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DECENTRALIZED STATIC OUTPUT TRACKING CONTROL OF INTERCONNECTED AND DISTURBED TAKAGI–SUGENO SYSTEMS

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This article describes a new procedure for the design of decentralized output-feedback tracking controllers for a class of interconnected Takagi–Sugeno (TS) fuzzy systems with external bounded disturbances and measurement noise. The main idea consists in transforming the decentralized tracking control problem, by using the descriptor redundancy formulation, to a robust decentralized stabilization one. The non-parallel distributed compensation (non-PDC) controllers proposed here are synthesized to satisfy robust H_∞ tracking performance with disturbance attenuation. The decentralized controllers design conditions are given in terms of LMIs via extended quadratic Lyapunov functions. Finally, simulations are presented: two numerical examples are dedicated to compare the conservatism of the proposed approach regarding the previous results available in the literature; then, the effectiveness of the decentralized controller design methodology is illustrated with a closed-loop simulation of two inverted pendulums connected by a spring.

Keywords: interconnected Takagi–Sugeno systems, decentralized static outputs tracking controllers, H_∞ criterion, LMIs.

1. Introduction

In the past decade, interconnected systems have been considered to describe various systems with rich interactions between a large number of state variables characterizing their dynamics (Guo *et al.*, 2013; 2015; 2000; Qu *et al.*, 2017; Wang and Ohmori, 2016; Benzaouia *et al.*, 2016; Deng and Yang, 2017; Wang and Yang, 2017; Zhong *et al.*, 2016; Yang *et al.*, 2016; Zhang and Yang, 2017; Jang *et al.*, 2017; Li and Tong, 2017). Decomposing a large-scale nonlinear system into a finite set of interconnected subsystems has found applications, for examples, in the areas of networked power systems, water transportation networks, traffic systems or ecological environment (Guo *et al.*, 2013; 2015). A glimpse at the literature shows that several works on interconnected systems have been proposed. They are related to many issues, such as modeling, stability analysis, control design, and so on (Qu *et al.*, 2017; Wang and Ohmori, 2016; Guo *et al.*, 2000; Kaczorek, 2018).

Among nonlinear control approaches, Takagi–Sugeno (TS) fuzzy systems have caught the attention of the control community due their ability to match nonlinear systems and to extend some control concepts that are originally dedicated to linear systems (Tagaki and Sugeno, 1985; Tanaka and Ohtake, 2001). Thus, several works dealing with the stability analysis and stabilization issues for interconnected TS fuzzy systems have appeared. For instance, recent works deal with the design of decentralized parallel-distributed-compensation (PDC) controllers design via common quadratic Lyapunov functions (Benzaouia *et al.*, 2016), considering

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nonlinear interconnections between subsystems (Deng and Yang, 2017), decentralized piecewise dynamic output-feedback controllers design (Wang and Yang, 2017), or robust H_∞ decentralized output-feedback controllers design (Zhong *et al.*, 2016).

Beside the above mentioned stabilization studies, tracking controller design is an important challenge in dealing with complex control problems. It consists in synthesizing controllers to ensure the convergence of the tracking error between the system's outputs (or states) and desired references. However, with regard to interconnected nonlinear systems, only few results have been reported on the tracking control problem (Zhao *et al.*, 2017; Kim *et al.*, 2017; Tong *et al.*, 2016). Kim *et al.* (2017) considered decentralized state-feedback tracking PDC controllers for interconnected TS systems from the point of view of sampled data. Furthermore, an observer-based decentralized tracking control approach was proposed by Tong *et al.* (2016) for a set of TS systems interconnected by their outputs. Recently, neural network algorithms have been proposed for decentralized tracking control of a class of interconnected nonlinear systems (Qu *et al.*, 2017; Li and Tong, 2017).

To improve robustness against external disturbances of decentralized TS tracking control problems, several solutions have been proposed (Guo *et al.*, 2013; 2015; Tseng and Chen, 2001; Wang and Tong, 2006; Liu *et al.*, 2014). Guo *et al.* (2013; 2015) set forth conditions available for specific classes of an application-oriented set of finite interconnected TS subsystems, reducing their applicability to a more generic class of large-scale systems. Furthermore, LMI-based conditions have been established by Tseng and Chen (2001). Then, inspired by that work LMI-based tracking control conditions have been proposed for a class of interconnected TS systems with uncertainties by Wang and Tong (2006), and with time-varying delay by Liu *et al.* (2014). However, the drawback of these approaches is the use of several restricting assumptions. Moreover, the LMI-based decentralized tracking control conditions proposed by Liu *et al.* (2014) cannot be solved for a set of interconnected subsystems of different orders.

