi \succeq^{\inf} \phi$ holds if and only if $I(\varphi) \geq I(\phi)$.

Property 4. The relation $\succeq^{\inf}$ is transitive.

Property 5. The relation $\succeq^{\inf}$ always defines the following partial sub-order over the set L_Φ:

$$\varphi_1 \succeq^{\inf} \varphi_5, \quad \varphi_2 \succeq^{\inf} \varphi_5, \quad \varphi_3 \succeq^{\inf} \varphi_5,$$
$$\varphi_4 \succeq^{\inf} \varphi_5, \quad \varphi_5 \succeq^{\inf} \varphi_6.$$

This order can also be extended with case-specific relations between core components $\varphi_i,\ i = 1, \dots, 4$.

It is important to stress that the semantic power of formulas influences the process of knowledge communication because the cognitive agent is always more inclined to utter formulas that are semantically richer. This phenomenon is taken into account when the procedures for language grounding and generation are defined.

5. The Grounding of the Language

5.1. The Necessity of Grounding the Language of Replies

According to cognitive linguistics (Fauconnier, 1997; Lakoff and Johnson, 1999) and the phenomenology of knowledge (Husserl, 1913; 1921), each external formula generated by the cognitive agent to communicate a particular content needs to be grounded in relevant structures of knowledge. These relevant structures are called the grounding experience of the related formula. In the case of the agents considered in this paper, formula grounding defines at each time point t the relation between this formula and particular parts of *KnowledgeState*(t).

Following the phenomenological and cognitive assumptions, it is assumed that any meaning assigned by the cognitive agent to modal formulas of the language of replies needs to have its origins in the empirical experience conceptualised and stored in *KnowledgeState*(t). In other words, it is possible for the cognitive agent to assign a particular meaning to an external formula of its language if and only if this meaning is extractable from the result of its own interactions with the external world. These basic pieces of data are treated as an ultimate source of this meaning and are the mental material in which the formulas are grounded. The way in which external language formulas are referred (grounded) to in the empirical content stored as *KnowledgeState*(t) is very specific for each of these formulas and in the case of the extended modal formulas it is indirect.

The consequence of accepting the assumption of necessary grounding is that each semantic formula can be accepted by the cognitive agent as an external representation of its knowledge if and only if the relevant content is extractable from the available set of data pieces. This fact will be further formalized by means of the epistemic satisfaction relation.

Two similar strategies for determining the relevant grounding experience are considered in the forthcoming sections. The first strategy is called context independent and involves all empirical data relevant to the grounded formulas. The second strategy is called context dependent and involves only these base profiles that are similar to the latest perception of the world. The second strategy is more advanced. The details of both strategies are given below.

5.2. The Context-Independent Strategy for Grounding Replies

The context-independent strategy for grounding replies is based on all data available in *KnowledgeState*(t). In particular, to ground a formula of the language of replies it uses the following sets:

$$Emp^{++}(P,Q,t) = \big\{(l,x):\ l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega \text{ and } Z^{++}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{+-}(P,Q,t) = \big\{(l,x):\ l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega \text{ and } Z^{+-}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{-+}(P,Q,t) = \big\{(l,x):\ l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega \text{ and } Z^{-+}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{--}(P,Q,t) = \big\{(l,x):\ l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega \text{ and } Z^{--}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\}.$$

These sets make it possible to summarize the overall strength of the empirical and stored experience in which particular formulas are grounded. The logic of the grounding is rather simple and can be stated as follows:

From the common-sense point of view the fact that the relation $(l,x)\in Emp^{++}(P,Q,t)$ holds means that at the time point $l\in T$ the agent experienced the object x as exhibiting both properties P and Q. In this particular sense $(l,x)\in Emp^{++}(P,Q,t)$ represents a piece of the stored empirical experience. This piece is an evidence for the cognitive agent that a particular distribution of P and Q can be an actual external event. Consequently, the set $Emp^{++}(P,Q,t)$ consists of the content in which formulas φ_1, $Pos(\varphi_1)$, $Bel(\varphi_1)$ are grounded by cognitive processes of the agent.

Similar arguments can be stated as regards the sets $Emp^{+-}(P,Q,t)$, $Emp^{-+}(P,Q,t)$ and $Emp^{--}(P,Q,t)$: The set $Emp^{+-}(P,Q,t)$ consists of the content in which formulas φ_2, $Pos(\varphi_2)$ and $Bel(\varphi_2)$ are grounded. The set $Emp^{-+}(P,Q,t)$ consists of the content in which formulas φ_3, $Pos(\varphi_3)$ and $Bel(\varphi_3)$ are grounded. The set $Emp^{--}(P,Q,t)$ consists of the content in which formulas φ_4, $Pos(\varphi_4)$ and $Bel(\varphi_4)$ are grounded.

The grounding data for formulas with exclusive alternative $\veebar$ and alternative $\vee$ are defined respectively as

$$Emp^{\veebar}(P,Q,t) = Emp^{+-}(P,Q,t)\cup Emp^{-+}(P,Q,t),$$

$$Emp^{\vee}(P,Q,t) = Emp^{+-}(P,Q,t)\cup Emp^{+(}(P,Q,t) \cup Emp^{-+}(P,Q,t).$$

In particular, the set $Emp^{\veebar}(P,Q,t)$ consists of the content in which formulas φ_5 and $Bel(\varphi_5)$ are grounded. In turn, the set $Emp^{\vee}(P,Q,t)$ consists of the content in which formulas φ_6 and $Bel(\varphi_6)$ are grounded.

