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OPTIMAL ERASURES IN DECISION–FEEDBACK EQUALIZATION
FOR THE GAUSSIAN NOISE

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A new method of optimizing decision feedback parameters for intersymbol interference equalizers is described. The coefficient existing in the decision feedback loop depends on risk qualification of the received decision. We prove that bit error probability can be decreased with this method for any channel with a single interference sample and small Gaussian noise. Experimental results are presented for selected channels. The dependences of optimal feedback parameters on channel interference samples and noise power are presented, too.

Keywords: data communication, decision feedback equalizer, intersymbol interference, error rate minimization

1. Introduction

The intersymbol interference equalization problem with the use of decision feedback has been described in many papers. It is a decision problem concerned with receiving incoming data. This is done by taking the recognized binary data to the tapped-delay line of the transversal filter and by using these data for incoming interference erasure, as shown in Fig. 1. If the discrete transfer function of the channel is

$$Y(z) = y_{-h} + y_{1-h}z^{-1} + \cdots + y_{-1}z^{-(h-1)}$$
$$+ y_0 z^{-h} + y_1 z^{-(h+1)} + \cdots + y_g z^{-(h+g)},$$

then the sample v_k of the equalizer input at the time t_k is

$$v_k = a_{k+h}y_{-h} + \cdots + a_{k+1}y_{-1} + a_k y_0 + a_{k-1}y_1$$
$$+ \cdots + a_{k-g}y_g + z_k,$$

where $a_{k+h}, \ldots, a_{k-g}$ are amplitudes of pulses representing the data being transmitted and z_k is the noise sample. Using v_k, we try to recover a_k. The part of v_k which depends on data, except for those represented by a_k, is called the *intersymbol interference* and has to be minimized. This problem is described in the books (Benedetto *et al.*, 1987; Clark, 1976; Dąbrowski, 1982; Dąbrowski and Dymarski, 2004).

To use the decision-feedback equalizer, the channel must be preequalized so that $y_{-h}, y_{1-h}, \ldots, y_{-1}$ is close to zero, and hence we can write $v_k = a_k y_0 +$ $a_{k-1}y_1 + a_{k-2}y_2 + \cdots + a_{k-g}y_g + z_k$. Data represented by $a_{k-1}, a_{k-2}, \ldots, a_{k-g}$ were received before a_k, so at the time t_k they are recognized as data represented by $d_{k-1}, d_{k-2}, \ldots, d_{k-g}$. Since the data symbol is recognized, its interference with the incoming next datum becomes known and can be subtracted from the incoming signal. The sample e_k at the decision unit input is

$$e_k = a_k y_0 + (a_{k-1} - d_{k-1})y_1 + (a_{k-2} - d_{k-2})y_2$$
$$+ \cdots + a_{k+g} + (a_{k-g} - d_{k-g})y_g + z_k.$$

The decision feedback equalizer works properly if the decisions in its delay line are correct, i.e., when $a_{k-1} = d_{k-1}, a_{k-2} = d_{k-2}, \ldots, a_{k-g} = d_{k-g}$, which means $e_k = a_k y_0 + z_k$. In the case of a received data error, the wrong data are taken to the feedback and the calculated incoming interference differs from the real one. For binary data, the existing interference is rather duplicated instead of being cancelled, so it increases the error probability for a_k. In this case no cancellation or partial cancellation are proposed, by subtracting a smaller value than the one calculated from the signal.

Partial cancellation is proposed only when the decision is risky, i.e., when the probability of the error is high. We qualify the decision to be risky if the signal value e_k at the decision unit input is close to the decision level S_*, i.e., if $|e_k - S_*| < \beta$. If the probabilities of sending 0 and 1 are the same and the signal mean value is equal to zero, the optimal decision level is $S_* = 0$.

