Int. J. Appl. Math. Comput. Sci., 2006, Vol. 16, No. 1, 27–35

FUZZY DIAGNOSTIC REASONING THAT TAKES INTO ACCOUNT THE UNCERTAINTY OF THE RELATION BETWEEN FAULTS AND SYMPTOMS

JAN MACIEJ KOŚCIELNY, MICHAŁ SYFERT

Institute of Automatic Control and Robotics, Warsaw University of Technology
ul. Św. Andrzeja Boboli 8, 02–525 Warsaw, Poland,
e-mail: {jmk,msyfert}@mchtr.pw.edu.pl

Knowledge about the relation between faults and the observed symptoms is necessary for fault isolation. Such a relation can be expressed in various forms, including binary diagnostic matrices or information systems. The paper presents the use of fuzzy logic for diagnostic reasoning. This method enables us to take into account various kinds of uncertainties connected with diagnostic reasoning, including the uncertainty of the faults-symptoms relation. The presented methods allow us to determine the fault certainty factor as well as certainty factors of the normal and unknown process states. The unknown process state factor groups all the states with unknown and multiple faults with the states with improper residual values, while the normal state factor indicates similarity between the observed state and the pattern fault-free state.

Keywords: fault isolation, fuzzy logic, diagnostic relation, uncertainty

1. Introduction

In recent years, growing interest in the development of diagnostic methods based on fuzzy logic as well as their application in industry can be observed. Fuzzy logic is a very efficient tool for the conversion of uncertain and inaccurate information. Most data in industrial practice have such a character (Frank, 1994; Frank and Marcu, 2000; Korbicz *et al.*, 2004) due to factors such as disturbances and measurement noise, uncertain and approximate process models or imprecise expert knowledge. Fuzzy logic is a natural way of taking these uncertainties into account.

One can distinguish two main groups of uncertainties that exist in diagnostic reasoning. The first one is connected with calculating and evaluating residual values. One cannot precisely define the ranges of the residual values that provide evidence for the existence of a fault in the system. This kind of uncertainty is caused by the following factors:

- measurement noise,
- process disturbances,
- inaccuracy or approximation of the models used, etc.

All of the above ingredients result in the fact that residual values do not have one (usually zero) value in the normal process state but they vary around that value. This kind of uncertainty is, in a natural way, taken into consideration by fuzzy logic. It is done during residual evaluation, when its qualitative representation is calculated (fuzzy variables with fuzzy sets and their membership functions). All of the above uncertainties can be interpreted together as the uncertainty of fault symptoms.

The second group of uncertainties is connected with the definition of the relation between faults and the observed symptoms, which is necessary for fault isolation. Such a relation can be written down as a binary diagnostic matrix, an information system or rules in various forms (Frank and Marcu, 2000; Korbicz *et al.*, 2004; Kościelny, 1999; 2001; Kościelny and Syfert, 2000; Kościelny *et al.*, 1999). Often, its absolute certainty is assumed. However, in many cases, the uncertainty of this relation exists and must be taken into account. The uncertainty of the definition of symptoms can be a result of the following factors:

- insufficient residual sensitivity to particular faults (e.g., due to a very small leakage scale, symptoms cannot be observed),
- lack of knowledge about the relation between faults and the observed symptoms.

The paper describes some of fuzzy logic based methods of diagnostic reasoning that can also take into account this kind of uncertainty. It must be emphasized that it is something more than writing the diagnostic relation in the form of fuzzy rules. The reasoning algorithm is modified with the use of various kinds of certainty factors.

As a result, the presented method determines certainty factors of particular faults as well as certainty factors of the normal and unknown process states. The unknown process state factor groups all the states with unknown and multiple faults with the states with improper residual values. The normal state certainty factor indicates similarity between the observed state and the normal process state. These additional factors help to make a decision when unambiguous fault isolation cannot be justified.

2. Relation between Faults and Symptoms

Inputs to isolation algorithms are diagnostic signals. They are a qualitative representation of residual values calculated during residual evaluation. Diagnostic signals are two- or multiple-valued, depending on the kind of quantization of residual values. Models used for isolation should map the space of diagnostic signal values onto a discrete space of faults (Korbicz *et al.*, 2004). One can distinguish the following kinds of models:

- models mapping the space of binary diagnostic signals onto the space of faults,
- models mapping the space of multi-valued diagnostic signals onto the space of faults,
- models mapping the space of continuous diagnostic signals onto the space of faults.

The last kind of models is not often used because in this case measurement data for states with faults are necessary for model determination. On the other hand, comparatively simple models, which consider two- or three-valued diagnostic signals, can be applied on the basis of expert knowledge. The proposed algorithm uses the second kind of models, which are a generalisation of the first kind and meet all the needs connected with real applications.