This way of defining the content for the latest two groups of formulas originates from the cognitive theory of mental models (Johnson-Laird, 1983). In this theory the meaning of both alternatives is assured by the co-existence of meanings for related conjunctions. Namely, the mental model for the exclusive alternative "either P or Q" is a system consisting of two mental models for conjunctions "not P and Q" and "P and not Q". The mental model for the classical alternative "P or Q" is a system consisting of three mental models for conjunctions "P and Q", "not P and Q" and "P and not Q".

5.3. The Context-Independent Strategy for Grounding Replies

The context-dependent strategy to determine grounding experience for meaning creation assumes that elements of the sets $Emp^{++}(P,Q,t)$, $Emp^{+-}(P,Q,t)$, $Emp^{-+}(P,Q,t)$, $Emp^{--}(P,Q,t)$, $Emp^{\veebar}(P,Q,t)$ and $Emp^{\vee}(P,Q,t)$ need to fulfil an additional common-sense requirement based on the idea of the cognitive distance of base profiles. The possible modification is

$$Emp^{++}(P,Q,t) = \big\{(l,x): l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega,\ \delta\big(\textit{BaseProfile}(l),\textit{BaseProfile}(t)\big)\leq\lambda_\delta, \text{ and } Z^{++}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{+-}(P,Q,t) = \big\{(l,x): l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega,\ \delta\big(\textit{BaseProfile}(l),\textit{BaseProfile}(t)\big)\leq\lambda_\delta, \text{ and } Z^{+-}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{-+}(P,Q,t) = \big\{(l,x): l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega,\ \delta\big(\textit{BaseProfile}(l),\textit{BaseProfile}(t)\big)\leq\lambda_\delta, \text{ and } Z^{-+}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{--}(P,Q,t) = \big\{(l,x): l\in T,\ l\leq^{\mathrm{TM}} t,\ x\in\Omega,\ \delta\big(\textit{BaseProfile}(l),\textit{BaseProfile}(t)\big)\leq\lambda_\delta, \text{ and } Z^{--}(P,Q,l,x) = \textit{PossibleWorlds}(l)\big\},$$

$$Emp^{\veebar}(P,Q,t) = Emp^{+-}(P,Q,t)\cup Emp^{-+}(P,Q,t),$$

$$Emp^{\vee}(P,Q,t) = Emp^{+-}(P,Q,t)\cup Emp^{+-}(P,Q,t) \cup Emp^{-+}(P,Q,t),$$

where the function

$$\delta: \textit{Universe}_{\text{Profiles}}\times\textit{Universe}_{\text{Profiles}}\to\mathbb{R}^{+}\cup\{0\}$$

is a distance measure between two base profiles, $\mathbb{R}^+ \cup \{0\}$ is the set of non-negative real numbers, and $\lambda_\delta \in \mathbb{R}^+ \cup \{0\}$. The role of λ_δ is crucial because it defines the cut point above which the base profiles are not considered as belonging to the grounding experience. The distance measure between base profiles can be defined in various ways. An example is given in (Katarzyniak and Pieczyńska-Kuchtiak, 2002; 2003). In general, the distance measures applicable to the case of base profiles are a subclass of a broader class of distance measures defined over the universe of ordered partitions (Daniłowicz and Nguyen, 1988).

In the case of the above modification of the grounding experience only this stored experience is used for building the meaning of the formulas which are collected in circumstances similar to the circumstances observed at the time point t. However, as will be given below, the rules for determining the strength of formula grounding remain the same for both context independent and context dependent strategies.

6. The Epistemic Satisfaction Relation and the Choice of External Messages

6.1. Epistemic Satisfaction and Grounding

The proposed procedures for verifying the epistemic satisfaction relation of formulas and for carrying out the choice of external replies (formulas) are based on both concepts of semantic power of the language (considered for core components of possible replies) and formula grounding. The general idea of the approach is that at each time point t these formulas are chosen by the cognitive agent as external representations of its knowledge that are preferred by this agent as having the most relevant core components and are well grounded in its stored experience. In this paper the determination of the core components' preference involves a simple measure of the relative strength of grounding based on the cardinalities of grounding sets $\mathit{Emp}^{++}(P,Q,t)$, $\mathit{Emp}^{+-}(P,Q,t)$, $\mathit{Emp}^{-+}(P,Q,t)$ and $\mathit{Emp}^{--}(P,Q,t)$. The preference of core components is determined according to the following definition:

Definition 7. (*λ-preference of core components*) Let the sets $\mathit{Emp}^{++}(P,Q,t)$, $\mathit{Emp}^{+-}(P,Q,t)$, $\mathit{Emp}^{-+}(P,Q,t)$, $\mathit{Emp}^{--}(P,Q,t)$, $\mathit{Emp}^{\underline{\vee}}(P,Q,t)$ and $\mathit{Emp}^{\vee}(P,Q,t)$ be given. The set $\mathit{Pref} \subseteq L_\Phi$, $\lambda \in (0,1]$, is the λ-preference of core components if and only if it is determined according to the following procedure:

