

## POWERFUL NONLINEAR OBSERVER ASSOCIATED WITH FIELD-ORIENTED CONTROL OF AN INDUCTION MOTOR

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In this paper, we associate field-oriented control with a powerful nonlinear robust flux observer for an induction motor to show the improvement made by this observer compared with the open-loop and classical estimator used in this type of control. We implement this design strategy through an extension of a special class of nonlinear multivariable systems satisfying some regularity assumptions. We show by an extensive study that this observer is completely satisfactory at low and nominal speeds and it is not sensitive to disturbances and parametric errors. It is robust to changes in load torque, rotational speed and rotor resistance. The method achieves a good performance with only one easier gain tuning obtained from an algebraic Lyapunov equation. Finally, we present results and simulations with concluding remarks on the advantages and perspectives for the observer proposed with the field-oriented control.

**Keywords:** induction motor, field-oriented control, nonlinear observer, rotor resistance

### 1. Introduction

Induction motors are widely used in industry due to their relatively low cost and high reliability. One way to obtain a speed or torque control with a dynamic performance similar to that of a more expensive DC-motor is to use Field-Oriented Control (FOC) (Blaschke, 1972; Bekkouche *et al.*, 1998; Mansouri *et al.*, 1997). Many other methods have been suggested but, in general, an estimate of the rotor flux is needed in most of these control schemes. Therefore a rotor flux observer must be employed. The dynamic behaviour of the induction motor is affected by time variations, mainly in the rotational speed and in the rotor resistance. The rotor flux observer must be robust with respect to these variations. The simplest flux estimation method is an open-loop observer based on stator current measurements (Grellet and Clerc, 1996). This method suffers from poor robustness and a slow convergence rate. Several methods have been suggested to overcome this, but most of them are hard to tune or difficult to implement. It is shown in (Gauthier and Bornard, 1981) that the major difficulty in implementing the high gain observer comes from the fact that the gain is depen-

dent on the coordinate transformation and it necessitates the inversion of its Jacobian. Another approach for designing nonlinear observers is to consider the properties of 'richness' or 'persistency' of inputs in the design strategy (Bornard *et al.*, 1988). In this respect, Bornard and Hammouri (1991) designed an observer for a class of nonlinear systems under 'locally regular inputs.' However, we obtain the gain of the observer from some differential equations which are not usually desirable for implementation purposes. For industrial purposes, the ideal observer scheme is easy to implement in hardware and does not require tuning.

In this paper a robust flux observer is developed using a multivariable systems approach (Busawon *et al.*, 1998). The observer does not require any kind of transformation to update its gain and is explicitly obtained from the solution of the algebraic Lyapunov equation. As a result, its implementation is greatly facilitated. In the first section, we present a model of an induction motor and field-oriented control. In the following section, the flux observer in both open and closed loops and the proposed nonlinear observer are introduced. Finally, a comparison in simulation between these three estimators is given. The

concluding remarks on the advantages and perspectives for the observer proposed with the field-oriented control are then given.

## 2. Motor Model and Field-Oriented Control

Modern control techniques often require a state-space model (Van Raumer *et al.*, 1994). The state-space representation of the asynchronous motor depends on the choice of the reference frame  $(\alpha, \beta)$  or  $(d, g)$  and on the state variables selected for the electric equations. We write the equations in the frame  $(d, g)$  because it is the most general and most complex solution, the frame  $(\alpha, \beta)$  being only its one particular case. Nevertheless, the use of the frame  $(d, g)$  implies exact knowledge of the position of this frame. The choice of the state variable  $x$  depends on the objectives of the control or observation. For a complete model, the mechanical speed  $\Omega$  is a state variable. The outputs to be independently controlled are the norm of the rotor flux and the torque. The rotor flux norm needs to be controlled for system optimization (e.g. power efficiency, torque maximization) while changing operating conditions and under inverter limits (Garcia *et al.*, 1994; Bodson and Chiasson, 1992). Torque control is essential for high dynamic performances. Once the torque is controlled, the speed and position can be controlled by simple outer linear loops, at least, if the load does not have significantly nonlinear dynamics (De Wit *et al.*, 1995).

As state variables, we choose the two components of stator currents, the two components of the rotor flux and the mechanical speed. As for the output  $y$ , the torque and the square of the rotor flux norm and for the input voltage, the stator voltage input  $u$  is selected. We can then write the model equations in the reference frame  $(d, g)$  as follows:

$$\dot{x} = f(x) + gu \quad (1)$$

and

$$y(x) = \begin{bmatrix} p \frac{M}{L_r} (\varphi_{rd} i_{sq} - \varphi_{rd} i_{sd}) \\ \varphi_{rd}^2 + \varphi_{rd}^2 \end{bmatrix}, \quad (2)$$

where

$$x = [i_{sd}, i_{sq}, \varphi_{rd}, \Omega]^T, \quad u = [u_{sd}, u_{sq}]^T,$$

$$f(x) = \begin{bmatrix} -\gamma i_{sd} + \omega_s i_{sq} + \frac{K}{T_r} \varphi_{rd} + p\Omega K \varphi_{rd} \\ -\omega i_{sd} - \gamma i_{sq} - p\Omega K \varphi_{rd} + \frac{K}{T_r} \varphi_{rd} \\ \frac{M}{T_r} i_{sd} - \frac{1}{T_r} \varphi_{rd} + (\omega_s - p\Omega) \varphi_{rq} \\ \frac{M}{T_r} i_{sq} - (\omega_s - p\Omega) \varphi_{rd} - \frac{1}{T_r} \varphi_{rq} \\ p \frac{M}{J_m L_r} (\varphi_{rd} i_{sq} - \varphi_{rq} i_{sd}) - \frac{f_m \Omega}{J_m} - \frac{\tau_L}{J_m} \end{bmatrix}$$

and

$$g = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix}^T$$

with

$$T_r = \frac{L_r}{R_r}, \quad \sigma = 1 - \frac{M^2}{L_s L_r},$$

$$K = \frac{M}{\sigma L_s L_r}, \quad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}.$$

$L_r$ ,  $L_s$  and  $M$  are the rotor, stator and mutual inductances, respectively,  $R_r$  and  $R_s$  are respectively rotor and stator resistances,  $\sigma$  is the scattering coefficient,  $T_r$  is the time constant of the rotor dynamics,  $J_m$  is the rotor inertia,  $f_m$  is the mechanical viscous damping,  $p$  is the pole pair induction, and  $\tau_L$  is the external load torque. We describe the induction motor in the stator fixed frame  $(\alpha, \beta)$  with the previous equations by setting  $\omega_s = 0$ , which is the pulsation of stator currents, and by replacing the indices  $(d, q)$  by  $(\alpha, \beta)$ , respectively. Good characteristics of the model  $(d, q)$  appear when we choose for  $\theta_s$  a particular orientation of the rotor flux such as

$$\varphi_{rd} = 0, \quad \text{with} \quad \theta_s = \int_0^t \omega_s d\tau.$$