We assume that the transmission path is composed of a partially equalized channel and a decision feedback
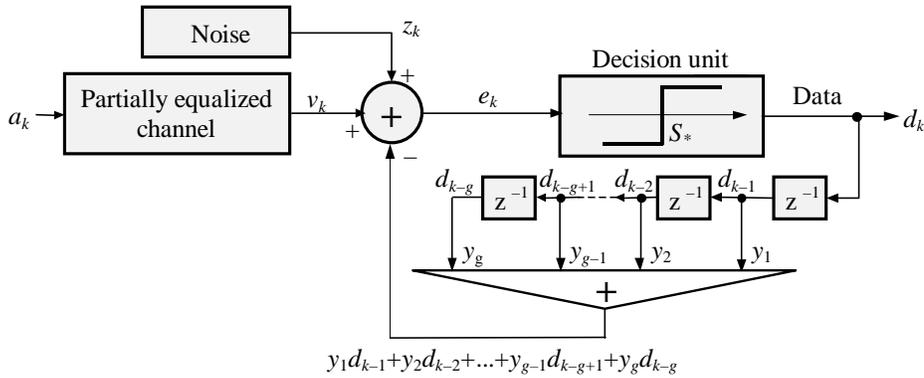
Fig. 1. Decision feedback and the equalizer.
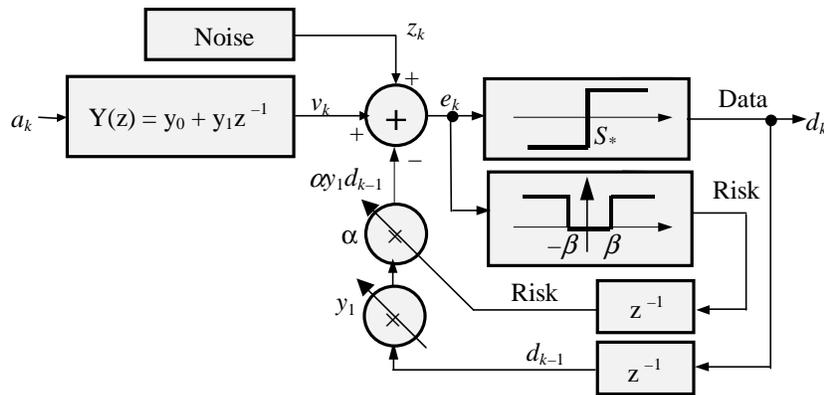
Fig. 2. Assumed channel and the equalizer.

equalizer, as shown in Fig. 2. The discrete transfer function of the channel is $Y(z) = y_0 + y_1 z^{-1}$ with the parameters $|y_0| > |y_1|$.

Decision feedback equalization with decision risk analysis is described in (Bergmans *et al.*, 1997; Chiani, 1997; Hacioglu and Amca, 1999). Similar methods, which do not include in equalization interference from the received signal when the received data were riskily detected, were proposed by Chiani (1997) and Hacioglu (1999). These methods do not subtract the calculated interference from the existing one if risky data have been used in calculations. This reduces the error extension phenomena described in (Altekar and Beaulieu, 1993; Choy and Beaulieu, 1997; Hacioglu and Amca, 1999; Labat and Laot, 2001). Chiani (1997) examined the dependence of the bit error rate (BER) on the risk threshold β, searching for the best values of β for chosen channels to obtain the minimum BER.

If the decision d_k is risky, the algorithm presented in this paper multiplies interference calculated from d_k by the factor of α and then it subtracts the result from the signal. In other words, if the value e_k at the decision unit input differs from the decision level $S_* = 0$ by less than β, i.e., $|e_k| < \beta$, the incoming interference calculated from d_k is multiplied by α (where $0 < \alpha < 1$) and the result is subtracted from the signal. In the case of an error $(d_k \neq a_k)$, interference at the decision unit input is multiplied by $1 + \alpha$, which is less than 2. The decision feedback contains an extra delay line which remembers risk qualifications of the decisions existing in a normal delay line of the feedback.

In this paper we will prove that it is possible to decrease the BER by properly choosing the parameters α and β. The optimal value of the risk level β as an analytic function of the factor α is given for channels with a single interference sample. The dependences of optimal α and β on the parameters y_0, y_1 and on the noise power σ^2 are presented graphically.

2. Calculation of the Risk Level

Assume that the noise probability density function is even and the probabilities of binary data values in the transmitting sequences are equal to 0.5. Let the bit 0 be represented by a negative pulse $(a_k = -1)$ and the bit 1 be represented by a positive pulse $(a_k = 1)$.

The first step is to find the dependence of bit error probability on the parameters α and β for the system shown in Fig. 2 with the discrete transfer function given by

$$Y(z) = y_0 + y_1 z^{-1}. \tag{1}$$

The sample of the channel response to the input signal at the time $t = kT$ is equal to $v_k = a_k y_0 + a_{k-1} y_1 + z_k$.