In a nutshell, although the available decentralized tracking controller design approaches have provided effective solutions for some class of interconnected nonlinear systems, the proposed LMI-based tracking control conditions still suffer from conservatism due to the use of a decentralized PDC control scheme and common quadratic Lyapunov functions. To the best of our knowledge, generic relaxed LMI conditions for decentralized output-tracking controller design of interconnected TS systems subject to external disturbances and measurement noise have not been explored yet, which motivates the present study. The contributions of this paper can be highlighted as follows:

- Providing a decentralized non-PDC output-feedback tracking controller design methodology which is able to ensure, on the one hand, the stability of the overall interconnected nonlinear systems and, on the other hand, robust H_∞ output-tracking performances with decentralized disturbances and measurement noise attenuation, such that each subsystem has a specific disturbance attenuation level.

- Providing new LMI conditions for the existence of output-feedback tracking controllers. To this end, the idea is to consider rewriting the interconnected closed-loop dynamics using a descriptor redundancy formulation (see, e.g., Schulte and Guelton, 2006; Tanaka *et al.*, 2007; Guelton *et al.*, 2009; Bouarar *et al.*, 2010; 2013).

- Providing relaxed LMI conditions by introducing slack decision variables with Peaucelle's transformations (Peaucelle *et al.*, 2000).

2. Preliminaries and the problem statement

Consider a set of n interconnected TS subsystems S_i ($i = 1, \dots, n$) given by

$$\begin{cases} \dot{x}_i(t) = \sum\limits_{j_i=1}^{m_i} h_{j_i}(z_i(t)) \big(A_{j_i} x_i(t) + B_{j_i} u_i(t) \\ \qquad + B_{j_i}^w w_i(t) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} h_{j_i}(z_i(t)) F_{j_i}^{\alpha} x_\alpha(t)\big), \\ y_i(t) = \sum\limits_{j_i=1}^{m_i} h_{j_i}(z_i(t)) \big(C_{j_i} x_i(t) + B_{j_i}^{\nu} \nu_i(t)\big), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{\eta_i}$, $y_i(t) \in \mathbb{R}^{\rho_i}$, $u_i(t) \in \mathbb{R}^{\upsilon_i}$ are respectively the i-th state, measurement (output) and input vectors, $w_i(t) \in \mathbb{R}^{\mu_i}$ and $\nu_i(t) \in \mathbb{R}^{\nu_i}$ are respectively a time-varying L_2-norm-bounded external disturbance and measurement noise signals associated with the i-th subsystem, m_i is the number of vertices of the i-th TS subsystem and, for $j_i = 1, \dots, m_i$, $A_{j_i} \in \mathbb{R}^{\eta_i \times \eta_i}$, $B_{j_i} \in \mathbb{R}^{\eta_i \times \upsilon_i}$, $B_{j_i}^w \in \mathbb{R}^{\eta_i \times \mu_i}$, $C_{j_i} \in \mathbb{R}^{\rho_i \times \eta_i}$ and $B_{j_i}^{\nu} \in \mathbb{R}^{\rho_i \times \nu_i}$ are constant matrices. The matrices $F_{j_i}^{\alpha} \in \mathbb{R}^{\eta_i \times \eta_\alpha}$ express the interconnection between the i-th subsystem and the α-th subsystem with $\alpha = 1, \dots, n$ and $\alpha \neq i$, $z_i(t)$ are the premise variables of the i-th TS subsystem, assumed to depend only on the component of the i-th output vector (i.e., $z_i(t) = y_i(t)$) for the static output feedback control purpose. Finally, $h_{j_i}(z(t)) \geq 0$ are the fuzzy membership functions of the i-th TS subsystem, which satisfy the convex sum proprieties $\sum_{j_i=1}^{m_i} h_{j_i}(z_{j_i}(t)) = 1$.

Remark 1. There are several ways to obtain the decomposition of a complex system into interconnected subsystems, depending on the nature of the global system. For instance, some physical systems are naturally interconnected (as in Section 4.3), some other complex or large-scale systems can be decomposed as subsystems based on mathematical subsets; see, e.g., the work of Bakule (2014) and the references therein for more details on these decomposition techniques.

For the clarity of mathematical expressions, in the sequel the following notation is adopted: $\mathcal{H}(W)$ stands for $W+W^T$; a star (*) in a matrix denotes a transpose quantity; I_{η_i} denotes the η_i-order identity matrix; the time t as the argument of functions will be omitted when there is no ambiguity; for convex combinations of matrices of appropriate dimensions, we set $M_{h_i}=\sum_{j_i}^{m_i} h_{j_i}(z_{j_i})M_{j_i}$, $M_{h_ih_i}=\sum_{k_i=1}^{m_i}\sum_{j_i=1}^{m_i} h_{k_i}(z_{k_i})h_{j_i}(z_{j_i})M_{k_ij_i}$, and so on.