Procedure *Preference*

Input: $\lambda \in (0,1]$, $\mathit{Emp}^{++}(P,Q,t)$, $\mathit{Emp}^{+-}(P,Q,t)$, $\mathit{Emp}^{-+}(P,Q,t)$, $\mathit{Emp}^{--}(P,Q,t)$, $\mathit{Emp}^{\underline{\vee}}(P,Q,t)$, $\mathit{Emp}^{\vee}(P,Q,t)$

Output: $\mathit{Pref} \subseteq L_\Phi$

begin

$\mathit{Pref} := \emptyset$;

$\mathit{Emp}(P,Q,t) := \mathit{Emp}^{++}(P,Q,t) \cup \mathit{Emp}^{+-}(P,Q,t) \cup \mathit{Emp}^{-+}(P,Q,t) \cup \mathit{Emp}^{--}(P,Q,t)$;

if $\left(\mathrm{card}(\mathit{Emp}^{++}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_1\}$;

if $\left(\mathrm{card}(\mathit{Emp}^{+-}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_2\}$;

if $\left(\mathrm{card}(\mathit{Emp}^{-+}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_3\}$;

if $\left(\mathrm{card}(\mathit{Emp}^{--}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_4\}$;

if $(\mathit{Pref} = \emptyset)$ **then**
if $\left(\mathrm{card}(\mathit{Emp}^{\underline{\vee}}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_5\}$;

if $(\mathit{Pref} = \emptyset)$ **then**
if $\left(\mathrm{card}(\mathit{Emp}^{\vee}(P,Q,t)) / \mathrm{card}\left(\mathit{Emp}(P,Q,t)\right) \geq \lambda\right)$ **then** $\mathit{Pref} := \mathit{Pref} \cup \{\varphi_6\}$;

end.

The rationale behind the above definition results from the idea of the semantic power of particular core components and the actual intensity of grounding experience. In the first step, the cognitive processes of language generation and verification take into account the grounding experience for the most informative core components φ_1, φ_2, φ_3 and φ_4. The relative intensity of the grounding experience is determined for these formulas and only the core components which are supported by the grounding experience higher than a given threshold λ are accepted as the ones preferred by the cognitive agent. If no core component is chosen from the set $\{\varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4\}$, then the processing of the exclusive alternative is launched. The relative strength of the core component φ_5 is determined and evaluated against the given threshold λ. If the core component φ_5 is found unaccepted as regards its relative strength of grounding, the procedure is repeated for the case of the remaining core component φ_6. If all core components are rejected as not preferred, no relevant external representation of the state of internal knowledge of the current distribution of P and Q in the object o exists, provided that some preference threshold λ is used.

In order to formalize the fact that a formula (reply) is the best representation of the agent's knowledge, the so-called epistemic satisfaction relation is introduced. The epistemic satisfaction relation is given separately for each modal formula. The general rule is that the formula is true (satisfied) in the epistemic sense if and only if it is grounded and its core component is preferred among all grounded core components. The result is as follows:

Definition 8. (*Epistemic satisfaction relation for the possibility with conjunction* φ_1) Let the set of base profiles *KnowledgeState*(t) be given. The epistemic satisfaction relation *KnowledgeState*$(t) \models_{\mathrm{E}}$ *Pos*$(P(o) \wedge Q(o))$ holds for the time point t if and only if the following requirements are fulfilled simultaneously:

1. $o \in P^{\pm}(t)$ and $o \in Q^{\pm}(t)$,
2. *Emp*$^{++}(P, Q, t) \neq \emptyset$,
3. $\mathrm{card}(\mathit{Pref}(t))(1$,
4. $P(o) \wedge Q(o) \in \mathit{Pref}(t)$.

The common-sense meaning of the requirement mentioned in the above definition is as follows: First, the use of *Pos*$(P(o) \wedge Q(o))$ is rational and acceptable from the cognitive agent's point of view at the time point t if and only if this agent is not able to get to know the current state of properties P and Q in the object o in a direct way. Therefore it is forced to approximate this current state on the base of its previous grounding experience. Second, the core component $P(o) \wedge Q(o)$ of the modal formula *Pos*$(P(o) \wedge Q(o))$ needs to be grounded in at least one piece of relevant empirical data. This piece of data is a proof for the cognitive agent that P and Q can be exhibited by an object simultaneously. From the phenomenological point of view this means that the cognitive agent was able to collect the core piece of data from which the meaning understood as the coexistence of P and Q in an object x is "extracted." In other words, this piece of grounding data is a carrier of the related meaning. Third, there are at least two core components with non-empty meanings developed from the grounding experience and chosen by the language cognitive processes as the preferred ones. Since two core components are preferred in the sense given in Definition 7, the modal operator of possibility is applicable instead of the modal operator of belief. Fourth, the core component $P(o) \wedge Q(o)$ is preferred as exhibiting a relatively acceptable level of the grounding experience.

The epistemic satisfaction relation for the remaining formulas *Pos*(φ_2), *Pos*(φ_3) and *Pos*(φ_4) is defined in the same way, provided that the relevant sets of the grounding experience are considered.

The epistemic satisfaction relation for conjunctions extended with the modal operator of belief is similar to the case of possibility formula satisfaction. However, belief extensions of conjunctions are satisfied by states of knowledge and are accepted as external representations of these states if and only if exactly one conjunction is preferred in the sense of Definition 7.

