Int. J. Appl. Math. and Comp. Sci., *1999, Vol.9, No.4, 883–897*

DESIGN AND STABILITY OF FUZZY LOGIC MULTI-REGIONAL OUTPUT CONTROLLERS

PAWEŁ DOMAŃSKI*, MIECZYSŁAW A. BRDYŚ**, PIOTR TATJEWSKI*

Design and stability analysis of fuzzy multi-regional digital controllers is considered in the paper. The controllers are based on a notion of NARMAX systems, very similar to the Takagi-Sugeno fuzzy model. The nonlinear system is approximated by a number of linear subsystems. Linear controllers are designed for all subsystems. It can be made in a classical way due to the subsystems linearity. The controllers are blended into one controller by employing fuzzy logic, the result being the fuzzy multi-regional controller (FuMR). The stability analysis of nonlinear systems with FuMR controllers composed of dynamic output feedback local linear controllers is provided. Examples illustrate the design procedure and the meaning of the stability criterion.

Keywords: nonlinear output control, fuzzy logic, Takagi-Sugeno models, stability conditions

1. Introduction

Nonlinear ARX or ARMAX models (called NARX or NARMAX (Johansen, 1994; Johansen and Foss, 1993)) can be used to extend the piecewise linear system by switching the subsystems in a fuzzy way. Such systems have the advantage of the possibility to deal with nonlinear processes. Following the idea of gain scheduling one can propose an algorithm which also switches different control actions in a similar way. The problem lies in the way of extracting proper subregions. It is natural to choose the construction of subsystems dividing the domains of input and output signals.

The idea of such controllers can be found in several papers (Cao *et al.*, 1996; Korba and Frank, 1997; Tanaka and Sugeno, 1992; Wang *et al.*, 1996; Zhao *et al.*, 1996). The authors consider processes and controllers in the form of Takagi-Sugeno fuzzy systems. The processes are described by linear state equations and the controllers are represented by a simple linear state-feedback law. In the paper, the idea of such multi-regional controllers is extended to cover dynamic output controllers including e.g. digital PID and predictive IMC structures that can be tuned in a standard way.

* Institute of Control and Computation Engineering, Warsaw University of Technology, ul. Nowowiejska 15/19, 00–665 Warszawa, Poland, e-mail: {domanski,tatjewski}@ia.pw.edu.pl

** School of Electronic and Electrical Engineering, The University of Birmingham, Edgbaston, Birmingham B15 2TT, UK, e-mail: mbrdys@bham.ac.uk

Another problem lies in the approach to the stability analysis. In the literature the Lyapunov function approach (Tanaka and Sugeno, 1992; Wang *et al.*, 1996), the uncertain linear system theory (Cao and Ress, 1996), quadratic stability (Marin and Titli, 1997) and some others approaches (Georgieva, 1995) can be found. In the present paper, the approach demonstrated in (Tanaka and Sugeno, 1992; Wang *et al.*, 1996) is applied.

The paper is constructed as follows. First, the idea of the proposed controller is presented. Afterwards, the stability analysis for the system under consideration is provided. The next section proposes a methodology for tuning FuMR controllers. Finally, examples illustrating the tuning methodology and the system performance are presented. Conclusions and indications for further research complete the paper. The idea of the paper was first presented at the European Control Conference in Brussels in 1997.

2. Controller Design

Let us consider the SISO control system presented in Fig. 1, with a nonlinear process G. For a large class of processes, an NARX input-output model of the process can be built (Chen and Billings, 1989). We shall follow the approach presented in (Johansen, 1994; Johansen and Foss, 1993; Takagi and Sugeno, 1985), where the NARX model is designed as a composition of local ARX models describing the process over certain subregions.

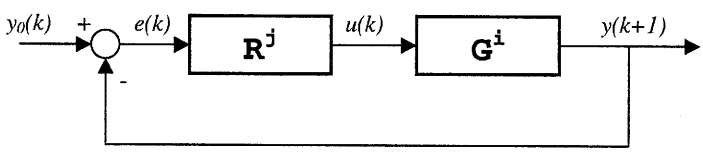

Fig. 1. Closed-loop control structure.

The composition of the ARX models is performed by using fuzzy reasoning due to Takagi and Sugeno (1985). In this approach, the fuzzy subregions Z^i are defined in the domains of input and output signals, see Fig. 2. Notice that the partitioning of the overall operating region is designed in such a way that the adjacent subregions overlap. This is an inherent feature of the fuzzy partitioning which enables us to achieve a smooth transfer between the submodels. In each of the subregions the ARX polynomial submodel $\mathbf{G}^i$ describes the process as

IF $z(k)$ is Z^i, THEN

$$y^i(k+1) = a_0^i + a_1^i y(k) + \cdots + a_{n_P}^i y(k - n_P + 1) + b_1^i u(k) + \cdots + b_{m_P}^i u(k - m_P + 1). \quad (1)$$

For the linear submodels digital linear controllers $\mathbf{R}^j$ of the following form are proposed:

IF $z(k)$ is Z^j, THEN

$$u^j(k+1) = c_0^j + c_1^j e(k) + \cdots + c_{n_C}^j e(k - n_C + 1) + d_1^j u(k) + \cdots + d_{m_C}^j u(k - m_C + 1). \quad (2)$$

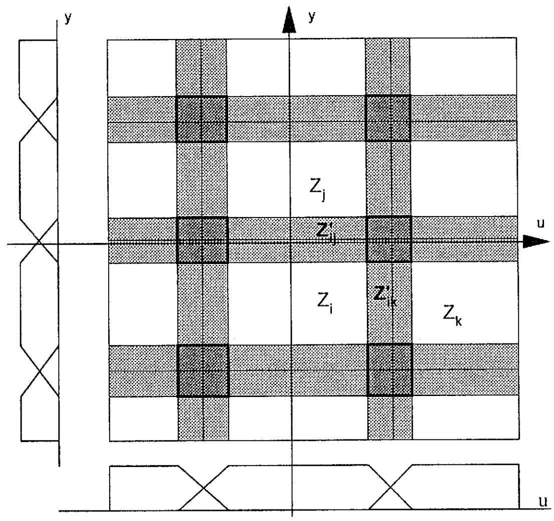

Fig. 2. Fuzzy clustering of the subregions.

It should be noticed that different orders of the process and controller submodels in each of the subregions were assumed (n_P, m_P and n_C, m_C, respectively). One-step-delay PID or IMC controller structures fall into this class. The delay is necessary to avoid the so-called 'logical inconsistency' (Zhao *et al.*, 1996) in operation of the resulting global controller, which occurs if the control signal appears in the premises of the controller rules (2), i.e. it is a component of the vector $\mathbf{z}(\mathbf{t})$. Bearing in mind the potential of available microprocessors allowing for a high sampling frequency which is needed due to process nonlinearities, this is not a limitation.

The composite (global, non-linear) controller will be constructed by using fuzzy reasoning due to Takagi and Sugeno, in the same way as the composite model is built (Takagi and Sugeno, 1985). It will be called the 'fuzzy multi-regional controller' (FuMR). The structure of the resulting control system with FuMR controller is shown in Fig. 3.

Using directly the input and output variables, the process model expressions (1) may be rewritten as

$$\text{IF } \boldsymbol{y}(k) \text{ is } P^i \text{ AND } \boldsymbol{u}(k) \text{ is } Q^i \text{ THEN}$$

$$y^i(k+1) = a_0^i + \sum_{p=1}^{n_P} a_p^i y(k-p+1) + \sum_{q=1}^{m_P} b_q^i u(k-q+1) \tag{3}$$

and the controller expressions (2) as

$$\text{IF } \boldsymbol{y}(k) \text{ is } P^i \text{ AND } \boldsymbol{u}(k) \text{ is } Q^i \text{ THEN}$$

$$u^j(k+1) = c_0^j + \sum_{p=1}^{n_C} c_p^j e(k-p+1) + \sum_{q=1}^{m_C} d_q^j u(k-q+1), \tag{4}$$

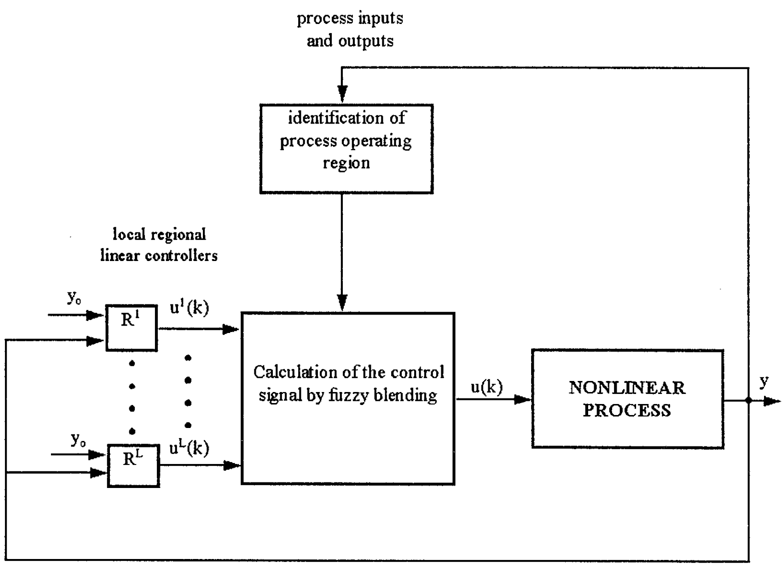

Fig. 3. Structure of the control system with the FuMR controller.