Our goal is to design a decentralized static output feedback control scheme that drives the interconnected TS systems (1) to track desired outputs, whose dynamics are specified, for each subsystems S_i, by the following set of n linear reference models ($i=1,\dots,n$):

$$\begin{cases} \dot{x}_{r_i}(t)=A_{r_i}x_{r_i}(t)+B_{r_i}r_i(t), \\ y_{r_i}(t)=C_{r_i}x_{r_i}(t), \end{cases} \tag{2}$$

where $x_{r_i}(t) \in \mathbb{R}^{\eta_i}$, $r_i(t) \in \mathbb{R}^{\upsilon_i}$ and $y_{r_i}(t) \in \mathbb{R}^{\rho_i}$ are respectively the i-th reference state vector, reference input vector and output vector, while $A_{r_i} \in \mathbb{R}^{\eta_i\times\eta_i}$, $B_{r_i} \in \mathbb{R}^{\eta_i\times\upsilon_i}$, $C_{r_i} \in \mathbb{R}^{\rho_i\times\eta_i}$ are real constant matrices specifying the dynamics to be tracked by each TS subsystems.

Remark 2. Theoretically speaking, it may be argued that, the plant dynamics (1) being nonlinear, the reference models (2) should also be chosen nonlinear. However, note that the dynamics of these reference models have to be pre-specified to design convenient tracking controller. Thus, choosing linear reference models makes their tuning by practitioners easier. Indeed, they can be chosen stable (i.e., each A_{r_i} Hurwitz) and their dynamics (tracking objectives) can be conveniently tuned, for instance, by linear pole placement techniques. For example, a convenient choice can be to design the reference models (2) as low-pass filters so that the reference outputs $y_{r_i}(t)$ are a smoothed replica of the reference signal $r_i(t)$ (see, e.g., the work of Seddiki *et al.* (2010) for an example of similar reference model tuning).

Assuming that all subsystems (1) are controllable, from the decentralized control topology depicted in Fig. 1, to drive the subsystem's outputs (1) to track the reference's outputs signals (2), a set of n decentralized non-PDC static output tracking controllers is introduced, for $i=1,\dots,n$, by

$$u_i(t)=K_{h_i}H_{h_ih_i}^{-1}e_i(t), \tag{3}$$

where $e_i(t)=y_i(t)-y_{r_i}(t) \in \mathbb{R}^{\rho_i}$ is the output tracking error of the i-th subsystem, K_{h_i} and $H_{h_ih_i}={H_{h_ih_i}}^T > 0$ are the decentralized static output tracking non-PDC controller gains to be synthesized.

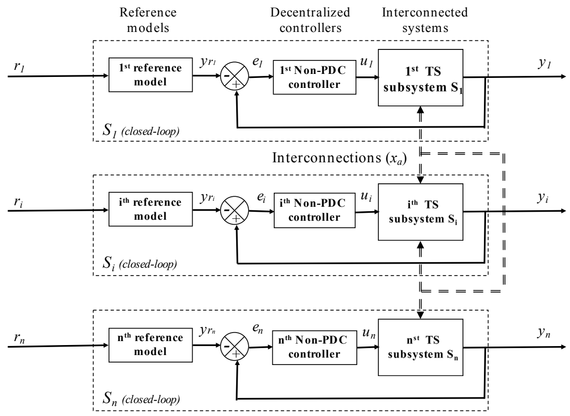

Fig. 1. Overall block diagram of the proposed control strategy.

The closed-loop dynamics can usually be expressed by substituting the control laws (3) into the open-loop subsystems (1) (see, e.g., Tanaka *et al.*, 2003). However, when dealing with static output feedback, this method introduces several crossing terms between the controllers' gains and the systems' output matrices (e.g., $B_{h_i}K_{h_i}H_{h_ih_i}^{-1}C_{h_i}$, $i=1,\dots,n$) and, unfortunately, lead to non-convex closed-loop stability conditions. Therefore, to avoid the occurrence these crossing terms, we will consider a descriptor redundancy formulation of the closed-loop dynamics (Tanaka *et al.*, 2007; Guelton *et al.*, 2009; Bouarar *et al.*, 2010; Jabri *et al.*, 2011; 2018b). The interest in this roundabout way is twofold: first, it allows us to decouple the systems' matrices from the controller gains; secondly, it introduces additional slack decision matrices, so as to reduce the conservatism of the stability conditions.