Definition 9. (*Epistemic satisfaction for the belief with conjunction* φ_1) Let the set of base profiles *KnowledgeState*(t) be given. The epistemic satisfaction relation *KnowledgeState*$(t) \models_E$ *Bel*$(P(o) \wedge Q(o))$ holds for the time point t if and only if the following requirements are fulfilled simultaneously:

1. $o \in P^{\pm}(t)$ and $o \in Q^{\pm}(t)$,
2. *Emp*$^{++}(P, Q, t) \neq \emptyset$,
3. $\mathrm{card}(\mathit{Pref}(t)) = 1$,
4. $P(o) \wedge Q(o) \in \mathit{Pref}(t)$.

The intuitive interpretation for Requirements 1, 2 and 4 is the same as the interpretation given for the epistemic relation for possibility satisfaction. The only difference is Requirement 3, which states that the core component $P(o) \wedge Q(o)$ is the only preferred formula chosen from among the core components φ_1, φ_2, φ_3 and φ_4. This requirement reflects the common-sense intuition known from the natural language processing. For instance, if a cognitive agent determines that both core components "object o exhibits properties P and Q" and "object o exhibits property P and does not exhibit property Q" are acceptable alternatives derived from its previous experiences, it will not be inclined to use both of them as core components of the relevant formulas. Therefore, it does not reject any of the two core components and combines them with the modal operator of belief. The result is "It is possible that object o exhibits properties P and Q" and "It is possible that object o exhibits property P and does not exhibit property Q." The situation is different if only one conjunction is chosen as the relevant core component in the sense of Definition 7. The cognitive state of the agent is different in this sense that it is surer about one and only one distribution of properties P and Q in object o. In such a situation in the natural language discourse, cognitive agents are generally inclined to use the modal operator of belief.

The definition of the epistemic satisfaction relation for the belief formula with an exclusive alternative as its core component is strictly related to the idea of the mental model for the exclusive alternative (Johnson-Laird, 1983). The details of the definition are as follows:

Definition 10. (*Epistemic satisfaction for belief with exclusive alternative*.) Let the set of base profiles *KnowledgeState*(t) be given. The epistemic satisfaction relation *KnowledgeState*$(t) \models_E$ *Bel*$(P(o) \veebar Q(o))$ holds for the time point t if and only if the following requirements are fulfilled simultaneously:

1. $o \in P^{\pm}(t)$ and $o \in Q^{\pm}(t)$,
2. *Emp*$^{++}(P, Q, t) = \emptyset$,
3. *Emp*$^{+-}(P, Q, t) \neq \emptyset$,
4. *Emp*$^{-+}(P, Q, t) \neq \emptyset$,

5. $Emp^{--}(P, Q, t) = \emptyset$,
6. $\mathrm{card}(Pref(t)) = 1$,
7. $P(o) \underline{\vee} Q(o) \in Pref(t)$.

The intuitive interpretation for the above requirements is as follows: Requirement 1 is the same as in the case of the previous formulas. Requirements 6 and 7 state that the only preferred core component is built from the connective of the exclusive alternative. Requirements 2–5 define a necessary distribution of the grounding experience in which simultaneous exhibition and lack of properties P and Q in the same object o are not allowed. At the same time, at least some grounding experience is needed to support the remaining two possible distributions of properties P and Q in the considered objects. This definition is compatible with the content of the theory of mental models in this sense that it assumes the meaning of $P(o)\underline{\vee}Q(o)$ to be a complex system of mental models for $P(o)\wedge\neg Q(o)$ and $\neg P(o)\wedge Q(o)$ (Johnson-Laird, 1983).

Definition 10 can be easily transformed into the following definition for the epistemic satisfaction relation for the belief operator with the classical alternative:

Definition 11. (*Epistemic satisfaction for belief with alternative.*) Let the set of base profiles *KnowledgeState*(t) be given. The epistemic satisfaction relation *KnowledgeState*$(t) \models_E$ *Bel*$(P(o) \vee Q(o))$ holds for the time point t if and only if the following requirements are fulfilled simultaneously:

1. $o \in P^{\pm}(t)$ and $o \in Q^{\pm}(t)$,
2. $Emp^{++}(P, Q, t) \neq \emptyset$,
3. $Emp^{+-}(P, Q, t) \neq \emptyset$,
4. $Emp^{-+}(P, Q, t) \neq \emptyset$,
5. $Emp^{--}(P, Q, t) = \emptyset$,
6. $\mathrm{card}(Pref(t)) = 1$,
7. $P(o) \vee Q(o) \in Pref(t)$.

This definition differs from Definition 10 as regards Requirement 2 and 7. In order to satisfy the belief formula with the classic alternative, the cognitive agent needs to be additionally supported with a non-empty set $Emp^{++}(P, Q, t)$. The role of Requirement 7 remains the same.

6.2. Epistemic Relations and the Classic Approach

It is important to notice that the above definitions for the epistemic satisfaction of formulas are not equivalent to the classical definitions for the satisfaction of logic formulas (Hunter, 1971; Tarski, 1935) and the related Kripke definitions for the satisfaction of belief and possibility (Kripke, 1963). The basic difference results from the nature of model structures against which all formulas are evaluated in both approaches. Namely, in the case of the classical approach the model structure represents objects external to cognitive agents and some logic formula is satisfied by this structure if and only if its intended content (its meaning) corresponds to an externally existing state of the world. In this sense the evaluated formula is true in the existing external conditions. Any cognitive state that coexists simultaneously to these conditions is not considered because it is treated as irrelevant to the idea of the classical truth. The model structure considered in the definitions of epistemic satisfaction does not represent external states of the world. Its function is to reflect internal states of cognitive agents, including the elements of internal structures that refer cognitive agents to external objects. It is the original assumption of the proposed solution. Moreover, such an approach assumes the basic role of cognitive agents as creators of meaning and conscious generators of messages. In this particular sense, the proposed definitions complete the relation between a particular formula of a semantic language and a related external object described by this formula. Namely, they describe this part of the semiotic triangle (Eco, 1991) which represents the subject of the language (not covered by the classical approach).