Assume that the system belongs to the class of fault isolation systems, denoted by *FIS* (Korbicz *et al.*, 2004; Kościelny, 1999; 2001; Kościelny *et al.*, 1999), as defined by Kościelny (1999). Define a finite set of faults:

$$F = \{f_k : k = 1, \dots, K\}, \qquad (1)$$

and a finite set of diagnostic signals:

$$S = \{s_j : j = 1, \dots, J\}. \qquad (2)$$

Each diagnostic signal $s_j \in S$ is associated with a set of its values V_j, called the domain of s_j. Define the mapping $S \times F \rightarrow V$, which assigns to each pair of $\langle s_j, f_k \rangle$ a subset of values $V_{j,k}$ of the diagnostic signal s_j, while ($V_{j,k} \subset V_j$):

$$V = \begin{bmatrix} V_{1,1} & \dots & V_{1,K} \\ \dots & V_{j,k} & \dots \\ V_{J,1} & \dots & V_{J,K} \end{bmatrix}. \qquad (3)$$

The *FIS* is defined by the above mapping, called the diagnostic relation. This definition complies with the rough information system defined by Pawlak (1983). An example of the *FIS* is shown in Tab. 1.

Table 1. Example of an *FIS*.

s/f	f_1	f_2	f_3	f_4	f_5	f_6	V_j
s_1	1	0	1	0	0	1	$\{0,1\}$
s_2	0	-1	0	$+1$	-1	0	$\{0,+1,-1\}$
s_3	-1	$+1$	$+1,-1$	0	$+1$	$+1$	$\{0,+1,-1\}$
s_4	0	$1,2$	$0,1$	0	$1,2$	$1,2$	$\{0,1,2\}$
s_5	$+1$	0	$+1$	$+1$	0	$+1,-1$	$\{0,+1,-1\}$

Such an approximate information system for fault isolation is a generalization of the notion of the binary diagnostic matrix. The extensions with respect to the binary diagnostic matrix are as follows:

- for each diagnostic signal s_j an individual set of its values V_j may exist,
- the set V_j of values for the j-th diagnostic signal can have several elements,
- any element of the *FIS* may include only one diagnostic signal value, as well as their subset.

If the sets of values for all diagnostic signals are identical and equal to $V_j = \{0, 1\}$, then the *FIS* can be simplified to a binary diagnostic matrix.

The columns of the *FIS* system, called fault signature, define pattern values of diagnostic signals in the case of the existence of particular faults:

$$V(f_k) = \begin{bmatrix} V_{1,k} \\ V_{2,k} \\ \vdots \\ V_{J,k} \end{bmatrix}. \qquad (4)$$

The signatures expressed by the relationship (4) are called complex fault signatures, in contrast to elementary signatures. In the case of multiple-valued evaluation, a subset of elementary signatures (more than one elementary signature) can correspond to one complex signature. Each elementary signature represents a different combination of diagnostic signal values. The relationship

$$V(f_3) = \begin{bmatrix} 1 \\ 0 \\ 1^+, 1^- \\ 0, 1 \\ 1^+ \end{bmatrix} \Rightarrow V(f_3) = \begin{bmatrix} 1 \\ 0 \\ 1^+ \\ 0 \\ 1^+ \end{bmatrix}$$

$$\text{or} \quad V(f_3) = \begin{bmatrix} 1 \\ 0 \\ 1^+ \\ 1 \\ 1^+ \end{bmatrix} \quad \text{or} \quad V(f_3) = \begin{bmatrix} 1 \\ 0 \\ 1^- \\ 0 \\ 1^+ \end{bmatrix}$$

$$\text{or} \quad V(f_3) = \begin{bmatrix} 1 \\ 0 \\ 1^- \\ 1 \\ 1^+ \end{bmatrix} \tag{5}$$

provides an example of the relation between complex signature and elementary ones.

Relations between faults and diagnostic signal values are usually expressed in the form of rules. They, most often, conform to the following pattern:

$$\text{If} \quad (s_1 = v_{1,a}) \quad \text{and} \ldots (s_j = v_{j,b}) \quad \text{and} \ldots (s_J = v_{J,c}) \quad \text{then} \quad (f_k) \tag{6}$$

for elementary signatures, called elementary rules, and

$$\text{If} \quad (s_1 \in V_{1,k}) \quad \text{and} \ldots (s_j \in V_{j,k}) \quad \text{and} \ldots (s_J \in V_{J,K}) \quad \text{then} \quad (f_k) \tag{7}$$

or

$$\text{If} \quad ([s_1 = v_{1,a}] \quad \text{or} \ldots [s_1 = v_{1,b}]) \quad \text{and} \ldots ([s_J = v_{J,a}] \quad \text{or} \ldots [s_J = v_{J,b}]) \quad \text{then} \quad (f_k) \tag{8}$$

for complex ones, called complex rules. Observe that the premise of (8) contains the symbol of conjunction. Such a rule is called a complex diagnostic rule (with complex premises).

It is possible that contradictory rules exist in the rule base, e.g., the rules with the same premises and different conclusions. Such rules correspond to unisolable faults. They can be replaced with one rule of the following form:

$$\text{If} \quad (s_1 = v_{1,a}) \quad \text{and} \ldots (s_j = v_{j,b}) \quad \text{and} \ldots (s_J = v_{J,c}) \quad \text{then} \quad (f_k) \quad \text{or} \ldots (f_n) \tag{9}$$

or

$$\text{If} \quad (s_1 \in V_{1,k}) \quad \text{and} \ldots (s_j \in V_{j,k}) \quad \text{and} \ldots (s_J \in V_{J,K}) \quad \text{then} \quad (f_k) \quad \text{or} \ldots (f_n). \tag{10}$$

3. Reasoning Using Fuzzy Logic

In the process of fuzzy reasoning, activation levels of particular rules from the knowledge base are determined. These rules define the diagnostic relation R_{FS}. The general scheme of diagnostic reasoning based on the rules (8) can be represented by the following steps:

- Each diagnostic signal is assigned several fuzzy sets describing residual values. The membership degrees $\mu(v_{j,i})$ of the residual values to these fuzzy sets are determined. They are called simple premise fulfilment factors. An example of evaluating a fuzzy, three-valued residual is shown in Fig. 1.