Therefore, to deal with the descriptor redundancy approach, consider first the augmented state vectors

$$\tilde{x}_i^T=\begin{bmatrix}(x_i-x_{r_i})^T & x_{r_i}^T & e_i^T\end{bmatrix}$$

and the extended disturbances

$$\tilde{w}_i^T=\begin{bmatrix}w_i^T & r_i^T & \nu_i^T\end{bmatrix}.$$

The open-loop TS subsystems (1) combined with the reference models (2) can be rewritten as the following descriptors:

$$E_i\dot{\tilde{x}}_i=\tilde{A}_{h_i}\tilde{x}_i+\tilde{B}_{h_i}u_i+\tilde{B}_{h_i}^{\tilde{w}}\tilde{w}_i+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{F}_{h_i}^{\alpha}\tilde{x}_{\alpha} \tag{4}$$

with

$$E_i=\begin{bmatrix} I_{\eta_i} & 0 & 0 \\ 0 & I_{\eta_i} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{A}_{h_i} = \begin{bmatrix} A_{h_i} & A_{h_i} - A_{r_i} & 0 \\ 0 & A_{r_i} & 0 \\ C_{h_i} & C_{h_i} - C_{r_i} & -I \end{bmatrix},$$

$$\tilde{B}^{\tilde{w}}_{h_i} = \begin{bmatrix} B^w_{h_i} & 0 & 0 \\ 0 & B_{r_i} & 0 \\ 0 & 0 & B^{\nu}_{h_i} \end{bmatrix},$$

$$\tilde{B}_{h_i} = \begin{bmatrix} B_{h_i} \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{F}^{\alpha}_{h_i} = \begin{bmatrix} F^{\alpha}_{h_i} & F^{\alpha}_{h_i} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Similarly, the decentralized non-PDC controllers (3) can be rewritten as

$$u_i = \tilde{K}_{h_i \overline{h_i h_i}} \tilde{x}_i \tag{5}$$

with $\tilde{K}_{h_i \overline{h_i h_i}} = \begin{bmatrix} 0 & 0 & K_{h_i} H^{-1}_{h_i h_i} \end{bmatrix}$.

Hence, substituting (5) into (4), the closed-loop dynamics of each subsystem S_i can be expressed as

$$E_i \dot{\tilde{x}}_i = \tilde{G}_{h_i h_i \overline{h_i h_i}} \tilde{x}_i + \tilde{B}^{\tilde{w}}_{h_i} \tilde{w}_i + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \tilde{F}^{\alpha}_{h_i} \tilde{x}_{\alpha}, \tag{6}$$

where

$$\begin{aligned} \tilde{G}_{h_i h_i \overline{h_i h_i}} &= \tilde{A}_{h_i} + \tilde{B}_{h_i} \tilde{K}_{h_i \overline{h_i h_i}} \\ &= \begin{bmatrix} A_{h_i} & A_{h_i} - A_{r_i} & B_{h_i} K_{h_i} H^{-1}_{h_i h_i} \\ 0 & A_{r_i} & 0 \\ C_{h_i} & C_{h_i} - C_{r_i} & -I \end{bmatrix} \end{aligned}$$

is free from crossing terms between the controller gains and the systems' output matrices.

Problem statement. The design objective considered in this study is summarized by the following requirements. For $i = 1, \ldots, n$, design the gain matrices K_{h_i} and $H_{h_i h_i}$ of the decentralized non-PDC output tracking controllers (3) such that

(i) the closed-loop dynamics (6) are globally asymptotically stable without external disturbances and measurement noise ($\tilde{w}_i(t) \equiv 0$);

(ii) each closed-loop subsystem (6) minimizes the transfer between the external disturbances (and measurement noise) $\tilde{w}_i(t)$ and the output tracking error $e_i(t)$ this can be achieved with the following H_∞ criterion, for $i = 1, \ldots, n$:

$$\int_0^{+\infty} \tilde{x}_i^T \Omega_i \tilde{x}_i \, \mathrm{d}t - \gamma_i^2 \int_0^{+\infty} \tilde{w}_i^T \tilde{w}_i \, \mathrm{d}t \leq 0, \tag{7}$$

where the scalars $\gamma_i > 0$ denote the disturbance attenuation levels of the closed-loop subsystem S_i with

$$\Omega_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{\eta_i} \end{bmatrix}.$$

To conclude these preliminaries, let us recall the following useful lemmas to be used to prove the main results.

Lemma 1. (Xie and de Souza, 1992) *For any matrices A and B with appropriate dimensions and any matrix $\mathcal{T} = \mathcal{T}^T > 0$, the following inequality is satisfied:*

$$A^T B + B^T A \leq A^T \mathcal{T} A + B^T \mathcal{T}^{-1} B. \tag{8}$$

Lemma 2. (Tuan *et al.*, 2001) *Let $\Gamma_{j_i k_i}$, $(j_i, k_i) \in \{1, \ldots, m_i\}^2$, be matrices of appropriate dimensions. $\Gamma_{h_i h_i} < 0$ is satisfied if both of the following conditions hold:*

$$\forall j_i \in \{1, \ldots, m_i\} : \Gamma_{j_i j_i} < 0, \tag{9}$$

$$\forall (j_i, k_i) \in \{1, \ldots, m_i\}^2 / k_i \neq j_i : \quad \frac{2}{r-1} \Gamma_{j_i j_i} + \Gamma_{j_i k_i} + \Gamma_{k_i j_i} < 0. \tag{10}$$