If the tap gain of the digital delay line is y_1, the value e_k at the decision module input is (Grzybowski and Kisilewicz, 1998):

$$e_k = a_k y_0 + (a_{k-1} - d_{k-1}) y_1 + z_k \tag{2}$$

for $\alpha = 1$, or

$$e_k = a_k y_0 + (a_{k-1} - \alpha d_{k-1}) y_1 + z_k \tag{3}$$

for $0 < \alpha < 1$.

In the case of $\alpha = 1$ and when the decisions (received data) are correct, $(d_{k-1} = a_{k-1})$, (2) gives

$$e_k = a_k y_0 + z_k = \begin{cases} y_0 + z_k & \text{if} \quad a_k = 1, \\ -y_0 + z_k & \text{if} \quad a_k = -1. \end{cases} \tag{4}$$

The sign of e_k determines the decision d_k. If $e_k > 0$, then $d_k = 1$, otherwise $d_k = -1$. If $|e_k| < \beta$, i.e., if e_k is close to zero, the decision d_k is assumed to be risky and in the next time period the calculated interference is compensated carefully, using (3) with a coefficient $0 < \alpha < 1$.

Let $f_z(z)$ be the probability density function of the noise z. We make the following assumptions:

- $f_z(z)$ is even, i.e., $f_z(-z) = f_z(z)$,

- the probabilities of $a_k = 1$ and $a_k = -1$ are equal.

Therefore, we will obtain the probabilities of the error in the decision d_k $(d_k \neq a_k)$ in case the decision d_{k-1} was assumed risky $(|e_{k-1}| < \beta)$ or not $(|e_{k-1}| > \beta)$.

In further investigations, we assume that y_0 and y_1 are positive. This assumption does not limit the generality of our deliberations. For negative y_0 and y_1, the appropriate signs preceding the variables y_0 and y_1 in the following equations will reverse all other signs, giving the same final result.

If the decision d_{k-1} was not risky $(|e_{k-1}| > \beta)$, then the probability of a wrong decision d_k is

$$p_{e1} = \int_{-\infty}^{-y_0} f_z(x) \, dx \tag{5}$$

if $d_{k-1} = a_{k-1}$, and

$$p_{e2} = \frac{1}{2} \int_{-\infty}^{-y_0 - 2y_1} f_z(x) \, dx + \frac{1}{2} \int_{-\infty}^{-y_0 + 2y_1} f_z(x) \, dx \tag{6}$$

if $d_{k-1} \neq a_{k-1}$.

If the correct decision $d_{k-1} = a_{k-1}$ was risky $(|e_{k-1}| < \beta)$, then (3) gives

$$e_k = \begin{cases} y_0 \pm (1 - \alpha) y_1 + z_k & \text{if} \quad a_k = 1, \\ -y_0 \pm (1 - \alpha) y_1 + z_k & \text{if} \quad a_k = -1, \end{cases} \tag{7}$$

and the probability of the binary error is

$$p_{e3} = \frac{1}{2} \int_{-\infty}^{-y_0 - (1-\alpha)y_1} f_z(x) \, dx + \frac{1}{2} \int_{-\infty}^{-y_0 + (1-\alpha)y_1} f_z(x) \, dx. \tag{8}$$

If the decision d_{k-1} was false $(d_{k-1} \neq a_{k-1})$ and risky $(|e_{k-1}| < \beta)$, then (3) gives

$$e_k = \begin{cases} y_0 \mp (1 + \alpha) y_1 + z_k & \text{if} \quad a_k = 1, \\ -y_0 \mp (1 + \alpha) y_1 + z_k & \text{if} \quad a_k = -1, \end{cases} \tag{9}$$

and the probability of the binary error is

$$p_{e4} = \frac{1}{2} \int_{-\infty}^{-y_0 - (1+\alpha)y_1} f_z(x) \, dx + \frac{1}{2} \int_{-\infty}^{-y_0 + (1+\alpha)y_1} f_z(x) \, dx. \tag{10}$$

Suppose that the correct decision d_{k-2} was not risky. Now we calculate the following probabilities:

(a) the probability of the correct and non-risky decision d_{k-1}:

$$q_1 = \int_{-y_0 + \beta}^{\infty} f_z(x) \, dx = \int_{-\infty}^{y_0 - \beta} f_z(x) \, dx, \tag{11}$$