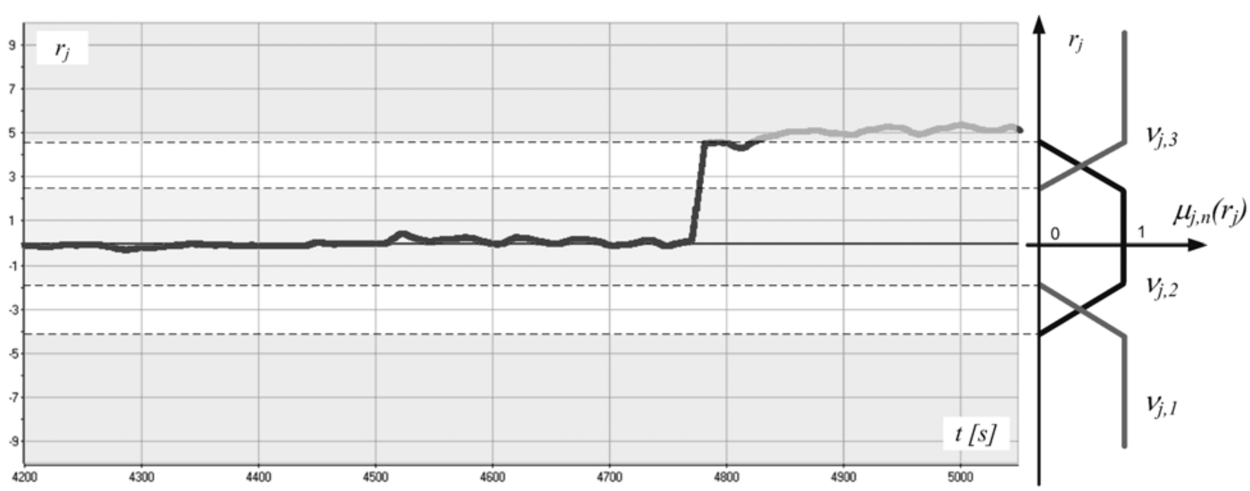

Fig. 1. Fuzzy, three-valued residual evaluation.

- The complex premise fulfilment factor is determined from

$$\mu\left(s_j \in V_{j,k}\right) = \mu\left(v_{j,a}\right) \oplus \cdots \oplus \mu\left(v_{j,n}\right), \quad (11)$$

where $\oplus$ is the symbol of the fuzzy alternative operator, $\mu(s_j \in V_{j,k})$ stands for the membership coefficient of the complex premise for the fault f_k and the diagnostic signal s_j, and $\mu(v_{j,i})$ signifies the membership coefficient of the i-th value (the i-th simple premise) of the j-th diagnostic signal.

- The rule premise fulfilment factors that correspond to the activation level of the rules are determined as

$$\mu\left(f_k\right) = \mu\left(s_1 \in V_{1,k}\right) \otimes \cdots \otimes \mu\left(s_J \in V_{J,k}\right), \quad (12)$$

where $\otimes$ is the symbol of the fuzzy conjunction operator, $\mu(f_k)$ means the activation level of the rule for the k-th fault, and $\mu(s_j \in V_{j,k})$ is the membership coefficient of the complex premise for the fault f_k and the diagnostic signal s_j.

- The diagnosis is formulated on the basis of the calculated rule activation levels, e.g., as a set of faults which were indicated by the rules with activation levels higher than some fixed threshold value H:

$$DGN = \left\{ \left\langle f_k, \mu\left(f_k\right)\right\rangle : \mu\left(f_k\right) > H \right\}. \quad (13)$$

Usually, t-norm operators are used as fuzzy conjunction operators and s-norm operators as fuzzy sum operators (Piegat, 2001). In the field of fault diagnosis themost commonly used s-norms are MAX or drastic sum (Sędziak, 2001) and t-norms: PROD or MIN operators (Korbicz *et al.*, 2004; Kościelny, 2001; Sędziak, 2001).

One of important issues is the way how the elaborated diagnosis is presented to the operators. Figure 2 shows exemplary solutions. Fault certainty factors are presented in the form of bar graphs. Fault indicators are placed near the components whose state is described by these faults. Such a placement can help the operators to react quickly and precisely.

The above reasoning method takes into account the uncertainty of symptoms, but omits the uncertainty of the relation between symptoms and faults. Some methods that take into account this uncertainty along with the appropriate extensions of the reasoning algorithm are presented in Section 5.