Consider the following feedback nonlinear state where  $v_d$  and  $v_q$  are auxiliary controls inputs:

$$\begin{pmatrix} u_{sd} \\ u_{sq} \end{pmatrix} = \sigma L_s \begin{pmatrix} -\frac{K}{T_r} \varphi_{rd} - p\Omega i_{sq} - \frac{M}{T_r} \frac{i_{sq}^2}{\varphi_{rd}} + v_d \\ pK\Omega \varphi_{rd} + p\Omega i_{sd} + \frac{M}{T_r} \frac{i_{sd} i_{sq}}{\varphi_{rd}} + v_q \end{pmatrix}. \quad (3)$$

Consequently, we obtain a simple system, with the dynamics of the module of linear flux,

$$\frac{d}{dt} \varphi_{rd} = -\frac{1}{T_r} \varphi_{rd} + \frac{M}{T_r} i_{sd},$$

$$\frac{d}{dt} i_{sd} = -\gamma i_{sd} + v_d. \quad (4)$$

As was shown in (Marino *et al.*, 1993; Van Raumer *et al.*, 1994), we can control the dynamics of the amplitude of the flux by  $v_d$  via two PI regulators  $H_1(s)$  and  $H_3(s)$  as shown in Fig. 1. Here we set

$$i_{sd}^* = H_1(s)(\varphi_{ref} - \varphi_{rd}),$$

$$v_d = H_3(s)(i_{sd}^* - i_{sd}), \quad (5)$$

so that  $i_{sd}^*$  and  $\varphi_{ref}$  represent respectively the reference stator current and the reference rotor flux, in the axis  $d$ .

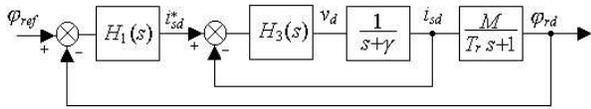


Fig. 1. Flux regulation.

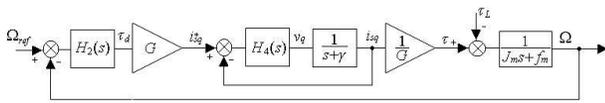
When the amplitude of the rotor flux  $\varphi_{rd}$  reaches its reference, which is constant, the dynamics rotor speed becomes linear too. For the following second subsystem, we have

$$\begin{aligned} \frac{d}{dt}\Omega &= p\frac{M}{J_m L_r}\varphi_{ref}i_{sq} - \frac{f_m}{J_m}\Omega - \frac{\tau_L}{J_m}, \\ \frac{d}{dt}i_{sq} &= -\gamma i_{sq} + v_q. \end{aligned} \quad (6)$$

The rotor speed can be controlled by  $v_q$  via two PI regulators,  $H_2(s)$  and  $H_4(s)$ , as shown in Fig. 2. Here we set

$$\begin{aligned} i_{sq}^* &= H_2(s)(\Omega_{ref} - \Omega), \\ v_q &= H_4(s)(i_{sq}^* - i_{sq}), \end{aligned} \quad (7)$$

$i_{sq}^*$  and  $\Omega_{ref}$  representing respectively the reference stator current in the axis  $q$  and the reference rotor speed. We take the PI regulator  $H_i(s) = k_p(s + k_i/k_p)/s$ .


 Fig. 2. Speed regulation where  $G = L_r/pM\varphi_{ref}$ .

### 3. Flux Observer in an Open and Closed Loops

In what follows, we present the classical flux observers existing in the literature.

#### 3.1. Flux Observer in an Open Loop

Until now we have assumed that all states including the rotor flux norm and the angle could be measured. In general, this assumption does not hold. This problem has been a longstanding research topic and generally there are two ways to solve it. The first one is to estimate the rotor flux angle and the amplitude, while the other is to use reference values for these two quantities. As an example of the first method, we estimate the rotor flux in an open loop from stator current measurements using the equations of the model  $(\alpha, \beta)$ . It is a version of the system equations

where we use only the flux estimate:

$$\frac{d}{dt}\hat{\varphi}_{r\alpha} = \frac{M}{T_r}i_{s\alpha} - \frac{1}{T_r}\hat{\varphi}_{r\alpha} - p\Omega\hat{\varphi}_{r\beta}, \quad (8)$$

$$\frac{d}{dt}\hat{\varphi}_{r\beta} = \frac{M}{T_r}i_{s\beta} + p\Omega\hat{\varphi}_{r\alpha} - \frac{1}{T_r}\hat{\varphi}_{r\beta}.$$

Expressing (8) in the reference frame  $(d, q)$ , through the transformation given in (Vas, 1990), we find

$$\dot{\hat{\varphi}}_{rd} = \frac{M}{T_r}\hat{i}_{sd} - \frac{1}{T_r}\hat{\varphi}_{rd}, \quad (9)$$

$$\dot{\hat{\theta}}_s = \omega_s = p\Omega + \frac{M}{T_r}\frac{\hat{i}_{sq}}{\hat{\varphi}_{rd}}. \quad (10)$$

The classical direct field-oriented control uses an estimate in an open loop, i.e. without gain. The disadvantage of this control is its sensitivity to perturbations and parametric errors, especially to changes in the rotor time constant  $T_r$ .