where

$$\boldsymbol{y}(k) = \big[y(k), y(k-1), \dots, y(k-n+1)\big]^T,$$
$$\boldsymbol{u}(k) = \big[u(k), u(k-1), \dots, u(k-m+1)\big]^T,$$
$$e(k) = y_0(k) - y(k),$$
$$P^i \triangleq \big[A_1^i, \dots, A_n^i\big], \quad Q^i \triangleq \big[B_1^i, \dots, B_m^i\big], \quad i = 1, \dots, L, \quad j = 1, \dots, L,$$

and the meaning of the composite sets is explained on the P^i example:

$$\boldsymbol{y}(k) \text{ is } P^i \iff y(k) \text{ is } A_1^i \text{ AND } \dots \text{ AND } y(k-n+1) \text{ is } A_n^i.$$

The same partitions of the output and control spaces have been assumed in (3) and (4) for the fuzzy rules determining the choices of both the local process models and local controllers. However, the lengths n and m of the vectors $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$, respectively, may be different than the corresponding orders n_P, m_P and n_C, m_C of the process submodels and local controllers.

Applying fuzzy reasoning due to Takagi-Sugeno yields the overall process model and controller outputs as

$$y(k+1) = \frac{\sum_{i=1}^{L} w^i y^i(k+1)}{\sum_{i=1}^{L} w^i}, \tag{5}$$

and

$$u(k+1) = \frac{\sum_{j=1}^{L} w^j u^j(k+1)}{\sum_{j=1}^{L} w^j}, \tag{6}$$

where

$$\sum_{i=1}^{L} w^i > 0, \quad w^i \geqslant 0 \quad \text{for} \quad i = 1, \dots, L,$$

and the firing strengths of the rules are

$$w^i = \prod_{p=1}^{n} A_p^i\big(y(k-p+1)\big) \times \prod_{q=1}^{m} B_q^i\big(u(k-q+1)\big). \tag{7}$$

Note that the above fuzzy blending of local models and controllers consists in taking weighted sums of their outputs. The weights depend on the current operating point $z(k) = (\boldsymbol{y}(k), \boldsymbol{u}(k))$ promoting contribution of the outputs associated with the subregions to which the operating point belongs with a high grade of membership. Also notice that the weights are the same in both the formulae (5) and (6), due to the same fuzzy clustering of the process output $\boldsymbol{y}(k)$ and input $\boldsymbol{u}(k)$ signal spaces for both the process model and controller design.

A natural approach to the control design is to employ good approximations to the process input-output mapping over the subregions by linear submodels and to design linear controllers using classical design techniques. In this way, good dynamic performance of the overall controller is achieved if the control system operates well within one of the subregions. The remaining problem is to achieve equally good dynamic performance when the operating point moves between subregions. Stability is the very first problem to be investigated.

3. Stability Analysis

In order to perform the stability analysis of the presented control system with the FuMR controller, the reasoning scheme will first be reformulated to obtain suitable formulae for the closed-loop system. Inserting local model equations into the reasoning scheme (5), we get the process description in the following form:

$$y(k+1) = \frac{\sum_{i=1}^{L} w^i \left\{ a_0^i + \sum_{p=1}^{n_P} a_p^i y(k-p+1) + \sum_{q=1}^{m_P} b_q^i u(k-q+1) \right\}}{\sum_{i=1}^{L} w^i}. \tag{8}$$

Analogously, inserting local controller equations into the controller reasoning scheme (6), we arrive at the controller description

$$u(k+1) = \frac{\sum_{j=1}^{L} w^j \left\{ c_0^j + \sum_{r=1}^{n_C} c_r^j e(k-r+1) + \sum_{s=1}^{m_C} d_s^j u(k-s+1) \right\}}{\sum_{j=1}^{L} w^j}. \tag{9}$$

After inserting the control error formula into the control algorithm (9), we obtain

$$u(k+1) = \frac{\sum\limits_{j=1}^{L} w^j \left\{ c_0^j + \sum\limits_{r=1}^{n_C} c_r^j \big[y_0(k-r+1) - y(k-r+1)\big] + \sum\limits_{s=1}^{m_C} d_s^j u(k-s+1) \right\}}{\sum\limits_{j=1}^{L} w^j}. \tag{10}$$

Inserting now the control rule (10) into the process description (8), we obtain the closed-loop system description in the form

$$y(k+1) = \sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j \Bigg[a_0^i + c_0^j \sum_{q=1}^{m_P} b_q^i + \sum_{q=1}^{m_P}\sum_{r=1}^{n_C} b_q^i c_r^j y_0(k-r-q+1)$$
$$+ \sum_{q=1}^{m_P} b_q^i \sum_{s=1}^{m_C} d_s^j u(k-s-q+1) + \sum_{p=1}^{n_P} a_p^i y(k-p+1)$$
$$- \sum_{q=1}^{m_P}\sum_{r=1}^{n_C} b_q^i c_r^j y(k-r-q+1)\Bigg] \Big/ \sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j. \tag{11}$$

Let us define the augmented state vector $\overline{\zeta}(k) \in \mathbb{R}^{n_Y+m_P+n_C-1}$, where $n_Y = \max(n_P, m_P+n_C)$, as follows:

$$\overline{\zeta}(k) = \Big[y(k), \dots, y(k-n_Y+1), u(k-1), \dots, u(k-m_P-m_C+1) \Big]^T. \tag{12}$$