This solution implements the idea of the grounding of a particular semantic language of communication in the cognitive agent. In an original formal way it captures the internal system of knowledge structures that are used by the cognitive agent to mediate between external representations of knowledge given as logic formulas and external objects described by these formulas. An external formula is treated as grounded if and only if the cognitive agent has collected sufficient empirical experience and can assign the relevant content of empirical experience to this formula. It is well grounded if and only if it is satisfied in the epistemic sense.

It is necessary to notice, too, that all definitions for epistemic satisfaction are strongly interrelated. For instance, if the formula *Know*(ϕ) is satisfied, then the remaining formulas *Bel*(ϕ) and *Pos*(ϕ) are not. The same rule is true for both *Bel*(ϕ) and *Pos*(ϕ). Therefore the proposed set of definitions covers a natural situation in which the choice of one linguistic representation excludes other external representations of knowledge states as less relevant.

The proposed set of definitions suggests a class of original implementations of processing some semantic language (in this particular case the language consisting of modal extensions of conjunctions). This set of definitions suggests language behavior that imitates the human language behavior, makes artificial systems understandable to humans and rationally involved in semantic communication. The assumptions underlying the definitions for epistemic satisfaction are suggested by cognitive science (cognitive linguistics) that deals with modeling cognitive agents as intentional systems (Denett, 1996) or knowledge systems (Newell, 1990).

The suggested implementations of epistemic satisfaction may differ according to the scope of the empirical experience stored in the agent’s knowledge bases and taken into account during formula verification. If a more rational way of language generation is considered, then the cognitive agent needs to be more context (situation) sensitive and a particular constraint needs to be applied to determine the relevant set of the stored empirical data. If additional constraints are applied, then the computational cost raises. However, in consequence, more appropriate and more natural behavior is achieved. If the constraints are not considered, the agent becomes less situated in the external world. Obviously, various sets of constraints can be designed and verified in order to achieve more situated reactions.

7. Computational Examples

7.1. The Knowledge Base and Grounding Experience

Let a simple cognitive agent be given. This agent is equipped with a conceptual system that makes it possible to represent and process perceptions of the world consisting of objects from the set $\Omega = \{o_1, o_2, o_3\}$. The states of the objects are given as distributions of properties P, Q, U and W. Let the set $\mathit{KnowledgeState}(t_5)$ be given by Table 2.

The context independent and context dependent strategies will be considered to illustrate the application of the empirical relation of satisfaction to modeling cognitive states in the generation of the semantic language.

7.2. The Case of the Context Independent Strategy

Let the context independent strategy be applied in order to verify the epistemic satisfaction of particular formulas at the time point t_5 described in Table 2. The related sets of grounding experience and their cardinalities are given as follows:

$$\mathit{Emp}^{++}(P,Q,t_5) = \big\{(t_1,o_1),(t_1,o_2)\big\},\quad \mathrm{card}\,\big(\mathit{Emp}^{++}(P,Q,t_5)\big) = 2,$$

$$\mathit{Emp}^{+-}(P,Q,t_5) = \big\{(t_2,o_1),(t_2,o_3),(t_3,o_3)\big\},\quad \mathrm{card}\,\big(\mathit{Emp}^{+-}(P,Q,t_5)\big) = 3,$$

$$\mathit{Emp}^{-+}(P,Q,t_5) = \big\{(t_3,o_1),(t_4,o_2),(t_4,o_3),(t_5,o_2)\big\},\quad \mathrm{card}\,\big(\mathit{Emp}^{-+}(P,Q,t_5)\big) = 4,$$

$$\mathit{Emp}^{--}(P,Q,t_5) = \big\{(t_3,o_2)\big\},\quad \mathrm{card}\,\big(\mathit{Emp}^{--}(P,Q,t_5)\big) = 1,$$

$$card\big(\mathit{Emp}(P,Q,t_5)\big) = 10.$$

The content of the set $\mathit{Emp}^{++}(P,Q,t_5) = \{(t_1,o_1),(t_1,o_2)\}$ tells us that the cognitive agent observed twice that an object can exhibit both properties P and Q at the same time. In consequence, the conceptual representation of these two observations became two pieces of the embodied meaning. For instance, these data contribute to the meaning of the modal formulas $\mathit{Bel}(P(o_1)\underline{\vee}Q(o_1))$, $\mathit{Pos}(P(o_2) \wedge Q(o_2))$ and the core component $P(o_2) \wedge Q(o_2)$. If these data were not present in a knowledge state at the time point t, the related meaning would not be accessible to the agent at the time point t. Obviously, another meaning could be assigned by the agent to both the formulas $\mathit{Bel}(P(o_1)\underline{\vee}Q(o_1))$ and $\mathit{Pos}(P(o_2) \wedge Q(o_2))$. However, this meaning would not be the same as the meaning traditionally assigned to them in human like contexts. Similar explanations are valid for other sets of grounding experience.