(b) the probability of the false and non-risky decision d_{k-1}:

$$q_2 = \int_{-\infty}^{-y_0 - \beta} f_z(x) \, dx, \tag{12}$$

(c) the probability of the correct and risky decision d_{k-1}:

$$q_3 = \int\limits_{-y_0}^{-y_0+\beta} f_z(x)\, \mathrm{d}x, \qquad (13)$$

(d) the probability of the false and risky decision d_{k-1}:

$$q_4 = \int\limits_{-y_0-\beta}^{-y_0} f_z(x)\, \mathrm{d}x. \qquad (14)$$

From (5), (6), (8), (10) and (11)–(14), we obtain the probability of getting a false decision d_k in the case of a correct and non-risky decision d_{k-2}:

$$P_e = q_1 p_{e1} + q_2 p_{e2} + q_3 p_{e3} + q_4 p_{e4}. \qquad (15)$$

For the assumed value of the parameter α, the optimal β satisfies

$$\frac{\mathrm{d}P_e}{\mathrm{d}\beta} = 0. \qquad (16)$$

Since p_{e1}, p_{e2}, p_{e3} and p_{e4} are not dependent on β, and the parameter β appears in (11)–(14) only in upper or lower limits of integrals, we have

$$\frac{\mathrm{d}P_e}{\mathrm{d}\beta} = (p_{e3}-p_{e1})f_z(-y_0+\beta)+(p_{e4}-p_{e2})f_z(-y_0-\beta),$$
$$\qquad (17)$$

when

$$p_{e3}-p_{e1} = \frac{1}{2}\left[\int\limits_{-y_0}^{-y_0+(1+\alpha)y_1} f_z(x)\, \mathrm{d}x - \int\limits_{-y_0-(1-\alpha)y_1}^{-y_0} f_z(x)\, \mathrm{d}x \right],$$

$$p_{e4}-p_{e2} = \frac{1}{2}\left[\int\limits_{-y_0-2y_1}^{-y_0-(1+\alpha)y_1} f_z(x)\, \mathrm{d}x - \int\limits_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\, \mathrm{d}x \right].$$

Substituting the above expressions into (16), we obtain

$$\frac{f_z(-y_0+\beta)}{f_z(-y_0-\beta)}$$

$$= \frac{\displaystyle\int\limits_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\, \mathrm{d}x - \int\limits_{-y_0-2y_1}^{-y_0-(1+\alpha)y_1} f_z(x)\, \mathrm{d}x}{\displaystyle\int\limits_{-y_0}^{-y_0+(1-\alpha)y_1} f_z(x)\, \mathrm{d}x - \int\limits_{-y_0-(1-\alpha)y_1}^{-y_0} f_z(x)\, \mathrm{d}x}$$

$$= C\,(\alpha). \qquad (18)$$

Assuming the white Gaussian noise $N(0,\sigma)$, i.e.,

$$f_z(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}},$$

we obtain

$$\frac{f_z(-y_0+\beta)}{f_z(-y_0-\beta)} = e^{\frac{(\beta+y_0)^2-(\beta-y_0)^2}{2\sigma^2}} = e^{\frac{2\beta y_0}{\sigma^2}}$$

and, finally, the optimal β is given by

$$\beta = \frac{\sigma^2}{2y_0} \ln C\,(\alpha). \qquad (19)$$

Lemma 1. *If $0 < y_1 < y_0$ and $0 < \alpha < 1$, then β given by (19) minimizes the error probability P_e.*

Proof. The minimization of P_e is implied by the positiveness of the second derivative $\mathrm{d}^2 P_e/\mathrm{d}\beta^2$. Following (16) and (17) for the optimal β, i.e., for

$$(p_{e2}-p_{e4})f_z(-y_0-\beta) = (p_{e3}-p_{e1})f_z(-y_0+\beta),$$

we get

$$\frac{\mathrm{d}^2 P_e}{\mathrm{d}\beta^2} = (p_{e2}-p_{e4})\frac{2y_0}{\sigma^3\sqrt{2\pi}} e^{-\frac{(y_0+\beta)^2}{2\sigma^2}}. \qquad (20)$$

The second derivative is positive if so is the difference $p_{e2} - p_{e4}$. It is easy to prove that this difference is positive for the white Gaussian noise $N(0,\sigma)$ and for $0 < y_1 < y_0$ and $0 < \alpha < 1$. From (17) we get

$$2\,(p_{e2}-p_{e4}) = \int\limits_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\, \mathrm{d}x - \int\limits_{-y_0-2y_1}^{-y_0-(1+\alpha)y_1} f_z(z)\, \mathrm{d}z.$$