Consequently, the closed-loop reasoning scheme (11) takes the form

$$y(k+1) = \frac{\sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j \big[\widetilde{a}_0^{ij} + \widetilde{A}^{ij}\overline{\zeta}(k) + \widetilde{B}^{ij}\boldsymbol{y}_0(k)\big]}{\sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j}, \tag{13}$$

where $\widetilde{a}_0^{ij} = a_0^i + c_0^j \sum_{q=1}^{m_P} b_q^i$ and $\widetilde{A}^{ij}, \widetilde{B}^{ij}$ are appropriate vectors resulting from the equivalence between (13) and (11).

For stability analysis only the internal dynamics of the augmented state vector is important, so we can investigate the autonomous fuzzy system in the following form:

$$y(k+1) = \frac{\sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j \widetilde{A}^{ij}\overline{\zeta}(k)}{\sum_{i=1}^{L}\sum_{j=1}^{L} w^i w^j}. \tag{14}$$

Since $w^i w^j = w^j w^i$, significant simplification of (14) can be obtained as

$$\frac{\sum\limits_{i=1}^{L} w^i w^i \widetilde{A}^{ii} + \sum\limits_{i,j=1, i\neq j}^{L} w^i w^j \widetilde{A}^{ij}}{\sum\limits_{i=1}^{L}\sum\limits_{j=1}^{L} w^i w^j} = \frac{\sum\limits_{i=1}^{L} w^i w^i \widetilde{A}^{ii} + \sum\limits_{i=1}^{L}\sum\limits_{j=i+1}^{L} 2 w^i w^j \frac{\widetilde{A}^{ij}+\widetilde{A}^{ji}}{2}}{\sum\limits_{i=1}^{L} w^i w^i + \sum\limits_{i=1}^{L}\sum\limits_{j=i+1}^{L} 2 w^i w^j}$$
$$= \frac{\sum\limits_{l=1}^{L} w^l \widetilde{A}^l + \sum\limits_{l=L+1}^{K} w^l \widetilde{A}^l}{\sum\limits_{l=1}^{K} w^l} = \frac{\sum\limits_{l=1}^{K} w^l \widetilde{A}^l}{\sum\limits_{l=1}^{K} w^l}, \tag{15}$$

where the definition of $\widetilde{A}^l$, $l = 1, 2, \dots, K$; $K = \sum_{k=1}^{L} k$ is obvious from the derivation presented above in (15).

The obtained simplification is important from a practical point of view, since all the matrices $\widetilde{A}^l$ will have to be checked to satisfy a certain condition in order to guarantee the control system stability (the criterion to be further developed).

In this way the autonomous system (14) has been transformed into the form

$$y(k+1) = \frac{\sum_{l=1}^{K} w^l \left(\widetilde{A}^l \cdot \overline{\zeta}(k)\right)}{\sum_{l=1}^{K} w^l}, \tag{16}$$

where $\sum_{l=1}^{K} w^l > 0$; $w^l \geq 0$ for $l = 1, \dots, K$. Further, the following state space form of the closed-loop system (16) will be used:

$$\overline{\zeta}(k+1) = \frac{\sum_{l=1}^{K} w^l \left(\overline{\widetilde{A}}^l \cdot \overline{\zeta}(k)\right)}{\sum_{l=1}^{K} w^l}, \tag{17}$$

where $\overline{\widetilde{A}}^l \in \mathbb{R}^{n_Y + m_P + m_C - 1} \times \mathbb{R}^{n_Y + m_P + m_C - 1}$ are the appropriate matrices containing vectors $\widetilde{A}^l$ as the first rows,

$$\overline{\widetilde{A}}^l = \begin{bmatrix} \widetilde{a}_1^l & \widetilde{a}_2^l & \cdots & \widetilde{a}_{n_Y+m_P+m_C-2}^l & \widetilde{a}_{n_Y+m_P+m_C-1}^l \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}. \tag{18}$$

Also notice that $\overline{\zeta}^l(k+1) = \overline{\widetilde{A}}^l \cdot \overline{\zeta}(k)$ can be defined as the state vectors resulting from the subsystems, $l = 1, \dots, K$.

Finally, we arrived at the system description (17) to which the stability criterion proposed by Tanaka and Sugeno (1992) can be applied. From this criterion it follows that *it is sufficient for the stability that there exists a positive definite matrix P satisfying the Lyapunov inequalities for all the local subsystems, i.e. the sufficient stability conditions are as follows:*

$$\left(\overline{\widetilde{A}}^l\right)^T \cdot P \cdot \overline{\widetilde{A}}^l - P < 0, \quad \text{for } l = 1, \dots, K. \tag{19}$$

4. Design Methodology

The design methodology for FuMR controllers is as follows:

1. Create an NARX model of the process, e.g. by performing a fuzzy clustering of the inputs and outputs to determine fuzzy regions and then the LS estimation of

local ARX models. There is also a possibility to apply the fuzzy neural network approach (Ishigami *et al.*, 1995).