The cardinalities of grounding sets are also important in determining the empirical satisfaction of modal formulas. The highest cardinality is assigned to the grounding experience of the modal formulas that are built from the core components $\neg P(o_1) \wedge Q(o_1)$, $\neg P(o_2) \wedge Q(o_2)$ and $\neg P(o_3) \wedge Q(o_3)$. The experience of objects not exhibiting the property P and exhibiting the property Q is relatively the strongest one among all stored experiences. In consequence, if the cognitive agent does not know the actual state of P and Q in an object x, it is

Table 2. Knowledge state.

T	Property P			Property Q			Property U			Property W		
	P^+	P^-	$P^\pm$	Q^+	Q^-	$Q^\pm$	U^+	U^-	$U^\pm$	W^+	W^-	$W^\pm$
t_1	o_1, o_2		o_3	o_1, o_2		o_3	o_3	o_2	o_1	o_2	o_3	o_1
t_2	o_1, o_3		o_2		o_1, o_3	o_2	o_1, o_3		o_2		o_1, o_3	o_2
t_3	o_3	o_1, o_2		o_1	o_2, o_3		o_1, o_3	o_2		o_2	o_1, o_3	
t_4		o_2, o_3	o_1	o_2, o_3		o_1	o_2		o_1, o_3		o_2	o_1, o_3
t_5		o_2	o_1, o_3	o_2, o_3		o_1	o_2		o_1, o_3		o_2	o_1, o_3

naturally inclined to project the strongest of all stored images onto the image of the object x. The strength of the projected material determines the choice of the modal operator to be used. If the grounding experience is stronger, the cognitive agent decides to use the modal operator of belief. Otherwise, it concentrates on the modal operator of possibility. Obviously, the role of the semantic power of connectives is also important.

In the context independent strategy, the numerical measures of the relative strength of the grounding experience are given as follows:

$$\mathrm{card}\,\big(\textit{Emp}^{++}(P,Q,t_5)\big)/\mu = 0.2,$$
$$\mathrm{card}\,\big(\textit{Emp}^{+-}(P,Q,t_5)\big)/\mu = 0.3,$$
$$\mathrm{card}\,\big(\textit{Emp}^{-+}(P,Q,t_5)\big)/\mu = 0.4,$$
$$\mathrm{card}\,\big(\textit{Emp}^{--}(P,Q,t_5)\big)/\mu = 0.1,$$

where

$$\mu = \mathrm{card}\,\big(\textit{Emp}^{++}(P,Q,t_5)\big) \cup \mathrm{card}\,\big(\textit{Emp}^{+-}(P,Q,t_5)\big) \cup \mathrm{card}\,\big(\textit{Emp}^{-+}(P,Q,t_5)\big) \cup \mathrm{card}\,\big(\textit{Emp}^{--}(P,Q,t_5)\big).$$

Let the level of preference be equal to $\lambda = 0.25$. The procedure for constructing the set $\textit{Pref}(t_5)$ of the preferred core components results in $\textit{Pref}(t_5) = \{\varphi_2, \varphi_3\}$. In consequence, the following is true:

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(o_1) \wedge Q(o_1)\big)$ holds,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(x) \wedge Q(x)\big)$ does not hold for $x \in o_2, o_3$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(\neg P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(o_1) \wedge \neg Q(o_1)\big)$ holds,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(x) \wedge \neg Q(x)\big)$ does not hold for $x \in o_2, o_3$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(\neg P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \underline{\vee} Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \vee Q(x)\big)$ does not hold for each $x \in \Omega$.

The state of knowledge is represented by formulas $\textit{Pos}(\neg P(o_1) \wedge Q(o_1))$ and $\textit{Pos}(P(o_1) \wedge \neg Q(o_1))$. From the practical point of view, this means that if the cognitive agent is asked the question "Does object o_1 exhibit properties P and Q?", it responds by uttering the formula $\textit{Pos}(\neg P(x) \wedge Q(x))$ or the formula $\textit{Pos}(P(o_1) \wedge \neg Q(o_1))$ or the conjunction of these formulas. It does not choose any of the other modal formulas mentioned above because these formulas are not satisfied in the sense of epistemic satisfaction.

The value of the threshold is important. This value determines the accuracy of external knowledge representations generated by the agent. If the value increases, then the agent is more inclined to use the more intensive grounding experience as the source of its beliefs on the current state of an object. Its external messages become more sound representations of the stored experience and knowledge. For instance, the threshold value $\lambda = 0.50$ results in the following set of conclusions:

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$ hold for $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(\neg P(x) \wedge Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(x) \wedge Q(x)\big)$ does not,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(x) \wedge \neg Q(x)\big)$ does not hold for $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(\neg P(x) \wedge \neg Q(x)\big)$ does not hold for each $x \in \Omega$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(o_1) \underline{\vee} Q(o_1)\big)$ holds,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \underline{\vee} Q(x)\big)$ does not hold for each $x \in \{o_2, o_3\}$,

$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(P(x) \vee Q(x)\big)$ does not hold for each $x \in \Omega$.

The result is that only the modal belief formula $\textit{Bel}(P(o_1) \underline{\vee} Q(o_1))$ is satisfied in the sense that it corresponds to the existing state of knowledge (episteme). It illustrates the role of the threshold in an apparent way. Namely, from the cognitive agent's point of view, the strength of the grounding experience becomes more important. If the experience is stronger, then the related external formulas become more adequate representations of

Table 3. A reduced vector—based representation of the knowledge state.