The ranges of changes in the variables x and z are the same and equal to $(1+\alpha)y_1$. But for $0 < y_1 < y_0$ and $0 < \alpha < 1$, we have $|x| \le y_0$ and $|z| \ge y_0$, and therefore $|x| \le |z|$. Since $f_z(x)$ is a Gaussian function, we obtain $f_z(x) \ge f_z(z)$. This gives the same inequality for the foregoing integrals, so $p_{e2} > p_{e4}$. This proves that the second derivative (20) is positive and the minimum of P_e exists for β which satisfies (19). ∎

The proposed method of optimizing the decision feedback can be useful for a large number of channels. Next we will prove that is possible to decrease error probability by this method for the white Gaussian noise if the noise power is sufficiently low.

Lemma 2. *For any channel with the samples y_0 and y_1 satisfying $0 < y_1 < y_0$, if the white Gaussian noise power is less than some positive value, there exist $\beta > 0$ and $0 < \alpha < 1$ that minimize the bit error probability P_e.*

Proof. From Lemma 1 we get the parameter β that minimizes the bit error probability P_e for fixed $0 < \alpha < 1$. We will prove Lemma 2 if we prove that the threshold β is positive ($\beta > 0$) for low noise ($\sigma \to 0$). In this case

we prove that the threshold β which minimizes P_e for the low noise is physically realizable.

To get positive β from (19), $C(\alpha)$ has to be greater than 1. From (18) we get

$$\int_{-y_0-(1-\alpha)y_1}^{-y_0} f_z(x)\,\mathrm{d}x + \int_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\,\mathrm{d}x$$
$$> \int_{-y_0-2y_1}^{-y_0-(1+\alpha)y_1} f_z(x)\,\mathrm{d}x + \int_{-y_0}^{-y_0+(1-\alpha)y_1} f_z(x)\,\mathrm{d}x, \quad (21)$$

where

$$f_z(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}.$$

From $0 < \alpha < 1$ and $0 < y_1 < y_0$ it is clear that the three integrals in (21) have negative limits. For such arguments the Gaussian function $f_z(x)$ is positive and increasing. Therefore

$$\int_{-y_0-2y_1}^{-y_0-(1+\alpha)y_1} f_z(x)\,\mathrm{d}x < \Delta_x f_z\big(-y_0-(1+\alpha)y_1\big),$$

$$\int_{-y_0}^{-y_0+(1-\alpha)y_1} f_z(x)\,\mathrm{d}x < \Delta_x f_z\big(-y_0+(1-\alpha)y_1\big),$$

$$\int_{-y_0-(1-\alpha)y_1}^{-y_0} f_z(x)\,\mathrm{d}x > \Delta_x f_z\big(-y_0-(1-\alpha)y_1\big),$$

where $\Delta_x = (1-\alpha)y_1$.

The estimation of the fourth integral is dependent on the absolute values of its limits. Therefore

$$\int_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\,\mathrm{d}x > \Delta_x f_z\big(-y_0+(1+\alpha)y_1\big),$$

when $|-y_0+(1+\alpha)y_1| > |-y_0+2y_1|$, i.e., when

$$0 < y_1 < \frac{2}{3+\alpha} y_0$$

or

$$\int_{-y_0+(1+\alpha)y_1}^{-y_0+2y_1} f_z(x)\,\mathrm{d}x > \Delta_x f_z(-y_0+2y_1),$$

when $|-y_0+(1+\alpha)y_1| \le |-y_0+2y_1|$, i.e., when $\frac{2}{3+\alpha} y_0 \le y_1 < y_0$.

The inequality (21) will be true if the following inequalities are true:

$$f_z\big(-y_0-(1-\alpha)y_1\big) + f_z\big(-y_0+(1+\alpha)y_1\big)$$
$$> f_z\big(-y_0-(1+\alpha)y_1\big) + f_z\big(-y_0+(1-\alpha)y_1\big) \tag{22a}$$

for $0 < y_1 < \frac{2}{3+\alpha}y_0$, and

$$f_z\big(-y_0-(1-\alpha)y_1\big) + f_z(-y_0+2y_1)$$
$$> f_z\big(-y_0-(1+\alpha)y_1\big) + f_z\big(-y_0+(1-\alpha)y_1\big) \tag{22b}$$

for $\frac{2}{3+\alpha}y_0 \le y_1 < y_0$.