#### 3.2. Flux Observer in a Closed Loop

We will present here some observers proposed in the literature, as well as an observer developed especially in the context of the nonlinear study which is going to be outlined. A classical reference on the flux observers is the observer proposed in (Verghese and Sanders, 1988) whose versions were presented in (De Luca and Ulivi, 1989; Garcia *et al.*, 1994; Mansouri *et al.*, 2002). The observer is of the form

$$\begin{aligned} \begin{bmatrix} \frac{d\hat{i}_s}{dt} \\ \frac{d\hat{\varphi}_r}{dt} \end{bmatrix} &= \left\{ \begin{bmatrix} -\gamma I & (K/T_r)I \\ (M/T_r)I & (-1/T_r)I \end{bmatrix} + \Omega \begin{bmatrix} 0 & -KJ \\ 0 & J \end{bmatrix} \right\} \\ &\times \begin{bmatrix} \hat{i}_s \\ \hat{\varphi}_r \end{bmatrix} + \begin{bmatrix} (1/\sigma L_s)I \\ 0 \end{bmatrix} u_s \\ &+ \begin{bmatrix} k_1 I + k_2 \Omega J \\ k_3 I + k_4 \Omega J \end{bmatrix} (\hat{i}_s - i_s), \end{aligned} \quad (11)$$

where

$$\hat{i}_s = [\hat{i}_{s\alpha}, \hat{i}_{s\beta}]^T, \quad \hat{\varphi}_r = [\hat{\varphi}_{r\alpha}, \hat{\varphi}_{r\beta}]^T, \quad u_s = [u_{s\alpha}, u_{s\beta}]^T,$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

the  $k_i$ 's being scalars. Note that the gains depend on the speed in (11). We show the diagram block of this observer

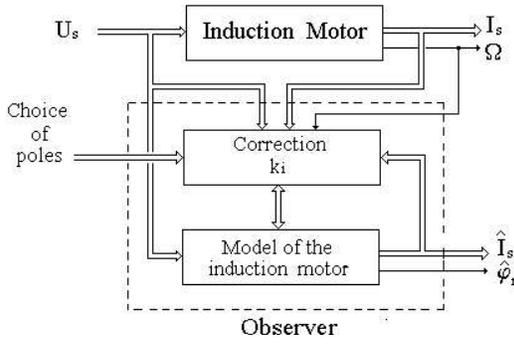


Fig. 3. Closed-loop observer block diagram.

in Fig. 3. The resulting model for the observer error dynamics is then

$$\frac{de}{dt} = \left\{ \begin{bmatrix} (k_1 - \gamma)I & (K/T_r)I \\ [k_3 + (M/T_r)I] & (-1/T_r)I \end{bmatrix} + \Omega \begin{bmatrix} k_2J & -KJ \\ k_4I & J \end{bmatrix} \right\} e, \quad (12)$$

where

$$e = \begin{bmatrix} \hat{i}_s - i_s \\ \hat{\varphi}_r - \varphi_r \end{bmatrix}.$$

Note that we can freely determine the scalar coefficients in the left-hand blocks of the two matrices in (12). If  $k_1$  and  $k_3$  are selected such that

$$k_1 - \gamma = -\frac{k_2}{T_r}, \quad k_3 + \frac{M}{T_r} = -\frac{k_4}{T_r},$$

the error dynamics become

$$\frac{de}{dt} = AQ(\Omega)e,$$

where

$$A = \begin{bmatrix} k_2I & -KI \\ k_4I & I \end{bmatrix},$$

$$Q(\Omega) = \begin{bmatrix} \left(-\frac{1}{T_r}\right)I + \Omega J & 0 \\ 0 & \left(-\frac{1}{T_r}\right)I + \Omega J \end{bmatrix}.$$

We select  $k_1$  and  $k_4$  to place the eigenvalues of  $A$  in arbitrary positions. Note that the characteristic polynomial of  $A$  is  $[p^2 - (1 + k_2)p + k_2 + k_4K]^2$ .

If the eigenvalues of  $A$  are  $p_1$  (twice) and  $p_2$  (twice), then the eigenvalues of  $AQ(\Omega)$  are

$$[(-1/T_r) \pm j\Omega] p_1, \quad [(-1/T_r) \pm j\Omega] p_2. \quad (13)$$

## 4. Observer Design for a Special Class of Nonlinear Systems

We present now extensions of the observer design strategy to the multi-output case (Busawon et al., 1998; Chenafa et al., 2002) and an application to the induction motor.

### 4.1. Extensions of the Observer Design Strategy to the Multi-Output Case

In this section, we show how the previous observer designs can be extended to a class of multi-output systems which may assume stronger nonlinear dependencies on state variables. Consider multi-output systems of the following form:

$$\begin{cases} \dot{z}_1 = F_1(s, y)z_2 + g_1(u, s, z_1), \\ \dot{z}_2 = F_2(s, y)z_3 + g_2(u, s, z_1, z_2), \\ \vdots \\ \dot{z}_{n-1} = F_{n-1}(s, y)z_n + g_{n-1}(u, s, z_1, \dots, z_{n-1}), \\ \dot{z}_n = g_n(u, s, z), \\ y = z_1, \end{cases} \quad (14)$$

where

$$z_l \in \mathbb{R}^q, \quad l = 1, \dots, n, \quad z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} \in \mathbb{R}^{n \times q},$$

$u \in \mathbb{R}^m$ ,  $y \in \mathbb{R}^q$  and  $s(t)$  is a known signal.  $F_l$  are  $q \times q$  square matrices and  $g_l = (g_{l1}, \dots, g_{lq})$ ,  $l = 1, \dots, n$ .

We can write the system (14) in the following compact form:

$$\begin{cases} \dot{z} = F(s, y)z + G(u, s, z), \\ y = Cz, \end{cases} \quad (15)$$

where

$$F(s, y) = \begin{bmatrix} 0 & F_1(s, y) & & 0 \\ \vdots & & \ddots & \\ & & & F_{n-1}(s, y) \\ 0 & \dots & \dots & 0 \end{bmatrix},$$

$$G(u, s, z) = \begin{bmatrix} g_1(u, s, z) \\ \vdots \\ g_n(u, s, z) \end{bmatrix}, \quad C = [I_q, 0, \dots, 0],$$

$C$  is of appropriate dimensions and  $I_q$  is the  $(q \times q)$  identity matrix.

We note that unlike in the previous sections, each of the matrices  $F_l$ ,  $l = 1, \dots, n - 1$  now stands for

any square matrix satisfying the assumptions below. The nonlinearities are block triangular and each block has the same dimension  $q$ . Also, all the outputs are regrouped in the first subsystem. Note that the block-triangular structure of the system (14) allows stronger coupling between the nonlinearities, for which the triangular coupling is found within each subsystem. To see this, consider the system

$$\begin{cases} \dot{z}_{11} = f_{11}(y)z_{21} + f_{12}(y)z_{22}, \\ \dot{z}_{12} = f_{21}(y)z_{21} + f_{22}(y)z_{22}, \\ \dot{z}_{21} = g_{21}(z), \\ \dot{z}_{22} = g_{22}(z), \\ y_1 = z_{11}, \\ y_2 = z_{12}. \end{cases}$$

Here, we make the following assumptions:

- (A1) There exists a class  $U$  of bounded admissible controls, a compact set  $K \subset \mathbb{R}^{n \times q}$  and positive constants  $\alpha, \beta > 0$  such that for every  $u \in U$  and every output  $y(t)$  associated with  $u$  and with an initial state  $z(0) \in K$ , we have  $0 < \alpha I_q \leq F_l^T(s, y)F_l(s, y) \leq \beta I_q$ ,  $l = 1, \dots, n-1$ .
- (A2)  $s(t)$  and its time derivative  $ds(t)/dt$  are bounded.
- (A3) The matrices  $F_l(s, y)$ ,  $l = 1, \dots, n-1$  are of class  $C^r$ ,  $r \geq 1$  with respect to their arguments.
- (A4) The functions  $g_l$ ,  $l = 1, \dots, n$  are global Lipschitz with respect to  $z$  uniformly in  $u$  and  $s$ .