2. Design linear controllers for each local model using standard methods, e.g. Ziegler-Nichols tuning for PID-type controllers.

3. Compose the overall controller forming the fuzzy multi-regional controller.

4. Check on stability. Inequalities (19) fall into a class of Linear Matrix Inequalities (LMI). The MATLAB LMI toolbox can be used to verify the existence of a solution and to calculate it.

5. Iterate the design if the sufficient condition for stability is not satisfied.

5. Examples

In this section, two examples will be presented. They illustrate the design methodology and the stability analysis.

Example 1. The process is modelled by two simple fuzzy rules:

$$\text{IF } y(k) \text{ is } \mathcal{A} \text{ THEN } y^1(k+1) = 0.7y(k) + 0.75u(k), \tag{20}$$

$$\text{IF } y(k) \text{ is } \mathcal{B} \text{ THEN } y^2(k+1) = 0.4y(k) + 0.1u(k). \tag{21}$$

Fuzzy clusters for the process are presented in Fig. (4). The PI controllers designed in a standard way for the linear models are as follows (notice that the one-step-delay control rules are used although it is not necessary):

$$\text{IF } y(k) \text{ is } \mathcal{A} \text{ THEN } u^1(k+1) = u(k) + e(k) + 0.75e(k-1), \tag{22}$$

$$\text{IF } y(k) \text{ is } \mathcal{B} \text{ THEN } u^2(k+1) = u(k) + 2.8e(k) + 2.6e(k-1). \tag{23}$$

Examplary step responses of these controllers are shown in Fig. 5. After closing the control loop we obtain the Takagi-Sugeno NARX system consisting of three rules:

$$\text{IF } y(k) \text{ is } \mathcal{A} \text{ THEN } \bar{\zeta}^1(k+1) = \bar{\tilde{A}}^1\bar{\zeta}(k) + B^1 y_0(k),$$

$$\text{IF } y(k) \text{ is } \mathcal{B} \text{ THEN } \bar{\zeta}^2(k+1) = \bar{\tilde{A}}^2\bar{\zeta}(k) + B^2 y_0(k),$$

$$\text{IF } y(k) \text{ is } (\mathcal{A} \text{ and } \mathcal{B}) \text{ THEN}$$

$$\bar{\zeta}^3(k+1) = \bar{\zeta}^{12}(k+1) = \bar{\tilde{A}}^{12}\bar{\zeta}(k) + B^{12} y_0(k).$$

with the following state space matrices:

$$\tilde{A}^{1} = \begin{bmatrix} 0.7 & -0.75 & -0.56 & 0.75 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \tilde{A}^{2} = \begin{bmatrix} 0.4 & -0.28 & -0.26 & 0.1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\tilde{A}^{12} = \begin{bmatrix} 0.55 & -1.1 & -1.01 & 0.43 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

For these three subsystems a positive matrix P which satisfies all the three local Lyapunov inequalities was found. The step response of the stable control system with the designed multi-regional fuzzy controller is presented in Fig. 5. ♦

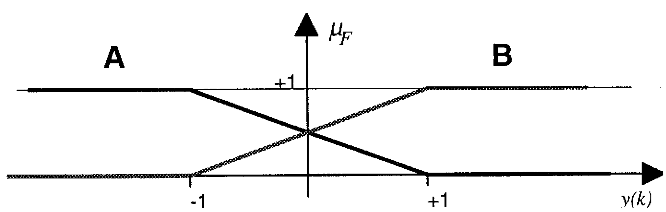

Fig. 4. Fuzzy clustering.

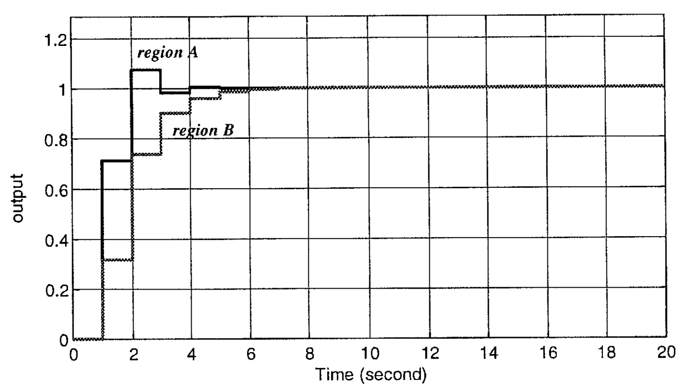

Fig. 5. Performance of local PI controllers.