Lemma 3. (Peaucelle *et al.*, 2000) *Let A, L, X, Q and R be matrices of proper size. The following inequalities are equivalent:*

$$\mathcal{H}(AX) + Q < 0, \tag{11}$$

$$\exists R, L : \begin{bmatrix} \mathcal{H}(AL) + Q & (*) \\ X - L + R^T A^T & -\mathcal{H}(R) \end{bmatrix} < 0. \tag{12}$$

3. Main results

The goal of this section is to propose sufficient LMI conditions to design the gain matrices K_{h_i}, $H_{h_i h_i}$ so that the robust H_∞ output-feedback tracking requirements defined above in the problem statement are satisfied. The first result is summarized by the following theorem.

Theorem 1. *For $i = 1, \ldots, n$, consider the interconnected and disturbed TS systems* (1) *and their respective reference models* (2)*. Under the decentralized static output tracking non-PDC control law* (3)*, the state and output tracking errors converge asymptotically to zero (without external disturbances) and satisfy the H_∞ criteria* (7)*, if there exist matrices $\mathcal{T}_{i\alpha} = \mathcal{T}^T_{i\alpha} > 0$, $X_{1i} = X^T_{1i}$, X_{2i}, $X_{3i} = X^T_{3i}$, K_{k_i} and $H_{j_i k_i}$, such that, minimizing γ_i^2, the LMI* (9), (10) *and* (13) *are satisfied with $\Gamma_{j_i j_i}$ defined in* (14)*:*

$$\begin{bmatrix} X_{1_i} & (*) \\ X_{2_i} & X_{3_i} \end{bmatrix} = \begin{bmatrix} X_{1_i} & (*) \\ X^T_{2_i} & X_{3_i} \end{bmatrix}^T > 0. \tag{13}$$

$$\Gamma_{j_ik_i} = \left[\begin{array}{cc|cccccc} \multicolumn{2}{c|}{\Psi_{j_ik_i}} & \multicolumn{6}{c}{(*)} \\ \hline X_{j_ik_i} & 0 & -\mathcal{T}_{1i} & 0 & \dots & \dots & \dots & 0 \\ \vdots & \vdots & 0 & \ddots & \ddots & & & \vdots \\ \vdots & \vdots & \vdots & \ddots & -\mathcal{T}_{i-1\,i} & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & -\mathcal{T}_{i+1\,i} & \ddots & \vdots \\ \vdots & \vdots & \vdots & & & \ddots & \ddots & 0 \\ X_{j_ik_i} & 0 & 0 & \dots & \dots & \dots & 0 & -\mathcal{T}_{ni} \end{array}\right], \tag{14}$$

$$\Psi_{j_ik_i} = \begin{bmatrix} \mathcal{H}\left(A_{j_i}X_{1i} + (A_{j_i}-A_{r_i})X_{2i}\right) + 2\mathcal{F}_{j_ik_i} & (*) & (*) & (*) & (*) & (*) & (*) \\ X_{2i}A_{j_i}^T + X_{3i}(A_{j_i}-A_{r_i})^T + A_rX_{2i} & \mathcal{H}\left(A_rX_{3i}\right) & (*) & (*) & (*) & (*) & (*) \\ C_{j_i}X_{1i} + (C_{j_i}-C_{r_i})X_{2i} + K_{k_i}^TB_{j_i}^T & C_{j_i}X_{2i}^T + (C_{j_i}-C_{r_i})X_{3i} & -\mathcal{H}\left(H_{j_ik_i}\right) & (*) & (*) & (*) & (*) \\ B_{j_i}^{wT} & 0 & 0 & -\gamma_i^2 I & (*) & (*) & (*) \\ 0 & B_{r_i}^T & 0 & 0 & -\gamma_i^2 I & (*) & (*) \\ 0 & 0 & B_{h_i}^{\nu T} & 0 & 0 & -\gamma_i^2 I & (*) \\ 0 & 0 & H_{j_ik_i} & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\mathcal{F}_{j_ik_i} = \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} \tilde{F}_{j_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{k_i}^{\alpha T}, \quad X_{j_ik_i} = \begin{bmatrix} X_{1_i} & (*) & 0 \\ X_{2_i} & X_{3_i} & 0 \\ 0 & 0 & H_{j_ik_i} \end{bmatrix}.$$

Proof. Consider the following extended quadratic Lyapunov function candidate:

$$V(x_1,\dots,x_n) = \sum_{i=1}^{n} \tilde{x}_i^T E_i X_{h_ih_i}^{-1}\tilde{x}_i, \tag{15}$$

$\forall (x_1,\dots,x_n) \neq 0$, $V(x_1,\dots,x_n) > 0$ if

$$E_iX_{h_ih_i}^{-1} = X_{h_ih_i}^{-T}E_i > 0 \Leftrightarrow X_{h_ih_i}^TE_i = E_iX_{h_ih_i} > 0. \tag{16}$$