We characterize the observer design for the system (15) in the following theorem (Busawon *et al.*, 1998).

**Theorem 1.** Assume that the system (15) satisfies Assumptions (A1) to (A4). Then there exists  $\theta > 0$  such that the system

$$\begin{aligned} \dot{\hat{z}} &= F(s, y)\hat{z} + G(u, s, \hat{z}) \\ &\quad - \Lambda^{-1}(s, y)S_\theta^{-1}C^T(C\hat{z} - y) \end{aligned} \quad (16)$$

is an exponential observer for the system (15), where

- $S_\theta$  is the unique solution of the algebraic Lyapunov equation

$$\theta S_\theta + A^T S_\theta + S_\theta A - C^T C = 0 \quad (17)$$

with  $\theta > 0$  as a parameter, and

$$A = \begin{bmatrix} 0 & I_q & & 0 \\ \vdots & & \ddots & \\ & & & I_q \\ 0 & \dots & \dots & 0 \end{bmatrix} r,$$

- the matrix  $\Lambda(s, y)$  is defined as

$$\Lambda(s, y) = \begin{bmatrix} C \\ CF(s, y) \\ \vdots \\ CF^{n-1}(s, y) \end{bmatrix}$$

$$= \begin{bmatrix} I_q & & & & 0 \\ & F_1(s, y) & & & \\ & & F_1(s, y)F_2(s, y) & & \\ & & & \ddots & \\ 0 & & & & \prod_{l=1}^{n-1} F_l(s, y) \end{bmatrix}.$$

Moreover, we can make the dynamics of this observer arbitrarily fast (Busawon *et al.*, 1998).

However, we carry out all the computations in a block-wise fashion, based essentially on the following facts:  $F(s, y) = \Lambda^{-1}(s, y)A\Lambda(s, y)$  and  $C\Lambda(s, y) = C$ . So, by multiplying the left- and right-hand sides of (17) by  $\Lambda^T(s, y)$  and  $\Lambda(s, y)$  respectively, the following algebraic equation holds:

$$\begin{aligned} \theta \bar{S}_\theta(s, y) + F^T(s, y)\bar{S}_\theta(s, y) \\ + \bar{S}_\theta(s, y)F(s, y) - C^T C = 0, \end{aligned} \quad (18)$$

where

$$\bar{S}_\theta = \Lambda^T(s, y)S_\theta\Lambda(s, y).$$

Note that the closed-form solution of (18) is

$$S_\theta(i, j) = \frac{(-1)^{i+j} C_{i+j-2}^{j-1}}{\theta^{i+j-1}} I_q, \quad 1 \leq i, j \leq n. \quad (19)$$

We can show the diagram block of this observer in Fig. 4.

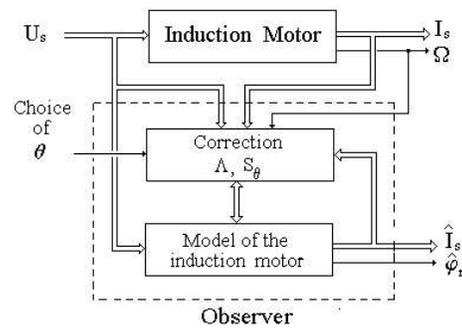


Fig. 4. Nonlinear observer diagram.

## 4.2. Application to the Induction Motor

In this section, we are going to apply the result given in the preceding part to construct a reduced flux observer for an induction motor written in the  $\alpha, \beta$  Park frame. The proposed observer uses the measurements of the stator voltage and current, and the rotor speed. More precisely, the observer is designed up to an injection of the speed measurements so that only electrical equations are considered. As will be seen below, the model is of the form given by (14). Consequently, the gain can be updated directly, as described in the theorem, without making use of any kind of transformation.

Consequently, the system (15) is of the form (14), where  $n = q = 2$ . We have

$$\begin{cases} \dot{z} = F(\Omega)z + G(u, \Omega, z), \\ y = Cz, \end{cases} \quad (20)$$

where

$$z_1 = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}, \quad z_2 = \begin{bmatrix} \varphi_{r\alpha} \\ \varphi_{r\beta} \end{bmatrix}, \quad u = \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix},$$

$$y = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}, \quad s = \Omega, \quad F_1(\Omega) = \begin{bmatrix} \frac{K}{T_r} & Kp\Omega \\ -Kp\Omega & \frac{K}{T_r} \end{bmatrix},$$

$$g_1(u, \Omega, z_1) = \begin{bmatrix} -\gamma i_{s\alpha} + \frac{1}{\sigma L_s} u_{s\alpha} \\ -\gamma i_{s\beta} + \frac{1}{\sigma L_s} u_{s\beta} \end{bmatrix}$$

and

$$g_2(u, \Omega, z) = \begin{bmatrix} \frac{M}{T_r} i_{s\alpha} - \frac{1}{T_r} \varphi_{r\alpha} - p\Omega \varphi_{r\beta} \\ \frac{M}{T_r} i_{s\beta} + p\Omega \varphi_{r\alpha} - \frac{1}{T_r} \varphi_{r\beta} \end{bmatrix}.$$

Now, assume that the speed and its time derivatives are bounded. Then Assumptions (A1) to (A4) can easily be checked. Hence we design an observer of the form (16) for the system (20) in Eqn. (21):

$$\dot{\hat{z}} = F(\Omega)\hat{z} + G(u, \Omega, \hat{z}) - \Lambda^{-1}(\Omega)S_\theta^{-1}C^T(C\hat{z} - y), \quad (21)$$

where

$$\Lambda(\Omega) = \begin{bmatrix} I_2 & 0 \\ 0 & F_1(\Omega) \end{bmatrix},$$

$$S_\theta^{-1}C^T = \begin{bmatrix} 2\theta I_2 \\ \theta^2 I_2 \end{bmatrix}.$$

The choice of  $\theta$  permits the pole placement of the motor and the observer according to the speed.