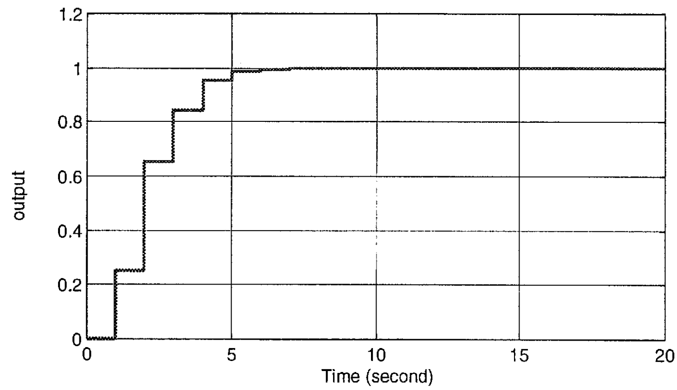

Fig. 6. Performance of the FuMR system.

Example 2. The nonlinear pH control continuous stirred tank chemical reactor (see Fig. 7) is now considered, with the dynamics described by the following equations:

$$\frac{\mathrm{d}V\xi}{\mathrm{d}t} = F_1 C_1 - (F_1 + F_2)\xi, \tag{24}$$

$$\frac{\mathrm{d}V\psi}{\mathrm{d}t} = F_2 C_2 - (F_1 + F_2)\psi, \tag{25}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = F_1 + F_2 - F, \tag{26}$$

$$[H^+]^3 + (K_a + \psi)[H^+]^2 + \big(K_a(\psi - \xi) - K_w\big)[H^+] - K_a K_w = 0,$$

where

$\xi \cong [HAC] + [AC^-]$,

$\psi \cong [Na^+]$,

$\mathrm{pH} = -\log_{10}[H^+]$,

$C_1 = 0.32\,[\mathrm{mol/l}]$ is the acid concentration in F_1 (flow rate of acid),

$C_2 = 0.05005\,[\mathrm{mol/l}]$ is the acid concentration in F_2 (flow rate of base),

$V = 1000\,[\mathrm{l}]$ is the volume of the tank,

K_a and K_w are acid and water equilibrium constants, respectively,

$K_a = 1.8\times10^{-5}$, $K_w = 1.0\times10^{-14}$, and $F_1(0) = 81\,[\mathrm{l/\,min}]$, $F_2(0) = 512\,[\mathrm{l/\,min}]$.

There are two control loops: a level (volume V) control loop and a pH control loop. A standard linear controller turned out to be sufficient for the volume control. However, the pH behaviour versus the flow F_1 is very nonlinear as we can see from the steady-state characteristics depicted in Fig. 8.

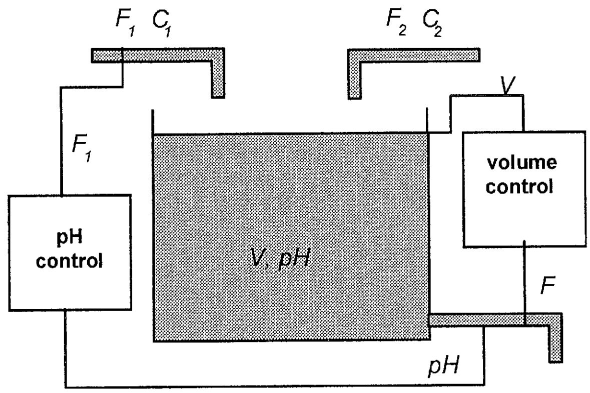

Fig. 7. The pH CSTR.

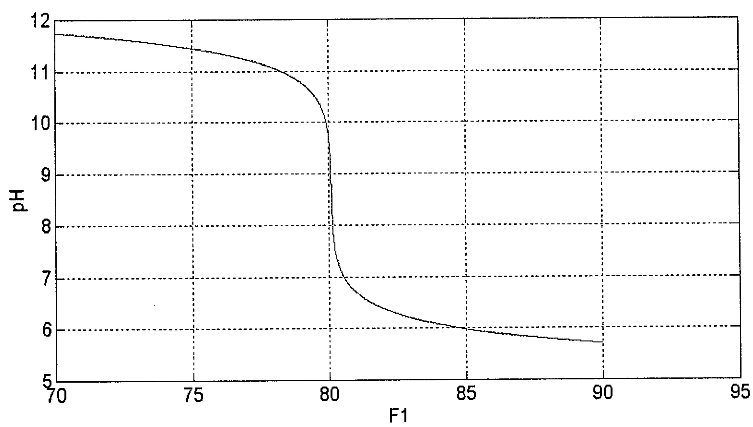

Fig. 8. Static pH curve.

The behaviour of the pH control loop with a single PI controller designed for average process dynamics is presented in Fig. 9. There are high oscillations of pH and V (not shown) in the region of the high gain. This is caused by the unstable controller performance in this subregion of a moderate pH. So it would be very reasonable to use another, nonlinear controller with its action dependent on the actual conditions. The fuzzy multi-regional controller possesses such an ability.

The pH dynamics model with three inputs $\mathrm{pH}(k-1)$, $\mathrm{pH}(k-2)$ and $F_1(k)$ and one output $\mathrm{pH}(k)$ was assumed. After fuzzy modelling the system, we obtained three local models in the domain of the pH signal (see Fig. 10).