4. Unknown State of the System

The rule base derived from the information system is not complete because it does not include rules for all combinations of the values of diagnostic signals. It has only those rules that correspond to fault signatures. It is usually supplemented with rules for a normal (fault free) state (Kościelny, 2001):

$$\text{If } \left(s_1 = 0\right) \text{ and} \ldots \left(s_J = 0\right) \text{ then } \left(\text{state OK}\right). \quad (14)$$

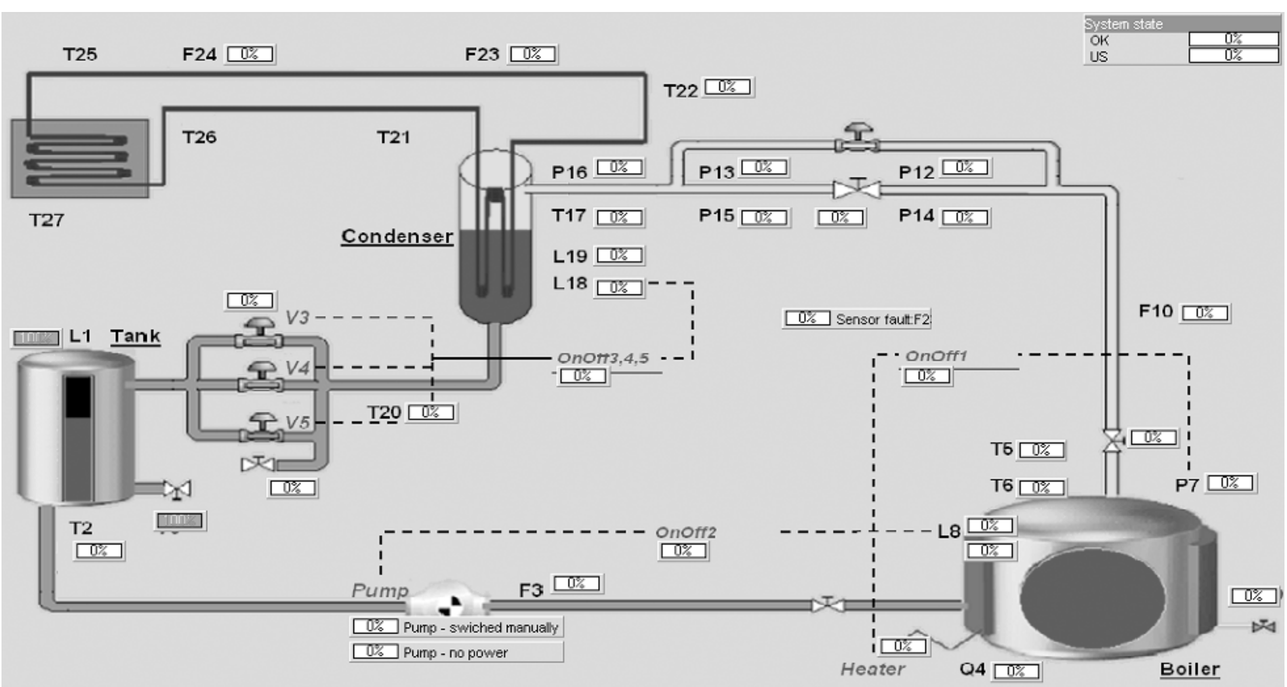

Fig. 2. Example of diagnosis visualisation on the operator's console. Specially designed indicators displaying fault certainty factors are placed close to the corresponding elements.

The number of all possible elementary rules, corresponding to elementary signatures, which exist in a complete rule base for the J-th diagnostic signal can be calculated as

$$N_{\text{full}} = \prod_{j=1}^{J} N_j, \tag{15}$$

where N_j denotes the number of fuzzy sets defined for the j-th diagnostic signal.

The rule base is usually created with the use of rules describing the normal state and the states with single faults. The rules that are not taken into account correspond to the following quantities:

- states with faults that have not been taken into account,
- states with multiple faults,
- physically impossible process states.

An improper combination of diagnostic signal values described by these rules can occur even if none of the above situations takes place. This is possible if improper diagnostic signal values are assigned due to factors such as disturbances, measurement noise or different dynamics of symptoms. Thus, it is advisable to introduce an additional complex rule that determines all other combinations of diagnostic signal values that are not present in the rule base. Such a rule is called the rule of the unknown process state f^{US} (Syfert, 2003).

Let us assume the use of normalised fuzzy sets describing diagnostic signals and the use of the PROD operator for calculating the activation level of the premises of the elementary rule (6). Then, the completeness and consistency of the rule base can be asserted. A rule base of N rules is complete and consistent (Piegat, 2001) if for all possible residual values we have

$$\sum_{n=1}^{N} \mu_n = 1, \tag{16}$$

where μ_n denotes the premise activation level of the n-th rule.

The sum of activation levels of all consistent rules μ_Σ in an incomplete rule base is less than or equal to 1, i.e.,

$$\mu_{OK} + \sum_{k=1}^{K} \mu(f_k) = \mu_\Sigma \leq 1. \tag{17}$$

The value of μ_Σ can be treated as a measure of diagnosis certainty. The diagnosis is more certain if μ_Σ is closer to 1. A lower value of μ_Σ indicates the existence of an unknown process state. The certainty factor of the unknown state can be calculated as

$$\mu_{\text{US}} = 1 - \mu_\Sigma \tag{18}$$

instead of calculating the sum of all rules that are absent in the rule base. The visualisation of this value enables us to evaluate the certainty of a diagnosis generated by the diagnostic system. An example of using normal and unknown process state factors on the operator's console is shown in Fig. 2.