Lemma 2 will be proved if we prove (22) for $\sigma \to 0$ and for the Gaussian function $f_z(x)$. Replacing $f_z(x)$ by $N(0,\sigma)$ in (22), from (22a) we have that

$$1 + \exp\left(\frac{2y_1(\alpha y_1 - y_0)}{\sigma^2}\right)$$
$$> \exp\left(\frac{-2(1+\alpha)y_1 y_0}{\sigma^2}\right) + \exp\left(\frac{2\alpha y_1(y_1-y_0)}{\sigma^2}\right) \tag{23a}$$

and, from (22b),

$$1 + \exp\left(\frac{y_1[4(y_1-y_0) - 2(1-\alpha)y_0 - (1-\alpha)^2 y_1]}{2\sigma^2}\right)$$
$$> \exp\left(\frac{y_1[4(y_1-y_0) - 2(1+\alpha)y_0 - (1+\alpha)^2 y_1]}{2\sigma^2}\right)$$
$$+ \exp\left(\frac{y_1[4(y_1-y_0) + 2(1-\alpha)y_0 - (1-\alpha)^2 y_1]}{2\sigma^2}\right). \tag{23b}$$

The left-hand sides of (23a) and (23b) are greater than one. For $0 < y_1 < y_0$ both exponents appearing on the right-hand side of (23a) are negative and they tend to $-\infty$ as $\sigma \to 0$. The right-hand side of the inequality decreases to zero. So (22a) is true for small σ.

The first exponent on the right-hand side of (23b) is negative. The second exponent is negative if $y_1 < \frac{2}{3+\alpha}y_0$, and for $\alpha = 1$, it is negative if $y_1 < y_0$. In any case, it is possible to choose α to get a negative value of this exponent. In this case (23b) is true. Accordingly, it is possible to find α making (21) true for low noise. ∎

It is easy to notice that Lemma 2 can be also generalized for the negative values of y_0 or y_1.

Lemma 3. *For any channel with the samples y_0 and y_1 satisfying $0 < |y_1| < |y_0|$, if the power of the white Gaussian noise is less than some positive value, there exist $\beta > 0$ and $0 < \alpha < 1$ that minimize the bit error probability P_e.*

Proof. Lemma 3 is nothing but Lemma 2 with the assumption that $|y_1| < |y_0|$ instead of $0 < y_1 < y_0$. Given bit error probability, (15) includes the probabilities p_{e1}, p_{e2}, p_{e3} and p_{e4}. They are expressed by the integrals (5), (6), (8) and (10) with the positive values $0 < y_1 < y_0$. If $y_1 < 0$, the replacement of y_1 by $-y_1$ does not change these equations, but the new y_1 will be positive. For negative y_0, the original equations (5), (6), (8), (10) and (11)–(14) will use negative y_0 in place of $-y_0$ and hence the same limits of integration can be used. The replacement of y_0 by $-y_0$, where the new y_0 is positive, converts the above equations to the same formulas which were used in (5), (6), (8) (10) and (11)–(14). Because the new positive quantities y_1 and y_0 satisfy the assumption of Lemma 2 and the equation for P_e is not changed, Lemma 3 is true. ∎

Lemma 3 gives information as to when decision feedback optimization by decision risk qualification can decrease bit error probability. We proved that this method can be used successfully for any channel (1) with $|y_1| < |y_0|$ if the noise power is sufficiently low.

The scale of decreasing the bit error probability P_e using the best values α and β depends on the noise power σ^2, and on y_0 and y_1. This will be illustrated with computational results for three selected channels.

3. Simulation Experiment

The aim of the experiment is to find out how much the optimization of decision feedback can decrease bit error probability and to find the optimal values of α and β as functions of channel parameters.

The channel chosen in the experiment was described by two pulse response samples, and therefore its transfer function is given by (1). These samples are normalized for their energy equal to one, so that $y_0^2 + y_1^2 = 1$. The sample y_1 is the interference sample and the sample y_0 is the main sample. If $|y_1| < |y_0|$, then $|y_1| = \sqrt{1 - y_0^2}$, where $1/\sqrt{2} < y_0 \leq 1$. First, the value of y_0 was increased by some constant and then y_1 was calculated.