The three local PI controllers were tuned using the standard Ziegler-Nichols procedure. As there is no control signal present in the fuzzy rule premises, no delay was needed. The closed-loop system (17) has six regions with the corresponding state-space matrices:

$$\tilde{\overline{A}}^{1} = \begin{bmatrix} 0.70 & -0.26 & 0.17 & -0.002 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \tilde{\overline{A}}^{2} = \begin{bmatrix} 0.92 & -0.03 & -0.51 & -0.14 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\tilde{\overline{A}}^{3} = \begin{bmatrix} 1.49 & -0.56 & -0.001 & 0.002 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\tilde{\overline{A}}^{12} = \begin{bmatrix} 0.81 & -22.11 & 6.35 & -0.07 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \tilde{\overline{A}}^{13} = \begin{bmatrix} 1.09 & -0.41 & 0.08 & -0.002 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\tilde{\overline{A}}^{23} = \begin{bmatrix} 1.21 & -0.24 & -0.01 & 0.07 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

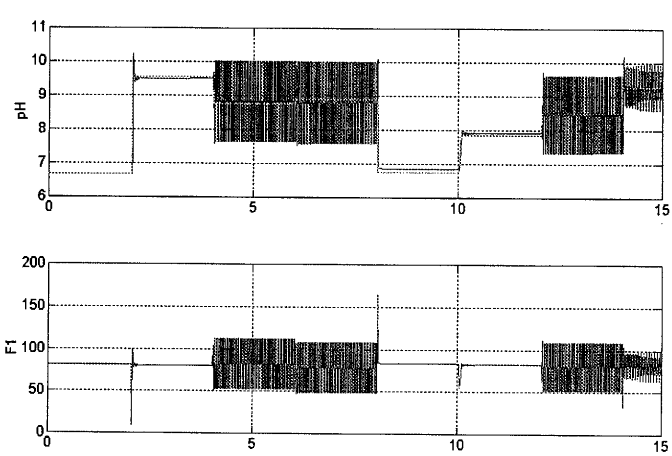

Fig. 9. Behaviour of the reactor system with a single PID controller.

For the above six subsystems the positive matrix

$$P = \begin{bmatrix} 7.6855 & 10.6140 & 2.1428 & 7.3875 \\ -1.6072 & 11.3143 & -4.3821 & -7.9680 \\ -10.2279 & -18.6881 & 6.2252 & -22.0545 \\ 8.5359 & 16.0419 & -11.1131 & 29.2553 \end{bmatrix}$$

which satisfies all the six local Lyapunov inequalities was found.

The performance of the fuzzy multi-regional PI controller (FuMR-PI) is presented in Fig. 11. As we can see, the controller allows us to obtain a satisfying performance with no oscillations in a wide area of possible set-points and thus pH changes. The FuMR controller is globally stable with a good dynamic response.

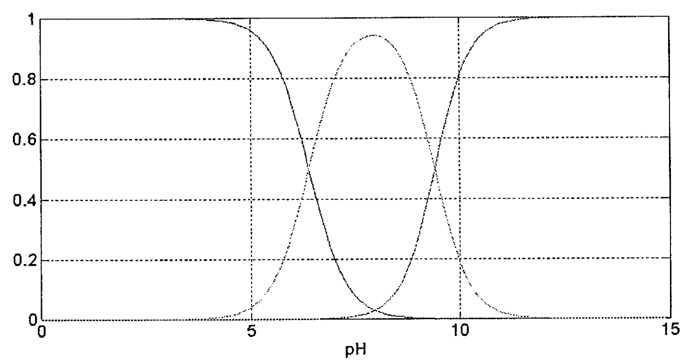

Fig. 10. Fuzzy clustering of the pH signal.

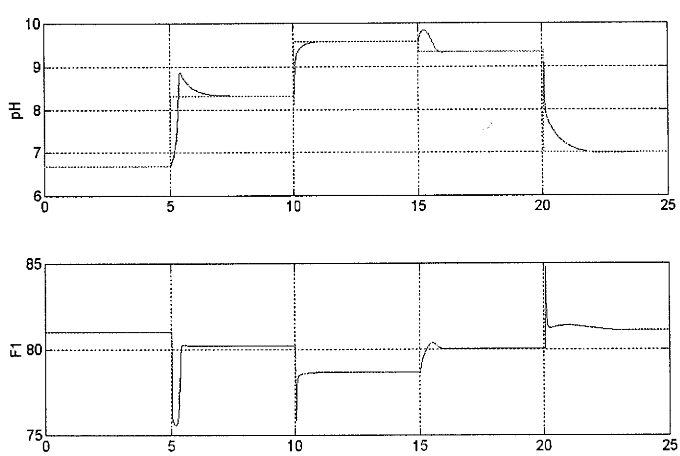

Fig. 11. Behaviour of the reactor system with FuMR PI controller.