## 5. Simulation Results

We have performed simulations using Matlab-Simulink on the benchmark of Fig. 9 and the motor parameters given in Table 1. We studied the performances of the three observers in open and closed loops associated with field-oriented control of the induction motor with an increase of 200% on the rotor constant  $T_r$ .

Table 1. Parameters of the induction motor.

Parameter	Notation	Value
Rotor resistance	$R_r$	4.3047 $\Omega$
Stator resistance	$R_s$	9.65 $\Omega$
Mutual inductance	$M$	0.4475 H
Stator inductance	$L_s$	0.4718 H
Rotor inductance	$L_r$	0.4718 H
Rotor inertia	$J_m$	0.0293 kg/m <sup>2</sup>
Pole pair	$p$	2
Viscous friction coefficient	$f_m$	0.0038 N·m·sec·rad <sup>-1</sup>

### 5.1. Simulations Block Diagrams, Motor Data and a Benchmark

We have designed block diagrams, as shown in Figs. 5–8. The parameters of the induction motor used in simulation (Cauët, 2001) are given in Table 1. The trajectories of the references speed, flux and load torque are given in Fig. 9. This benchmark shows that the load torque appears at the nominal speed. In spite of a varying speed, the resistive torque is zero. The desired flux remains constant in the asynchronous machine to satisfy the objectives of the field-oriented control.

### 5.2. Motor and Observer Poles Depending on the Speed

In the first case, we consider a closed-loop observer. The behaviour of the observers varies considerably depending on whether the eigenvalues are real (Verghese and Sanders, 1988) or complex (De Luca, 1989; Bellini et al., 1988). Indeed, in the latter case, the convergence speed, which is a function of the speed and the damping ratio, can be improved. To illustrate this, we simulated the trajectory of the poles of the motor and the observer, cf. (11), by taking account of the experimental values given in Section 5. We took real poles  $p_1 = 0.7$  and  $p_2 = 1$  in Fig. 10, and complex poles  $p_1 = 1 + 0.15j$  and  $p_2 = 0.5 + 0.2j$  in Fig. 11.

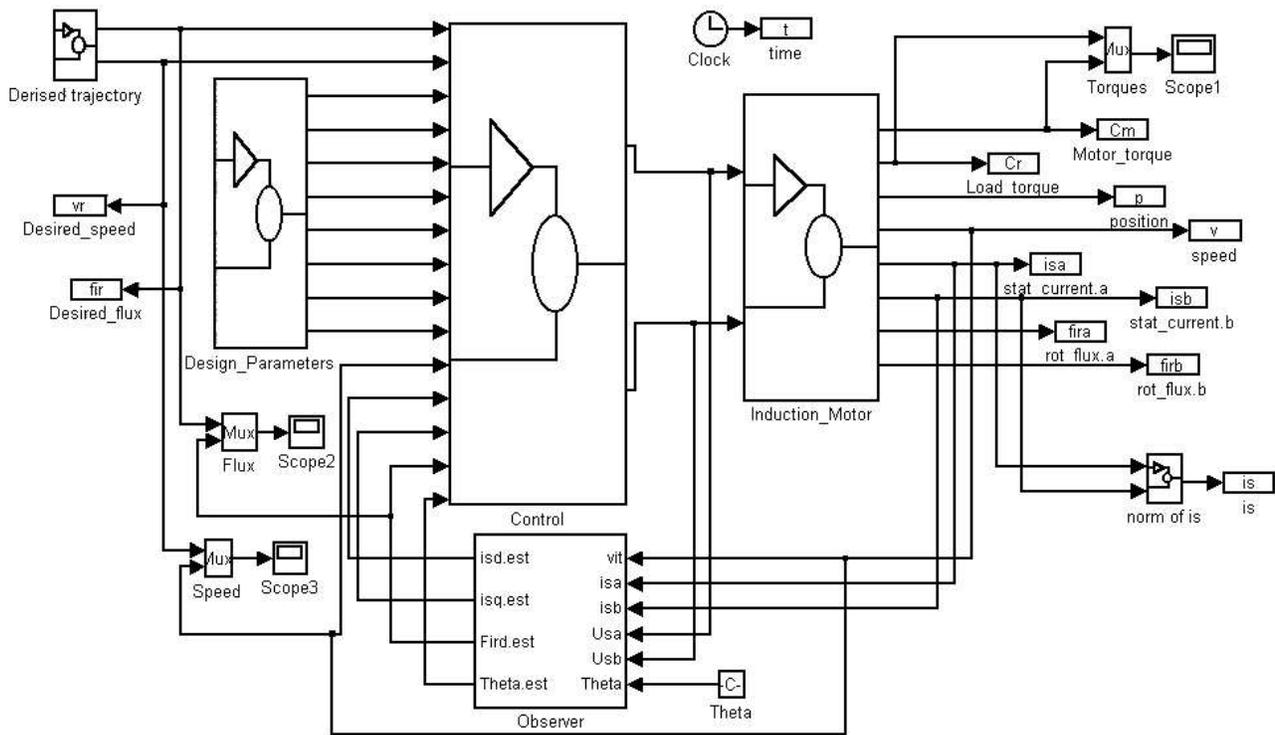


Fig. 5. General block diagram in Simulink.