For such channels, the assumed noise power σ^2 and the assumed values α ($0 \leq \alpha < 1$), the optimal risk qualification threshold β was calculated to minimize the bit error probability P_e. Optimal α^* and β^* were find by changing α and calculating optimal β and P_e for α. The main result of this experiment is to find optimal α^* and β^* as functions of the quotient y_1/y_0 and the noise power σ^2, $\alpha^*(y_1/y_0, \sigma)$ and $\beta^*(y_1/y_0, \sigma)$.

4. Results and Conclusion

Three channels were chosen for experiments. The parameter α was increased from 0.1 to 0.9 for each channel.

For each α the optimal level β was calculated using (19) and (20), and then the bit error rate P_e was obtained from (15). For each channel the bit error rate P_{e_0} was calculated using (15) with the assumption that $\beta = 0$ and $\alpha = 1$. Bit error rate reduction in the effect of decision feedback optimization is given by the quotient P_e/P_{e_0}. These experiments were repeated for several values of the noise power σ^2. The resulting bit error rate reductions are shown in Figs. 3–5 for three selected channels.

The minimum of this dependence represents the best reduction and gives the optimal point (α^*, β^*). The minimum P_e is reached at this point. These approximate points (α^*, β^*) are shown as crosses, triangles, squares or circles.
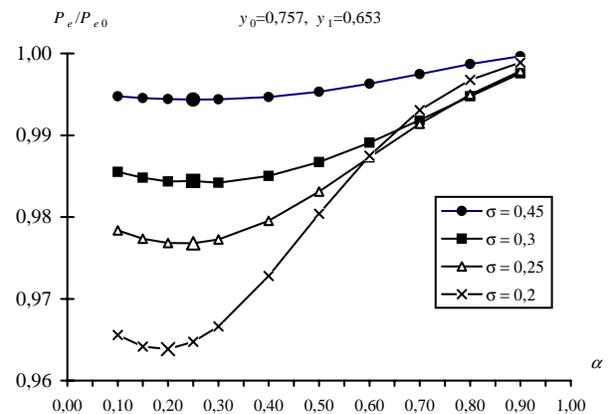
Fig. 3. BER reduction for the channel with $y_0 = 0,757$, $y_1 = 0,653$.
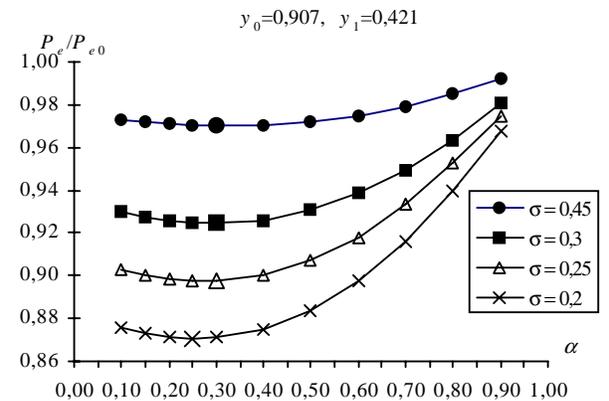
Fig. 4. BER reduction for the channel with $y_0 = 0,907$, $y_1 = 0,421$.

Optimal α^* depends on the noise power only to some extent, as seen in Figs. 3–5. The results for particular channels are as follows:
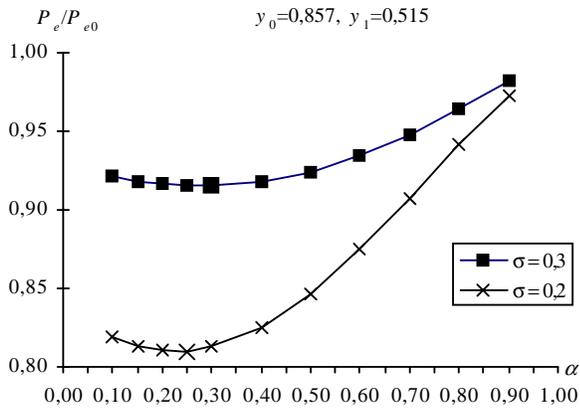
Fig. 5. BER reduction for the channel with
$y_0 = 0,857, \ y_1 = 0,515$.