5. Uncertainty of the Diagnostic Relation

5.1. Signature Certainty Factor

In the case of the relation between faults and symptoms, an assumption about its absolute certainty is usually made. In practice, one can expect cases when the conviction that it is certain is unfounded. Attributing a certainty factor to each fault signature is the simplest way of taking account the uncertainty of the relation between faults and symptoms. Such a certainty factor expresses the conviction about the correctness of the signature

$$V(f_k) \rightarrow \delta_k. \tag{19}$$

The certainty factor corresponds to the rules (7), which can be generalised to

$$\text{If } \ (s_1 \in V_{1,k}) \ \text{ and} \ldots (s_J \in V_{J,k}) \quad \text{then } \ (f_k \text{ with } \delta_k). \tag{20}$$

In diagnostic reasoning, the uncertainty of the relation is taken into account by the following formula:

$$\mu^*(f_k) = [\mu(s_1 \in V_{1,k}) \otimes \ldots \mu(s_J \in V_{J,k})] \otimes \delta_k. \tag{21}$$

The activation level of a fault rule, taking into account the uncertainty of the diagnostic relation, is a fuzzy conjunction of the fulfilment factor of all complex premises and the rule certainty factor.

5.2. Fuzzy Information System

To take into account the uncertainty of the relation between faults and symptoms, a fuzzy diagnostic relation was applied in the study (Kościelny, 2001). The fuzzy information system *FFIS*, introduced by Sędziak (2001), constitutes a generalisation of the fuzzy diagnostic relation and the information system *FIS*.

A fuzzy information system can be represented as the pair

$$FFIS = \langle FIS, FR_{FS} \rangle, \tag{22}$$

where FR_{FS} denotes the fuzzy diagnostic relation defined for the *FIS*.

The relation FR_{FS} describes the certainty factor $\delta_R(f_k, s_j)$ stating that, if the fault f_k occurs, then the diagnostic signal s_j will take one of the values belonging to the set $V_{j,k}$. The relation FR_{FS} is defined as

$$FR_{FS} = \{ \langle\langle f_k, s_j \rangle, \ \delta_R(f_k, s_j)\rangle : \langle f_k, s_j\rangle \in F \times S, \ \delta_R(f_k, s_j) \in [0,1] \}. \quad (23)$$

The values of the certainty factors $\delta_R(f_k, s_j)$ are usually defined based on the experts' knowledge. Values close to zero express the certainty that any of the values belonging to the set V_j,k will not appear in the case when f_k occurs. A value close to 1 expresses the certainty that one of the values belonging to the set $V_{j,k}$ will appear in the case when f_k occurs.

The fulfilment factor of a complex premise, taking into account the uncertainty of the diagnostic relation, is determined according to

$$\mu^*(s_j \in V_{j,k}) = [\mu(v_{j,a}) \oplus \dots \mu(v_{j,n})] \otimes \delta_R(f_k, s_j). \quad (24)$$

The rule activation level is calculated as a fuzzy conjunction of the complex premise fulfilment factors:

$$\mu^{**}(f_k) = \mu^*(s_1 \in V_{1,k}) \otimes \dots \otimes \mu^*(s_J \in V_{J,k}). \quad (25)$$

As a result of taking into account the fuzzy diagnostic relation, faults indicated by uncertain rules will be pointed out with adequately lower certainty factors.

In this case, the rules (7) can be expressed in the form

$$\text{If} \ (s_1 \in V_{1,k}) \ \text{and} \dots (s_J \in V_{J,k}) \ \text{Then} \ \big(f_k \ \text{with} \ [\delta_R(f_k, s_1), \dots, \delta_R(f_k, s_J)]\big). \quad (26)$$

5.3. Extended Form of the Fuzzy Information System

In the fuzzy information system the certainty factors $\delta_R(f_k, s_j)$ are attributed to the subsets of the values $V_{j,k}$ defined for the j-th diagnostic signal and the k-th fault: $\langle f_k, s_j\rangle \rightarrow \delta_R(f_k, s_j)$. Attributing the certainty factors $\delta_R(f_k, s_j, v_{j,i})$ to each value of the diagnostic signal $v_{j,i} \in V_{j,k}$: $\langle f_k, s_j, v_{j,m,i}\rangle \rightarrow \delta_R(f_k, s_j, v_{j,i})$ is an extension of such an approach (Syfert, 2003).

A modified relation FR^*_{FS} describes the certainty factor expressing the assumption that for a particular fault f_k the diagnostic signal s_j will take a value $v_{j,i}$ belonging to the set $V_{j,k}$. The relation FR^*_{FS} is defined as

$$FR^*_{FS} = \{ \langle\langle f_k, s_j, v_{j,i}\rangle, \ \delta_R(f_k, s_j, v_{j,i})\rangle : \langle f_k, s_j\rangle \in F \times S, \ v_{j,i} \in V_{j,k}, \ \delta_R(f_k, s_j, v_{j,i}) \in [0,1] \}, \quad (27)$$

where $\delta_R(f_k, s_j, v_{j,i})$ denotes the conviction factor stating that, if the fault f_k occurs, then the diagnostic signal s_j will take on the value of $v_{j,i}$.