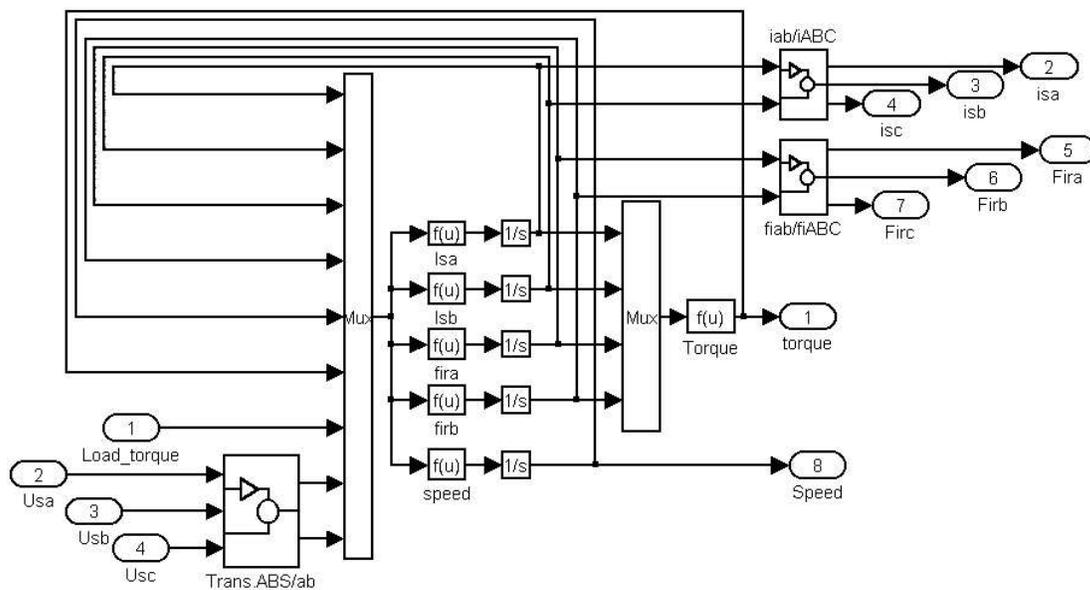


Fig. 6. Induction motor in Simulink.

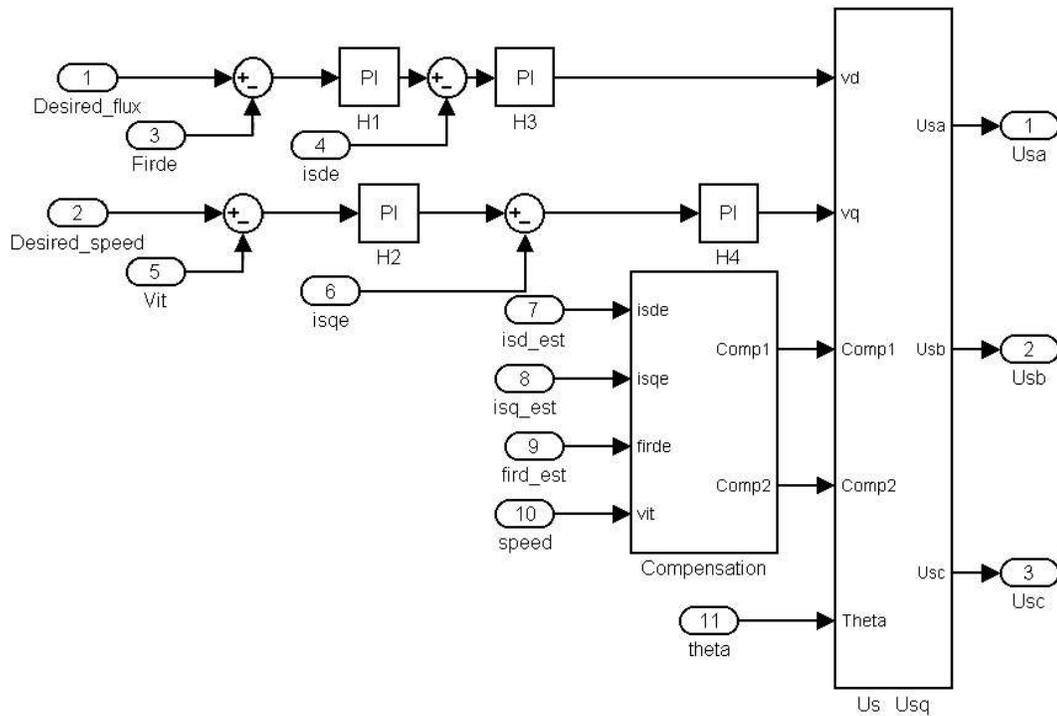


Fig. 7. Control block in Simulink.

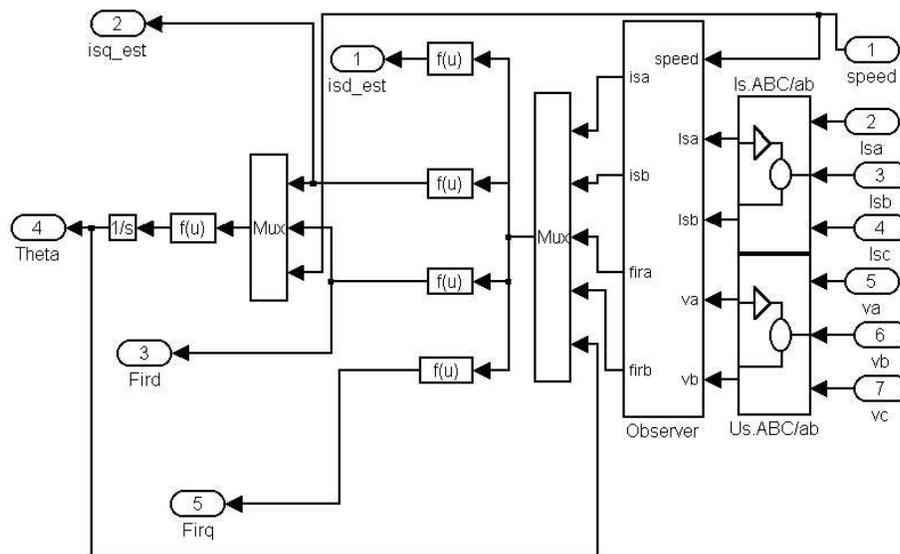


Fig. 8. Observer block in Simulink.

In the second case, we simulated the pole placement of the motor and the observer as a function of the speed resulting of the choice of  $\theta$ . For example, the values of

$\theta$  equal to three and five were selected for simulations of Fig. 12.

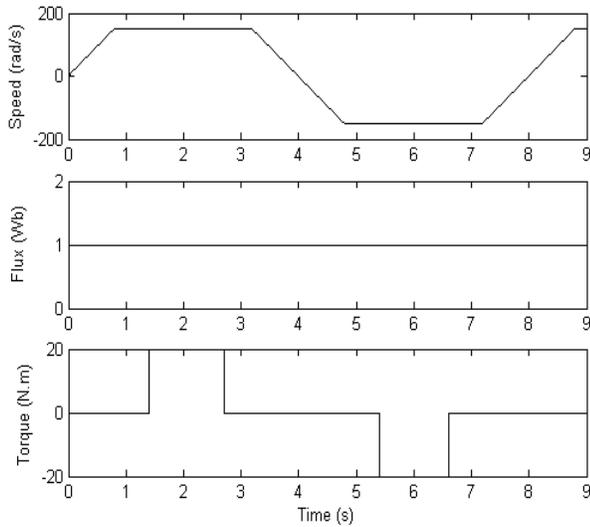


Fig. 9. Reference trajectories.