T	Property P			Property Q			Property U			Property W		
	P^+	P^-	$P^\pm$	Q^+	Q^-	$Q\pm$	U^+	U^-	$U^\pm$	W^+	W^-	$W^\pm$
t_1	o_1, o_2		o_3	o_1, o_2		o_3	o_3	o_2	o_1	o_2	o_3	o_1
t_4		o_2, o_3	o_1	o_2, o_3		o_1	o_2		o_1, o_3		o_2	o_1, o_3
t_5		o_2	o_1, o_3	o_2, o_3		o_1, o_3	o_2		o_1, o_3		o_2	o_1, o_3

the existing state of knowledge. However, at the knowledge state given as *KnowledgeState*(t_5), only the complex grounding experience related to the exclusive alternative fulfils the requirement of being relatively higher than the threshold $\lambda = 0.50$, provided that the most informative formula is considered in the first step of language processing. The choice of a modal belief or possibility formula with one of the conjunctions is not acceptable because it is related to a too lower strength of the grounding experience. At the same time, the choice of the modal belief formula with alternative is not acceptable although the relative strength of grounding experience is high enough. The reason is the existence of a more informative exclusive alternative that is also grounded with a desirable relative strength. In this particular sense this belief formula is the best (the most adequate) external representation of the cognitive agent's view on unknown distribution of the properties P and Q in the object o_1.

7.3. The Case of the Context Dependent Strategy

In order to apply the context dependent strategy for grounding, it is necessary to define a distance function over the universe of all base profiles. In this example, the following normalized distance function δ will be used: For any base profiles *BaseProfile*(t_m) and *BaseProfile*(t_n), their distance $\delta(\textit{BaseProfile}(t_m), \textit{BaseProfile}(t_n))$ is equal to the normalized and minimal number of object movements that are necessary to translate all sets $P_i^+(t_m)$, $P_i^-(t_m)$, $P_i^\pm(t_m)$ into the related sets $P_i^+(t_n)$, $P_i^-(t_n)$, $P_i^\pm(t_n)$, $i = 1, 2, \ldots, K$.

For the knowledge state given in Table 2, the following values of the distances can be computed:

$$\delta\big(\textit{BaseProfile}(t_5), \textit{BaseProfile}(t_1)\big) = 8/12 = 0.67,$$

$$\delta\big(\textit{BaseProfile}(t_5), \textit{BaseProfile}(t_2)\big) = 12/12 = 1.00,$$

$$\delta\big(\textit{BaseProfile}(t_5), \textit{BaseProfile}(t_3)\big) = 11/12 = 0.92,$$

$$\delta\big(\textit{BaseProfile}(t_5), \textit{BaseProfile}(t_4)\big) = 2/12 = 0.17,$$

$$\delta\big(\textit{BaseProfile}(t_5), \textit{BaseProfile}(t_5)\big) = 0/12 = 0.00.$$

These values are used to reject the base profiles in *KnowledgeState*(t_5) that are not close enough to the base profile related to the current time point t_5. Let only the base profiles be accepted as close enough that are assigned the distance value not higher than $\lambda_\delta = 0.75$. The result is given in Table 3.

The related sets of the grounding experience and their cardinalities are

$$\textit{Emp}^{++}(P,Q,t_5) = \emptyset,\ \ \mathrm{card}\,\big(\textit{Emp}^{++}(P,Q,t_5)\big) = 0,$$

$$\textit{Emp}^{+-}(P,Q,t_5) = \{(t_1,o_3),(t_4,o_2),(t_5,o_2)\},\quad \mathrm{card}\,\big(\textit{Emp}^{+-}(P,Q,t_5)\big) = 3,$$

$$\textit{Emp}^{-+}(P,Q,t_5) = \{(t_1,o_2)\},\quad \mathrm{card}\,\big(\textit{Emp}^{-+}(P,Q,t_5)\big) = 1,$$

$$\textit{Emp}^{--}(P,Q,t_5) = \emptyset,\ \ \mathrm{card}\,\big(\textit{Emp}^{--}(P,Q,t_5)\big) = 0,$$

$$\mathrm{card}\,\big(\textit{Emp}(P,Q,t_5)\big) = 4.$$

The related values of the relative strength of this grounding experience are

$$\mathrm{card}\,\big(\textit{Emp}^{++}(P,Q,t_5)\big)/\,\mathrm{card}\,\big(\textit{Emp}(P,Q,t_5)\big) = 0.00,$$

$$\mathrm{card}\,\big(\textit{Emp}^{+-}(P,Q,t_5)\big)/\,\mathrm{card}\,\big(\textit{Emp}^{(}P,Q,t_5)\big) = 0.75,$$

$$\mathrm{card}\,\big(\textit{Emp}^{-+}(P,Q,t_5)\big)/\,\mathrm{card}\,\big(\textit{Emp}(P,Q,t_5)\big) = 0.25,$$

$$\mathrm{card}\,\big(\textit{Emp}^{--}(P,Q,t_5)\big)/\,\mathrm{card}\,\big(\textit{Emp}^{(}P,Q,t_5)\big) = 0.00.$$

The remaining steps of verifying epistemic satisfaction is the same as in the case of the context independent strategy for the determination of the grounding experience. In particular, for the grounding preference level $\lambda = 0.25$ the proposed strategies of language processing result in the conclusion that the only epistemic satisfaction relations that hold are

$$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(\neg P(o_1) \wedge Q(o_1)\big),$$

$$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Pos}\big(P(o_1) \wedge \neg Q(o_1)\big).$$

However, if the grounding preference $\lambda = 0.50$ is taken into account by the cognitive agent, then the set of satisfied relations is a singleton. Namely, the only satisfied relation is

$$\textit{KnowledgeState}(t_5) \models_{\mathrm{E}} \textit{Bel}\big(\neg P(o_1) \wedge Q(o_1)\big).$$

The rules of common-sense interpretation of the above result are the same as in the case of the context independent strategy for the determination of the grounding experience.