In this case the complex premise fulfilment factor is calculated according to the formula

$$\mu^\wedge(s_j \in V_{j,k}) = \big(\mu(v_{j,a}) \otimes \delta_R(f_k, s_j, v_{j,a})\big) \oplus \dots \oplus \big(\mu(v_{j,n}) \otimes \delta_R(f_k, s_j, v_{j,n})\big). \quad (28)$$

The rule activation level is calculated as the fuzzy conjunction of complex premise fulfilment factors:

$$\mu^{\wedge\wedge}(f_k) = \mu^\wedge(s_1 \in V_{1,k}) \otimes \dots \otimes \mu^\wedge(s_J \in V_{J,k}). \quad (29)$$

5.4. Extended Interpretation of the Fuzzy Diagnostic Relation

It is possible to introduce an additional extension of the interpretation for the fuzzy diagnostic relation when an unknown state is considered. It was proposed in (Syfert, 2003). However, its detailed description exceeds the scope of this paper. Only a general idea of such an extension will be presented.

The way of taking into account the fuzzy diagnostic relation described in Sections 5.1 and 5.2 considers only one direction of information influence, i.e., fault certainty factors pointed out by uncertain rules have lower values. Two additional methods that take into account the extended interpretation of this relation are described below.

The first modification was marked as an *extension of the fuzzy diagnostic relation*. It is based on the following reasoning:

> *"If the uncertainty of the relation between a signature and fault f_k is taken into account, then the relation between that signature and the unknown state of the object f^{US} should be simultaneously considered."*

If the uncertainty of the signature is expressed by the certainty factor $\delta_R(f_k, s_j)$, then the relation between that signature and the unknown state of the object f^{US} should be taken into account with certainty factor $[1 - \delta_R(f_k, s_j)]$. The rule (26) will take the following form:

$$\text{If} \ (s_1 \in V_{1,k}) \ \text{and} \ \dots \ \text{and} \ (s_J \in V_{J,k}) \ \text{Then} \ \big(f^{US} \ \text{with} \ [(1 - \delta_R(f_k, s_1)), \dots, \delta_R(f_k, s_J)]\big). \quad (30)$$

The second modification was marked as the *complement of the fuzzy diagnostic relation*. It is based on the following reasoning:

> *"If the uncertainty of the relation between a signature and fault f_k exists as a result of the uncertainty of the symptoms for the j-th diagnostic signal, then the relation between the signature where uncertain symptoms do not appear and the fault f_k should be simultaneously considered."*

For example, if the uncertainty of the signature is expressed by the certainty factor $\delta_R(f_k, s_1)$, then the rule (26) will be extended to

$$\text{If} \quad (s_1 \in V_1 - V_{1,k}) \quad \text{and} \quad \ldots \quad \text{and} \quad (s_J \in V_{J,k})$$
$$\text{Then} \ \big(f_k \text{ with } [(1 - \delta_R(f_k, s_1)), \ldots, \delta_R(f_k, s_J)]\big). \tag{31}$$

The functioning of the *extension* and the *complement of a fuzzy diagnostic relation* gives, in some sense, opposite effects. However, it is also justified using both these methods simultaneously. The reasoning that is based on the *complement of a fuzzy diagnostic relation* constitutes a natural extension of the fuzzy diagnostic relation. Its use is justified in all cases when the uncertainty of expert knowledge about the diagnostic relation is taken into account. The justification for the use of the *extension of a fuzzy diagnostic relation* is dependent on the assumed reasoning strategy:

- In the case when there is a very small probability that faults not belonging to the set F occur (when the *FIS* includes all possible system states), then the application of the extension can have disadvantageous influence.

- In the case when there is a high probability that faults not belonging to the analysed set F occur, the application of the extension is fully justified. Its use underlines the connection between the uncertainty and unknown state of the object.

6. Example

Methods tackling the diagnostic relation uncertainty given in Sections 5.1, 5.2 and 5.3 will be illustrated using a simple example. Table 2 shows an *FIS* describing the relation between faults and symptoms of some technical system. The fuzzy set built up on residual values close to zero is denoted by 0. Fuzzy sets assigned to negative residual values are denoted by negative numbers (see Fig. 1), and fuzzy sets assigned to positive residuals are denoted by positive numbers.

Table 2. Example of an *FIS*.

s/f	f_1	f_2	f_3	f_4	V
s_1	-2,-1	1,2	1,2	-2	-2,-1,0,1,2
s_2	+1	+1	-1	+1	-1,0,+1
s_3	0	+1	+1	+1	-1,0,+1

Consider the following values of fuzzy diagnostic signals obtained in the diagnosing phase:

$$s_1 = \{ \langle -2, 0.8 \rangle, \langle -1, 0.2 \rangle, \langle 0, 0.0 \rangle, \langle +1, 0.0 \rangle, \langle +2, 0.0 \rangle \},$$
$$s_2 = \{ \langle -1, 0.0 \rangle, \langle 0, 0.0 \rangle, \langle +1, 1.0 \rangle \},$$
$$s_3 = \{ \langle -1, 0.0 \rangle, \langle 0, 0.4 \rangle, \langle +1, 0.6 \rangle \}.$$

(a) Now, consider the case described in Section 5.1. Assume that the values of the certainty factor assigned to the signatures of particular faults are known and given in Tab. 3.