The inequality (16) holds if (13) holds and

$$X_{h_ih_i} = \begin{bmatrix} X_{1_i} & (*) & 0 \\ X_{2_i} & X_{3_i} & 0 \\ 0 & 0 & H_{h_ih_i} \end{bmatrix}.$$

Note that, according to (16), the third row of $X_{h_ih_i}$ may contain free decision variables. However, to further obtain LMI conditions, the blocs (3,1) and (3,2) of $X_{h_ih_i}$ are set to zero and the Lyapunov matrices X_{1i}, X_{2i} and X_{3i} are chosen constant to avoid the occurrence of the time derivatives of the membership functions in the stability conditions. By considering (15), the closed-loop dynamics (6) are globally asymptotically stable (without external disturbances) and satisfy the H_∞ criteria (7) if, $\forall \tilde{x}_i(t) \neq 0$,

$$\dot{V}(x_1,\dots,x_n) + \sum_{i=1}^{n}\left(\tilde{x}_i^T\Omega_i\tilde{x}_i - \gamma_i^2\tilde{w}_i^T\tilde{w}_i\right) = \sum_{i=1}^{n}\left(2\tilde{x}_i^TE_iX_{h_ih_i}^{-1}\dot{\tilde{x}}_i + \tilde{x}_i^T\Omega_i\tilde{x}_i - \gamma_i^2\tilde{w}_i^T\tilde{w}_i\right) < 0. \tag{17}$$

That is to say, from (16) and (6),

$$\sum_{i=1}^{n}\Big(2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{G}_{h_ih_i\overline{h_ih_i}}\tilde{x}_i + 2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{B}_{h_i}^{\tilde{w}}\tilde{w}_i + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{F}_{h_i}^{\alpha}\tilde{x}_\alpha + \tilde{x}_i^T\Omega_i\tilde{x}_i - \gamma_i^2\tilde{w}_i^T\tilde{w}_i\Big) < 0. \tag{18}$$

By applying Lemma 1 to the terms $2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{F}_{h_i}^{\alpha}\tilde{x}_\alpha$, (18) is satisfied, for any matrices $\mathcal{T}_{i\alpha} = \mathcal{T}_{i\alpha}^T > 0$, if

$$\sum_{i=1}^{n}\Big(2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{G}_{h_ih_i\overline{h_ih_i}}\tilde{x}_i + 2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{B}_{h_i}^{\tilde{w}}\tilde{w}_i + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{F}_{h_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{h_i}^{\alpha T}X_{h_ih_i}^{-1}\tilde{x}_i + \tilde{x}_\alpha^T\mathcal{T}_{i\alpha}^{-1}\tilde{x}_\alpha\right) + \tilde{x}_i^T\Omega_i\tilde{x}_i - \gamma_i^2\tilde{w}_i^T\tilde{w}_i\Big) < 0. \tag{19}$$

Note that

$$\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{x}_\alpha^T\mathcal{T}_{i\alpha}^{-1}\tilde{x}_\alpha = \sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{x}_i^T\mathcal{T}_{\alpha i}^{-1}\tilde{x}_i.$$

Hence, the inequality (19) can be rewritten as

$$\sum_{i=1}^{n}\Big(\tilde{x}_i^T\Big(\mathcal{H}\left(X_{h_ih_i}^{-T}\tilde{G}_{h_ih_i\overline{h_ih_i}}\right) + \Omega_i$$

$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(X_{h_ih_i}^{-T}\tilde{F}_{h_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{h_i}^{\alpha T}X_{h_ih_i}^{-1}+\mathcal{T}_{\alpha i}^{-1}\right)\Big)\tilde{x}_i$$
$$+2\tilde{x}_i^TX_{h_ih_i}^{-T}\tilde{B}_{h_i}^{\tilde{w}}\tilde{w}_i-\gamma_i^2\tilde{w}_i^T\tilde{w}_i\Big)<0, \tag{20}$$

or equivalently,

$$\sum_{i=1}^{n}\begin{bmatrix}\tilde{x}_i\\ \tilde{w}_i\end{bmatrix}^T\begin{bmatrix}\Theta_{h_ih_i\overline{h_ih_i}} & (*)\\ \tilde{B}_{h_i}^{\tilde{w}T}X_{h_ih_i}^{-1} & -\gamma_i^2I\end{bmatrix}\begin{bmatrix}\tilde{x}_i\\ \tilde{w}_i\end{bmatrix}<0 \tag{21}$$

with

$$\Theta_{h_ih_i\overline{h_ih_i}}=\Big(\mathcal{H}\left(X_{h_ih_i}^{-T}\tilde{G}_{h_ih_i\overline{h_ih_i}}\right)+\Omega_i$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(X_{h_ih_i}^{-T}\tilde{F}_{h_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{h_i}^{\alpha T}X_{h_ih_i}^{-1}+\mathcal{T}_{\alpha i}^{-1}\right)\Big).$$