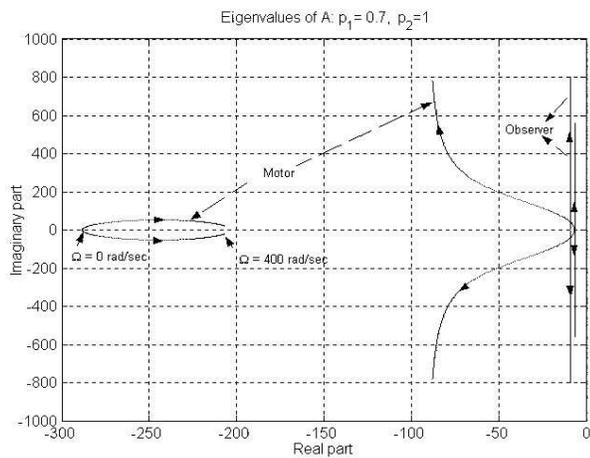


Fig. 10. Motor and observer poles as functions of speed with real eigenvalues of  $A$ .

### 5.3. Implementation of the PI Controllers

By imposing a time constant  $T_d = 0.3$  ms and  $T_q = 5$  ms for dynamic currents  $i_{sd}$  and  $i_{sq}$ , respectively, with a unit static gain, and by compensating their poles with the zeros of their respective regulators, for the current  $i_{sd}$  we found  $k_{p3} = 1/T_d$ ,  $k_{i3} = \gamma/T_d$ . The same procedure for the current  $i_{sq}$  gives  $k_{p4} = 1/T_q$ ,  $k_{i4} = \gamma/T_q$ .

For the synthesis of the corrector flux and speed, we replace the two dynamic components of the currents by their transfer functions imposed previously. Finally, by compensating the poles of both systems (flux and speed) by the zeros of the PID controller and by imposing a pole which is  $a$  times faster than that of the flux, and in the same way for the current  $i_{sq}$ ,  $b$  times faster than that

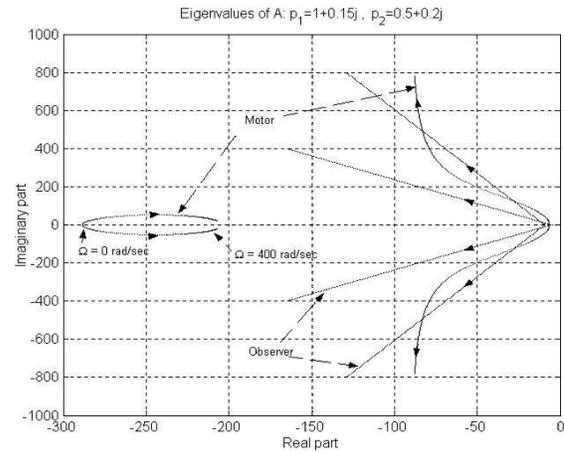


Fig. 11. Motor and observer poles as functions of speed with complex eigenvalues of  $A$ .

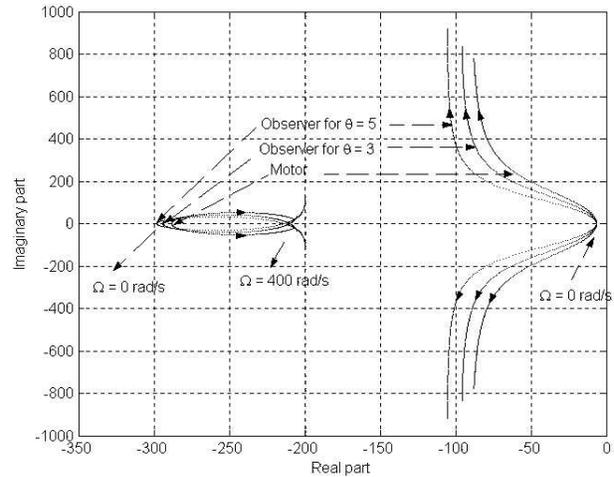


Fig. 12. Pole trajectory of the motor and the proposed nonlinear observer.

of the speed, we found for the flux  $k_{p1} = T_r/(aMT_d)$ ,  $k_{i1} = 1/(aMT_d)$ . In the same way we also found for the speed  $k_{p2} = J_m/(bT_q)$ ,  $k_{i2} = f_m/(bT_q)$ . We chose  $a = 100$  and  $b = 10$ .

### 5.4. Open-Loop Observer Performance

We simulated an error in estimation of the three observers simultaneously at a low (230 tr/mm) and a nominal (1500 tr/mm) speed. The results of the simulation, cf. Figs. 13 and 14, clearly show good transient performances of the proposed nonlinear observer compared with the other observers.

We chose the gain  $\theta = 500$  to clearly show the advantage of this observer valid at low and nominal speeds. For the closed-loop observer, we chose poles at  $p_1 =$

$p_2 = 2$  in the nominal case, to obtain good dynamics at a nominal speed and to suppress the transitory mode. On the other hand, only one adjustment of  $\theta$  enables us to obtain good performances within the range of the speed variation. The test permits to simulate the convergence of the three observers with different values from the actual values of the flux in the motor (variation of 1 Wb in the flux).

### 5.5. Performances of the Observers Associated with the Field-Oriented Control

#### Tracking speed

The magnitude of the error speed is lowest in the case of the nonlinear observer associated with the field-oriented control. Its sign is opposed to the sign of the load torque. Peaks appear at the times of 4.2 sec and 8.4 sec, i.e.

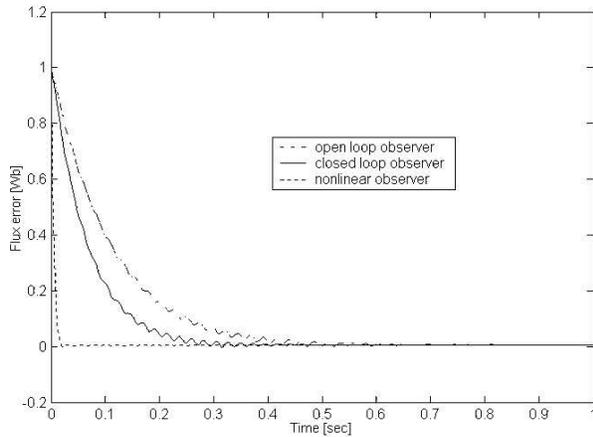


Fig. 13. Observation errors of the flux at a low speed of 230 tr/mn.