- For Channel 1 $(y_0 = 0.757, \ y_1 = 0.653)$ we obtain $0.20 \leq \alpha^* \leq 0.25$,

- For Channel 2 $(y_0 = 0.907, \ y_1 = 0.421)$ we obtain $0.25 \leq \alpha^* \leq 0.30$,

- For Channel 3 $(y_0 = 0.857, \ y_1 = 0.515)$ we obtain $0.25 \leq \alpha^* \leq 0.30$.

The functions $\alpha^*(\sigma)$ and $\beta^*(\sigma)$ are shown in Figs. 6 and 7 for all three channels. The optimal α^* and β^* depend on the channel and increase if σ^2 increases for small σ. The functions $\alpha^*(y_1/y_0)$ and $\beta^*(y_1/y_0)$ are shown in Figs. 8–9 for some σ. Usually, optimal α^* and β^* are smaller for larger interference. These diagrams can be helpful in the optimization of decision feedback given a channel with known noise.
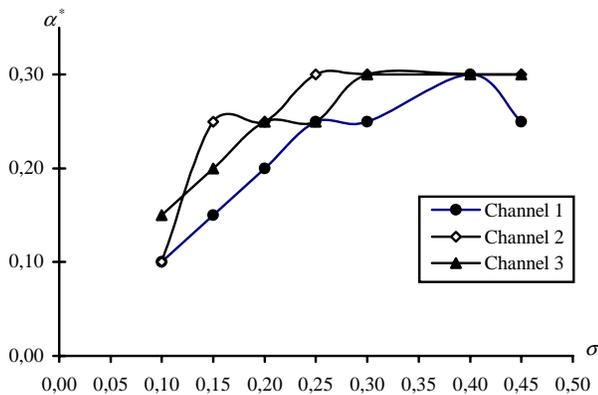
Fig. 6. Dependence of the parameter α^* on σ.

Fig. 7. Dependence of the parameter β^* on σ.

Fig. 8. Optimal values of the parameter α.

Fig. 9. Optimal values of the parameter β.

5. Summary
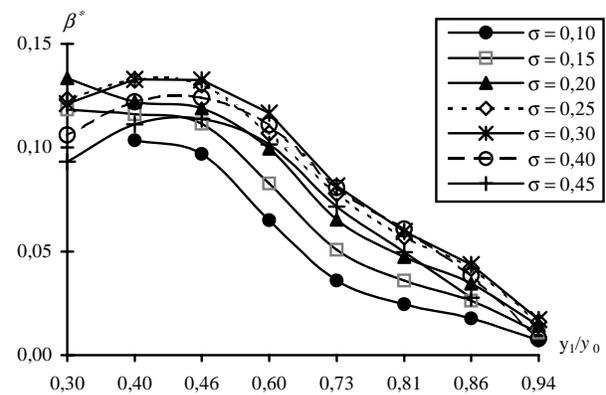
The presence of errors in decision feedback of an equalizer involves a high probability of new errors. This is called the *error extension*. The reduction of *error extension* is still an open problem. The method presented in this paper allows us to reduce error extension for some kind of channels. It was proved in an analytical way that the presented method has an optimal solution. It was also proved for channels with discrete transfer functions $Y(z) = y_0 + y_1 z^{-1}$, i.e., with one interference sample $y_1 \ (|y_1| < |y_0|)$ and with the white Gaussian noise, that the optimization of decision feedback using the risk

threshold β and the erasure factor α decreases the bit error rate if the noise power is suitably low. Under these assumptions, the optimal values for α and β were found numerically. The results are presented in Figs. 8 and 9.

BER reduction depends on channel parameters and the signal-to-noise ratio. The proposed optimization of decision feedback decreases the bit error rate by 1% to 19% depending on the channel (y_1/y_0) and the noise power. For lower noise (a higher signal-to-noise ratio), the bit error rate decreases more than for higher noise. In the presented experiments, the BER decreases from 3.6% to 19.0% for $\sigma = 0.2$ (SNR $= 5$), and from 1.6% to 8.5% for $\sigma = 0.3$ (SNR $= 3.3$).

The presented theoretical deliberations were performed for the optimization of the second decision following error absence in decision feedback of the equalizer. They have to be generalized for any decision assuming the Markov chain model for this decision process.

Another problem that has to be solved is the implementation or extension of the presented results for channels with more than one interference sample, i.e., for longer decision feedback. We can test BER reduction for separate optimization of each tap of the feedback using more than one decision, as in Fig. 1. Next we can test simultaneous optimization of all parameters of feedback with many taps.

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