Table 3. Certainty factors.

s/f	f_1	f_2	f_3	f_4
δ_k	0.8	0.8	0.7	1.0

Activation levels for fault rules are determined from (11) and (12). The MAX operator was applied as the fuzzy union and the PROD operator as the fuzzy intersection:

$$\mu(f_1) = \text{MAX}\{0.8, 0.2\} \otimes 1.0 \otimes 0.4 = 0.32,$$
$$\mu(f_2) = \text{MAX}\{0.0, 0.0\} \otimes 1.0 \otimes 0.6 = 0.0,$$
$$\mu(f_3) = \text{MAX}\{0.0, 0.0\} \otimes 0.0 \otimes 0.6 = 0.0,$$
$$\mu(f_3) = 0.8 \otimes 1.0 \otimes 0.6 = 0.48.$$

From (21) we can determine fault activation levels after correction by considering certainty factor values:

$$\mu^*(f_1) = 0.32 \otimes 0.8 = 0.256,$$
$$\mu^*(f_2) = 0.0 \otimes 0.8,$$
$$\mu^*(f_3) = 0.0 \otimes 0.7,$$
$$\mu^*(f_4) = 0.48 \otimes 1.0 = 0.48.$$

The diagnosis (assuming $H = 0.1$) is $DGN = \{\langle f_4, 0.48\rangle, \langle f_1, 0.256\rangle\}$.

(b) Consider the case described in Section 5.2. Assume that the fuzzy diagnostic relation FR_{FS} (23) is known and given in Tab. 4.

Fault rule activation levels are determined from (24) and (25):

$$\mu^{**}(f_1) = (MAX\{0.8, 0.2\} \otimes 0.8) \otimes (1.0 \otimes 0.9) \otimes (0.4 \otimes 1.0) \approx 0.23,$$
$$\mu^{**}(f_2) = (MAX\{0.0, 0.0\} \otimes 0.9) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 0.8) = 0.0,$$

Table 4. Fuzzy diagnostic relation FR_{FS}.

s/f	f_1	f_2	f_3	f_4
s_1	0.8	0.9	0.9	1.0
s_2	0.9	1.0	1.0	1.0
s_3	1.0	0.8	0.7	1.0

$$\mu^{**}(f_3) = (MAX\{0.0, 0.0\} \otimes 0.9) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 0.7) = 0.0,$$

$$\mu^{**}(f_4) = (0.8 \otimes 1.0) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 1.0) \approx 0.48.$$

In this case the diagnosis is $DGN = \{\langle f_4, 0.48\rangle, \langle f_1, 0.23\rangle\}$.

(c) Consider case described in Section 5.3. Assume that the extended fuzzy diagnostic relation FR^*_{FS} (27) is defined by Tab. 5.

Table 5. Extended fuzzy diagnostic relation FR^*_{FS}.

s/f	f_1	f_2	f_3	f_4
s_1	0.9, 0.8	0.9, 0.9	0.9, 1.0	1.0
s_2	0.9	1.0	1.0	1.0
s_3	1.0	0.8	0.7	1.0

Corrected faults rule activation levels are determined from (28) and (29):

$$\mu^{\wedge\wedge}(f_1) = (\mathrm{MAX}\{0.8 \otimes 0.9, 0.2 \otimes 0.8\}) \otimes (1.0 \otimes 0.9) \otimes (0.4 \otimes 1.0) \approx 0.26,$$

$$\mu^{\wedge\wedge}(f_2) = (\mathrm{MAX}\{0.0 \otimes 0.9, 0.0 \otimes 0.9\}) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 0.8) = 0.0,$$

$$\mu^{\wedge\wedge}(f_3) = (\mathrm{MAX}\{0.0 \otimes 0.9, 0.0 \otimes 1.0\}) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 0.7) = 0.0,$$

$$\mu^{\wedge\wedge}(f_4) = (0.8 \otimes 1.0) \otimes (1.0 \otimes 1.0) \otimes (0.6 \otimes 1.0) \approx 0.48.$$

In this case the diagnosis is $DGN = \{\langle f_4, 0.48\rangle, \langle f_1, 0.26\rangle\}$.

7. Concluding Remarks

The paper presents a method of fuzzy diagnostic reasoning that takes into account uncertainties connected with the process of diagnosing. The known methods (Frank, 1994; Frank and Marcu, 2000; Garcia *et al.*, 1997; Combaste *et al.*, 2003) use fuzzy residual evaluation, which is an efficient tool for neutralising uncertainties connected with disturbances, measurement noise and modelling inaccuracy. The paper puts together and extends different approaches of taking into account the uncertainty of the relation between symptoms and faults that were described in the previous papers (Kościelny, 2001; Kościelny and Syfert, 2004; Syfert, 2003). Such an uncertainty may result from the following factors:

- insufficient residual sensitivity to particular faults (e.g., due to a very small leakage size, symptoms will not be observed),
- lack of knowledge about the relation between faults and the observed symptoms.

The proposed reasoning algorithms that enable us to take into account several kinds of uncertainties connected with the knowledge about the relation between symptoms and faults form a basis for the diagnostic reasoning algorithm.

Various methods of taking into account the uncertainty of the relation between faults and diagnostic signal values were described. The idea of signatures for certainty factors and an extended version of the fuzzy information system were proposed for the first time. Fuzzy logic was applied for taking into account that uncertainty. The rules of reasoning were given for the cases considered.