The inequality (21) holds, for all $\tilde{x}_i\neq 0$ and $\tilde{w}_i\neq 0$, if

$$\forall i=1,\dots,n:\begin{bmatrix}\Theta_{h_ih_i\overline{h_ih_i}} & (*)\\ \tilde{B}_{h_i}^{\tilde{w}T}X_{h_ih_i}^{-1} & -\gamma_i^2I\end{bmatrix}<0. \tag{22}$$

Pre- and post-multiplying the inequalities (22) respectively by $\begin{bmatrix}X_{h_ih_i}^T & 0\\ 0 & I\end{bmatrix}$ and its transpose yields

$$\forall i=1,\dots,n:\begin{bmatrix}\Phi_{h_ih_i\overline{h_ih_i}} & (*)\\ \tilde{B}_{h_i}^{\tilde{w}T} & -\gamma_i^2I\end{bmatrix}<0 \tag{23}$$

with

$$\Phi_{h_ih_i\overline{h_ih_i}}=\Big(\mathcal{H}\left(\tilde{G}_{h_ih_i\overline{h_ih_i}}X_{h_ih_i}\right)+X_{h_ih_i}^T\Omega_iX_{h_ih_i}$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tilde{F}_{h_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{h_i}^{\alpha T}+X_{h_ih_i}^T\mathcal{T}_{\alpha i}^{-1}X_{h_ih_i}\right)\Big).$$

Finally, to deal with the terms $X_{h_i}^T\Omega_iX_{h_i}$ and $X_{h_i}^T\mathcal{T}_{\alpha i}^{-1}X_{h_i}$, we apply the Schur complement. After matrices expansions and the application of Lemma 2, the inequality (23) is satisfied if there exists a solution to the LMI conditions expressed in Theorem 1. ∎

Let us recall that the conditions of Theorem 1 are only sufficient. Hence there is still space for conservatism reduction. To relax the proposed conditions, Peaucelle's LMI transformations given by Lemma 3 are considered to introduce additional slack decision variables. The obtained relaxed conditions are summarized in the following theorem.

Theorem 2. *For $i=1,\dots,n$, consider the interconnected and disturbed TS systems* (1) *and their respective reference models* (2)*. Under the decentralized static output tracking non-PDC control law* (3)*, the state and output tracking errors converge asymptotically to zero (without external disturbances) and satisfy the H_∞ criteria* (7)*, if there exist matrices $\mathcal{T}_{i\alpha}=\mathcal{T}_{i\alpha}^T>0$, $X_{1i}=X_{1i}^T$, X_{2i}, $X_{3i}=X_{3i}^T$, K_{k_i} and $H_{j_ik_i}$ such that, minimizing γ_i^2, the LMIs* (9), (10) *and* (13) *are satisfied with $\Gamma_{j_ij_i}$ defined in* (24).

Proof. Follow the same mathematical developments in Theorem 1 until Eqn. (23). Applying Lemma 3, (23) is satisfied if there exist $R_{h_ih_i}$ and $L_{h_ih_i}$ such that

$$\begin{bmatrix}\mathcal{H}(\tilde{A}_{h_i}L_{h_ih_i})+Q_{h_ih_i\overline{h_ih_i}} & (*) & (*)\\ X_{h_ih_i}-L_{h_ih_i}+R_{h_ih_i}^T\tilde{A}_{h_i}^T & -\mathcal{H}(R_{h_ih_i}) & 0\\ \tilde{B}_{h_i}^{\tilde{w}T} & 0 & -\gamma_i^2I\end{bmatrix}<0 \tag{25}$$

with

$$Q_{h_ih_i\overline{h_ih_i}}=\Big(\mathcal{H}\left(\tilde{B}_{h_i}\tilde{K}_{\overline{h_ih_i}}X_{h_ih_i}\right)+X_{h_ih_i}^T\Omega_iX_{h_ih_i}$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tilde{F}_{h_i}^{\alpha}\mathcal{T}_{i\alpha}\tilde{F}_{h_i}^{\alpha T}+X_{h_ih_i}^T\mathcal{T}_{\alpha i}^{-1}X_{h_ih_i}\right)\Big)<0.$$

Assume that

$$L_{h_ih_i}=\begin{bmatrix}L_{h_i}^1 & L_{h_i}^2 & L_{h_i}^3\\ L_{h_i}^4 & L_{h_i}^5 & L_{h_i}^6\\ L_{h_ih_i}^7 & L_{h_ih_i}^8 & L_{h_ih_i}^9\end{bmatrix}$$

and

$$R_{h_ih_i}=\begin{bmatrix}R_{h_i}^1 & R_{h_i}^2 & R_{h_i}^3\\ R_{h_i}^4 & R_{h_i}^5 & R_{h_i}^6\\ R_{h_ih_i}^7 & R_{h_ih_i}^8 & R_{h_ih_i}^9\end{bmatrix}.$$