8. Final Remarks

In this study some basic concepts related to the idea of semantic languages for multiagent communication have been discussed. In recent years this class of languages has been extensively studied and applied for theoretical and practical reasons. However, the symbol grounding problem that is claimed by the cognitive linguistics and phenomenology of knowledge to be one of the most important aspects of interpreted languages has not been studied extensively enough in the context of these languages. In this paper an original approach to the analysis and realization of language grounding was proposed for the case of a language of logic-like formulas with modal operators of belief and possibility. Although this language is not extensive, the study points at the multidimensional nature of the processes that grounds it in the cognitive structures and knowledge bases of communicative agents.

The model of formula grounding was considered for a class of relatively simple agents. In particular, an original way of implementing the grounding was presented. The cognitive agent was situated in a dynamic world consisting of atom objects exhibiting particular properties. The cognitive agent was equipped with the temporal database of the so-called base profiles representing the simplest observations of the external world carried out by the agent. These perceptions were assumed to be the basic source of any meaning that can be assigned to particular formulas. It was also assumed that the intention of the cognitive agent is to choose the formula of the grounded language that is its most relevant response to the information query given as "*Does the object o exhibit properties P and Q?*". It was been assumed that, while being asked, the cognitive agent is not able to verify the state of the properties P and Q directly in the external world. Therefore, in order to choose the best response, it refers itself to its previous experience given by stored perceptions that constitute the source of any meaning assigned to external language messages. The proposed solution is that this generation of the best and most relevant responses is a multistep procedure that involves the determination of the degrees of particular formulas grounding in the stored experience.

All situations (all states of the knowledge of this cognitive agent), in which a particular formula is considered by the agent as the most relevant representation of its knowledge of a current state of an object, were defined in this paper by the epistemic satisfaction relation. This relation is complementary to the classical satisfaction relation for logic-like languages (Tarski, 1935). In this approach each model structure satisfying the formula of the response language is not a representation of the state of the world objective and external to the cognitive agent but a representation of the actually existing state of the agent's knowledge, provided that the agent is assumed to interpret and ground this formula. In this particular sense, the epistemic satisfaction relation covers the part of the semiotic triangle (Eco, 1991) different from the part covered by the classical satisfaction relation (Tarski, 1935).

It is necessary to stress that the epistemic satisfaction relation is also based on the idea of the semantic power of formulas. In particular, the semantic power of logic connectives influences the procedures for determining the relevance of core components of modal formulas. The concept of the semantic power of connectives is known from the philosophy of language and formal logic where it is used in an informal way (Ajdukiewicz, 1956). In this paper an original approach to modeling and measuring this characteristic of formulas was proposed. This approach is purely formal and based on the basic concepts of the mathematical theory of information. Therefore it makes the idea of the semantic power of formulas from at least some classes more precise and useful in technical fields.

The results presented in the paper are interesting in the sense that, first, they define a model of an artificial cognitive agent that imitates to a visible extent some aspects of natural language processing in humans and, second, suggest an original way of implementing these phenomena related to the natural language in artificial cognitive systems. This model makes it possible to preserve the common-sense (intentional) interpretation of the artificial language behavior and its compatibility with the previous sense giving experiences. In particular, this model for the language behavior preserves the common-sense meaning of modal operators of belief and possibility and logic connectives for the conjunction, exclusive alternative and classic alternative.

The formal concepts used in this study point at an original approach to the implementation of semantic languages in the case of a class of artificial cognitive agents. The examples given at the end of the paper illustrate such implementations in a simple way. There exist a few directions of further research in this field. First, it is possible to extent the formal language for which the epistemic satisfaction relation is defined (Katarzyniak, 2000; 2001a; 2001b; 2002). The target of such an extension could be to make the agent more precise and elastic as regards the description of its own state of knowledge. Obviously, the richer the language, the richer the list of internal states distinguishable and named on the level of external representations by the cognitive agent. Secondly, it is possible to extent the list of the requirements accepted in the definitions for the epistemic satisfaction relation in order to choose the most relevant formula from all the formulas available for the cognitive agent. In particular, in order to determine the degree of relevance, the structural and temporal characteristics of the overall body of temporal data representing stored perceptions can also be considered and used. In fact, some steps towards this direction have already been made (Katarzyniak and Pieczyńska-Kuchtiak, 2002; 2003). The resulting models are more complex and require advanced computational techniques to be implemented. Thirdly, it is interesting to study the relation between the epistemic satisfaction relation and the possible worlds semantics applicable to the same class of modal formulas. In particular, it is necessary to study the relation between Kripke model structures and the content of temporal databases given as $KnowledgeState(t)$. Some elements interesting for such a study have already been suggested in this paper, namely, when sets of possible worlds related to base profiles and accessible for the cognitive agent have been defined.

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