The diagnostic reasoning conducted on the basis of the above rules enables us to take into account the fault-symptom relation uncertainty, as well as symptom uncertainty. Fuzzy residual evaluation is used for that reason. The proposed reasoning algorithm enables us to calculate certainty factors for the states with particular faults and the normal process state. Additionally, certainty factor of the unknown process states can be calculated. The unknown state groups all the cases with unknown faults (omitted during system configuration), with multiple faults and with improper diagnostic signal values, which were distorted by measurement noise, disturbances, etc. Calculating that factor increases the reliability of the diagnosis generated by the diagnostic system. An example of reasoning with the use of the proposed diagnostic methods was shown.

Fuzzy logic is an efficient and natural way of taking into account uncertainties that occur during the diagnostics of industrial processes. Many parameters are set up arbitrarily based on experts' knowledge and experience. However, such an approach seems to have advantages in the case of industrial applications. In the case of an alternative approach based on the Bayesian theory, many serious difficulties occur. They are mainly connected with defining probabilities necessary for calculations.

Given the above, reasoning methods for faults are concerned with the diagnostics of technical systems. However, identical approaches can be applied in medical diagnostics, too. These approaches may benefit from considering symptoms of an illness with, generally, imprecise knowledge about the relation between the symptoms and the illness. Consider the example of Section 6 in the context of medical diagnostics. The faults f_k are equivalent to illness units. Fuzzy diagnostic signals define fuzzy symptoms of the illness. In this case, 0 is assigned to the measured parameter qualified as normal, and negative and positive numbers are assigned to fuzzy sets of parameter values below or above the values within the normal range. The *FIS* column describes the set of illness symptoms. The certainty factor or the fuzzy relation FR_{FS}, cf. Tab. 3, describes in various ways the uncertainty of medical knowledge about the illness and its symptoms.

Acknowledgments

This work was supported by the Polish Ministry of Education and Science under Grant 1527/T11/2005/29 entitled *Diagnosis algorithms of industrial processes taking into account problems of multiple faults and the dynamic, of symptoms—ALDIAG*.

References

Combastel C., Gentil S. and Rognon J.P. (2003): *Toward a better integration of residual generation and diagnostic decision*. — Proc. 5-th IFAC Symp. *Fault Detection, Supervision and Safety for Technical Process, SAFEPROCESS'2003*, Washington, USA, CD-ROM.

Frank P.M. (1994): *Fuzzy supervision. Application of fuzzy logic to process supervision and fault diagnosis*. — Proc. Workshop *Fuzzy Technologies in Automation and Intelligent Systems*, Duisburg, Germany, pp. 36–59.

Frank P.M. and Marcu T. (2000): *Diagnosis strategies and system: principle, fuzzy and neural approaches*, In: Intelligent Systems and Interfaces (H.N. Teodorescu, D. Mlynek, A. Kandel and H.-J. Zimmermann, Eds.). — The Kluwer International Series in Intelligent – International Series in Intelligent Technologies, Springer, Chap. 11.

Garcia F.J., Izquierdo V., de Miguel L., and Peran J. (1997): *Fuzzy Identification of Systems and its Applications to Fault Diagnosis Systems*. — Proc. IFAC Symp. *Fault Detection Supervision and Safety for Technical Processes, SAFEPROCESS'97*, Kingston Upon Hull, UK, Vol. 2., pp. 705–712.

Korbicz J., Kościelny J.M., Kowalczuk Z. and Cholewa W. (2004): *Fault Diagnosis. Models. Artificial intelligence. Applications*. — Berlin: Springer.

Kościelny J.M. (1999): *Application of fuzzy logic fault isolation in a three-tank system*. — Proc. 14-th IFAC World Congress, Bejing, China, pp. 73–78.

Kościelny J.M. (2001): *Diagnostics of Automated Industrial Processes*. — Warsaw: Exit, (in Polish).

Kościelny J.M., Sędziak D. and Zakroczymski K. (1999): *Fuzzy logic fault isolation in large scale systems*. — Int. J. Appl. Math. Comput. Sci., Vol. 9, No. 3, pp. 637–652.

Kościelny J.M. and Syfert M. (2000): *Current diagnostics of power boiler system with use of fuzzy logic*. — Proc. 4-th IFAC Symp. *Fault Detection, Supervision and Safety for Technical Processes, SAFEPROCESS'2000*, Budapest, Hungary, Vol. 2, pp. 681–686.

Kościelny J.M. and Syfert M. (2004): *Fuzzy logic application for the description of diagnostic relation uncertainty*. — Proc. 10-th IEEE Int. Conf. *Methods and Models in Automation and Robotics*, Międzyzdroje, Poland, pp. 757–762.

Pawlak Z. (1983): *Information Systems. Theoretical Background*. — Warsaw: WNT, (in Polish).

Piegat A. (2001): *Fuzzy Modelling and Control*. — Berlin: Springer.

Sędziak D. (2001): *Methods of fault isolation in industrial processes*. — Ph.D. thesis, Faculty of Mechatronics, Warsaw University of Technology, Warsaw, Poland, (in Polish).

Syfert M. (2003): *The diagnostics of industrial processes with the use of partial models and fuzzy logic*. — Ph.D. thesis, Faculty of Mechatronics, Warsaw University of Technology, Warsaw, Poland, (in Polish).

Received: 7 November 2005
Revised: 16 February 2006