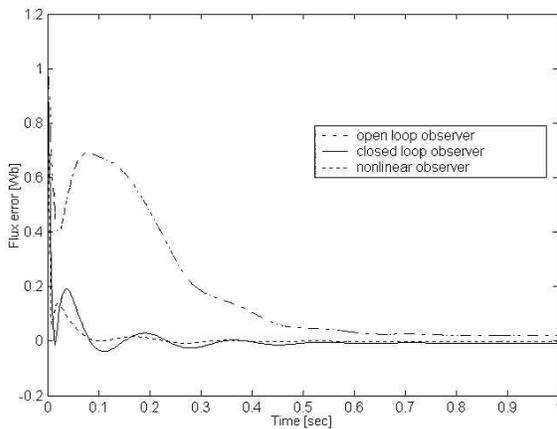


Fig. 14. Observation errors of the rotor flux at a nominal speed (1500 tr/mn).

at the times of the change in the speed sign as shown in Fig. 15.

#### Torque

Between  $t = 0$  and 0.8 sec, during the linear growth of the speed, the load torque corresponds to better damping for the nonlinear observer with the control, whereas the resistive torque is zero. Between  $t = 0.8$  and 3.2 sec, the speed is constant and the motor torque follows the load torque, no matter whether it is zero or equal to 20 N·m. The cycle begins again between 4 and 9 sec in the opposite direction as shown in Fig. 16.

#### Flux estimation error

At a constant speed and zero torque, we note the cancellation of the observation error. On the other hand, the effect of the load torque appears in Fig. 17 between 1.5 and 2.5 sec and between 5.5 and 6.5 sec where the flux

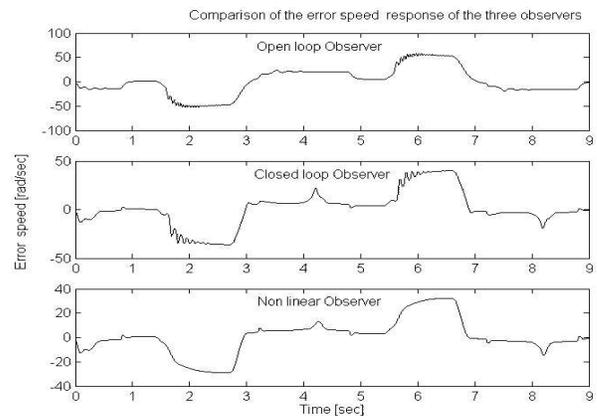


Fig. 15. Comparison of the error speed for 200% variation in  $T_r$ , at  $\theta = 50$ ,  $p_1 = p_2 = 2$ .

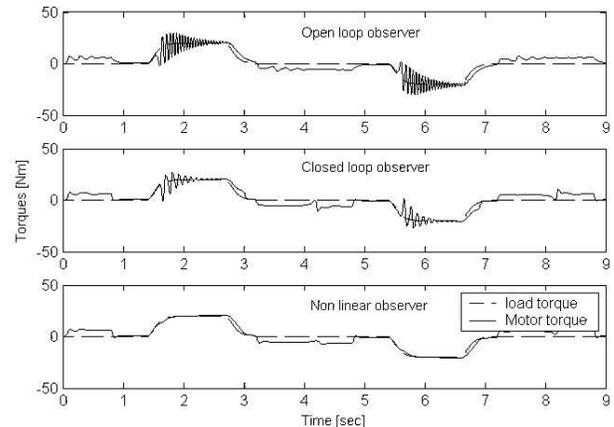


Fig. 16. Motor and load torques of field-oriented control.

estimation error has a non-zero constant value. A peak appears when the speed is zero. The nonlinear observer shows the best characteristics.

### Stator current norm

We note that the norm of the stator currents is significant when a couple of loads are applied. A peak also appears when the speed changes the sign. The amplitude of this current norm is least significant and of a smooth form for the nonlinear observer, as shown in Fig. 18.

## 6. Conclusion

We have proposed a nonlinear observer of a special class associated with field-oriented flux control. The evaluation regarding the robustness of its performances compared with the traditional estimator in an open loop and

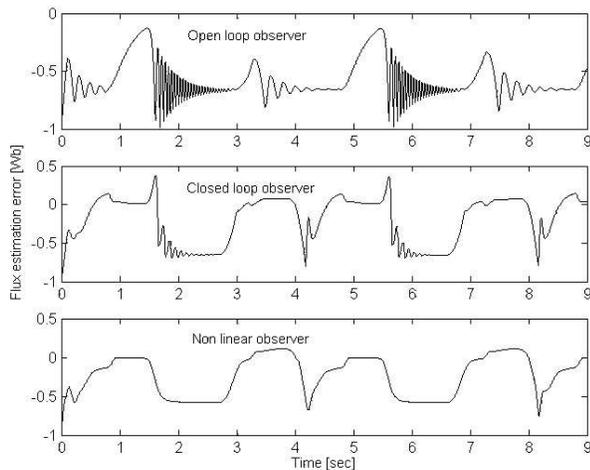


Fig. 17. Errors in flux estimation.

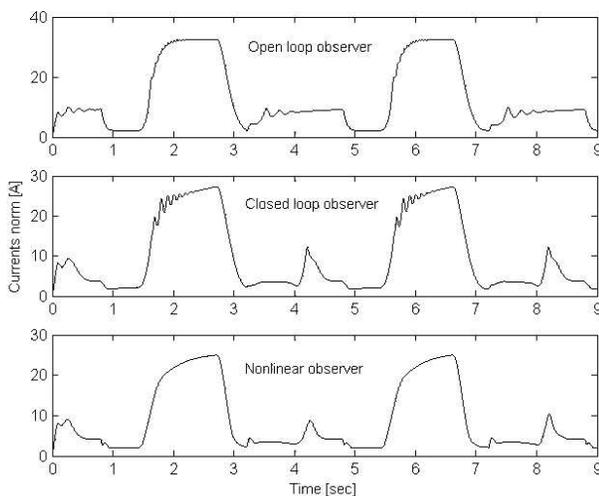


Fig. 18. Currents norm stator.

the observer in a closed loop was made when the rotor resistance varied considerably. The results show that this nonlinear observer offers better performances while tracking the torque, speed and estimating the flux. It presents only one adjustment of the gain in the range of the varying speed and it is easy to control, compared with that proposed in the closed loop, which requires the adjustment of two gains under the constraint on the speed at low values or in the nominal case. A major advantage of the method is that very little tuning was required to obtain the convergence of the observation at low speeds. We hope to perform experiments on-line to validate these theoretical results.

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