The design methodology was also tried with predictive local controllers of IMC structure (Domański, 1996). Again, the performance of a single IMC controller with fixed parameters was not acceptable. On the other hand, the FuMR IMC controller revealed a good dynamic performance over the whole range of set-point changes, although it was slightly worse than for the presented FuMR PI controller.

6. Conclusions

The idea of fuzzy multi-regional controllers has been pursued in the paper. The controller is composed of a number of local linear controllers blended by using fuzzy logic. The description of the composite fuzzy control system in the augmented state-space domain has been derived that enabled the stability analysis on the basis of the Lyapunov function approach. The original idea of state augmentation has allowed us to extend the Takagi-Sugeno type stability analysis valid for local controllers of state feedback type to cover the dynamic output feedback controllers. The important class of such controllers including PID and predictive IMC ones can now be used in multi-regional controller design.

The sufficient criterion for the stability of the closed-loop system with the plant represented by its fuzzy model has been derived in the LMI form. Thus, the stability criterion can be checked efficiently using e.g. the MATLAB LMI toolbox. Hence the fuzzy multi-regional controller is an attractive option in a non-linear control design. Its design makes use of the well-developed linear dynamic output control technology and involves the efficient LMI technique.

Of course, the approach also has disadvantages. It leads only to a sufficient stability criterion. Using LMI methods we can quickly solve the Lyapunov inequalities, but if a solution does not exist, finding a right direction of the design modifications is not supported.

There is a noticeable conservatism in the derived stability criterion. Namely, the membership functions of the fuzzy sets used in the design do not affect the criterion. Sharpening the sufficient stability conditions by incorporating parameters of the fuzzy clusters would allow us to improve the FuMR controller performance by tuning not only local controller parameters but also by optimising fuzzy clustering.

The theory presented in the paper is illustrated with two rather simple examples. It has also found its first industrial application in one of the Polish power plants, where a multi-regional PID controller based on the fuzzy process modelling (Domański *et al.*, 1998) has been successfully applied to SO_2 emission control.

Acknowledgment

This work was partly supported by the Polish State Committee for Scientific Research under grant No. 8T11A 003 10.

References

Cao S.G., Rees N.W. and Feng G. (1996): *Stability analysis of fuzzy control systems.* — IEEE Trans. Syst. Man Cybern., Vol.26, No.1, pp.201–204.

Chen S. and Billings S.A. (1989): *Representation of non-linear systems: The NARMAX model.* — Int. J. Contr., Vol.49, No.3, pp.1013–1032.

Domański P.D. (1996): *Application of qualitative methods to modelling and design of control systems.* — Ph.D. Thesis, Warsaw University of Technology (in Polish).

Domański P.D., Włoszek A., Kucharski M., Misiurek D. and Rzewuski M. (1998): *Qualitative approach to the process modelling — Fluidized bed boiler example.* — Proc. ISIAC'98 during World Automation Congress'98 WAC'98, Anchorage, CD ISIAC.

Georgieva O. (1995): *Stability of quasilinear fuzzy systems.* — Fuzzy Sets Syst., Vol.73, pp.249–258.

Ishigami H., Fukuda T., Shibata T. and Arai F. (1995): *Structure optimization of fuzzy neural network by genetic algorithm.* — Fuzzy Sets Syst., Vol.71, pp.227–264.

Johansen T.A. and Foss B.A. (1993): *Constructing NARMAX models using ARMAX models.* — Int. J. Contr., Vol.58, No.5, pp.1125–1153.

Johansen T.A. (1994): *Fuzzy model based control: Stability, robustness, and performance issues.* — IEEE Trans. Fuzzy Syst., Vol.2, No.3, pp.221–234.

Korba P. and Frank P.M. (1997): *Controller design for a class of non-linear systems based on the fuzzy Takagi-Sugeno model.* — Proc. 7-th Workshop *Fuzzy Control*, Dortmund, Germany, pp.80–92.

Marin J.P. and Titli A. (1997): *Robust quadratic stabilizability of non-homogoneous Sugeno's systems ensuring completeness of the closed loop system.* — Proc. *FUZZ-IEEE 1997 Conference*, Vol.1, pp.185–192.

Takagi T. and Sugeno M. (1985): *Fuzzy identification of systems and its application to modeling and control.* — IEEE Trans. Syst. Man Cybern., Vol.15, pp.116–132.

Tanaka K. and Sugeno M. (1992): *Stability analysis and design of fuzzy control systems.* — Fuzzy Sets Syst., Vol.45, pp.135–166.

Wang H.O., Tanaka K. and Griffin M.F. (1996): *An approach to fuzzy control of nonlinear systems: Stability and design issues.* — IEEE Trans. Fuzzy Syst., Vol.4, No.1, pp.14–23.

Zhao J., Wertz V. and Gorez R. (1996): *Dynamic fuzzy state-feedback controller and its limitations.* — Proc. 13-th IFAC Congress, San Francisco, pp.121–1261.

Received: 16 April 1999
Revised: 23 March 1999