To deal with the terms $X_{h_i}^T\Omega_iX_{h_i}$ and $X_{h_i}^T\mathcal{T}_{\alpha i}^{-1}X_{h_i}$, we apply the Schur complement. After matrix expansions and the application of Lemma 2, the inequality (25) is satisfied if there exists a solution to the LMI conditions expressed in Theorem 2. ∎

4. Simulation examples

In this section, three simulation examples illustrate the effectiveness and the performances of the proposed decentralized tracking controllers. The first example is dedicated to comparison of the conservatism of the proposed conditions with previous results (Wang and Tong, 2006; Liu *et al.*, 2014), which are suitable for interconnected systems having the same order. The second example aims at comparing the conservatism of the proposed conditions with previous results available for interconnected TS systems with different orders (Wang and Tong, 2006). Note that, in these first two examples, measurement noise is not considered ($\nu_i=0$) for fair comparison with the results of Wang and Tong (2006) or Liu *et al.* (2014). Finally, to show the effectiveness of the proposed results on systems having physical meaning, the last example concerns decentralized output feedback stabilization of two inverted pendulums interconnected

$$
\Gamma_{j_ik_i}=\left[\begin{array}{ccc|c}
\begin{matrix}\ominus^1_{j_ik_i} & (*) & (*)\\ \ominus^2_{j_ik_i} & -\mathcal{H}(R_{h_ih_i}) & 0\\ \ominus^3_{j_ik_i} & 0 & \ominus^4_{j_ik_i}\end{matrix} & & & (*)\\
\hline
\begin{matrix} X_{j_ik_i} & 0 & \dots & \dots & \dots & 0\\ \vdots & \vdots & \ddots & & & \vdots\\ \vdots & \vdots & & 0 & & \vdots\\ \vdots & \vdots & & 0 & & \vdots\\ \vdots & \vdots & & & \ddots & \vdots\\ X_{j_ik_i} & 0 & \dots & \dots & \dots & 0\end{matrix} & & & \begin{matrix} -\mathcal{T}_{1i} & 0 & \dots & \dots & \dots & 0\\ 0 & \ddots & \ddots & & & \vdots\\ \vdots & \ddots & -\mathcal{T}_{i-1\,i} & \ddots & & \vdots\\ \vdots & & \ddots & -\mathcal{T}_{i+1\,i} & \ddots & \vdots\\ \vdots & & & \ddots & \ddots & 0\\ 0 & \dots & \dots & \dots & 0 & -\mathcal{T}_{ni}\end{matrix}
\end{array}\right], \tag{24}
$$

$$
\ominus^1_{j_ik_i}=\begin{bmatrix}
\mathcal{H}\left(A_{j_i}L^1_{k_i}+(A_{j_i}-A_{r_i})L^4_{k_i}\right)+2\mathcal{F}_{j_ik_i} & (*) & (*)\\
L^{2T}_{k_i}A^T_{j_i}+L^{5T}_{k_i}(A_{j_i}-A_{r_i})^T+A_rL^4_{k_i} & \mathcal{H}\left(A_rL^5_{k_i}\right) & (*)\\
\begin{pmatrix}C_{j_i}L^1_{k_i}+(C_{j_i}-C_{r_i})L^4_{k_i}-L^7_{j_ik_i}\\ +L^{3T}_{k_i}A^T_{j_i}+L^{6T}_{k_i}(A_{j_i}-A_{r_i})^T+K^T_{k_i}B^T_{j_i}\end{pmatrix} & \begin{pmatrix}C_{j_i}L^2_{k_i}+(C_{j_i}-C_{r_i})L^5_{k_i}\\ -L^8_{j_ik_i}+L^{6T}_{k_i}A^T_r\end{pmatrix} & -\mathcal{H}\begin{pmatrix}C_{j_i}L^3_{k_i}-L^9_{j_ik_i}\\ +(C_{j_i}-C_{r_i})L^6_{k_i}\end{pmatrix}
\end{bmatrix},
$$

$$
\ominus^2_{j_ik_i}=\begin{bmatrix}
X_{1_i}-L^1_{h_i}+R^{1T}_{k_i}A^T_{j_i}+R^{4T}_{k_i}(A_{j_i}-A_{r_i})^T & X^T_{2_i}-L^2_{h_i}+R^{4T}_{k_i}A^T_{r_i} & \begin{pmatrix}-L^3_{h_i}+R^{1T}_{k_i}C^T_{j_i}\\ +R^{4T}_{k_i}(C_{j_i}-C_{r_i})^T-R^{7T}_{j_ik_i}\end{pmatrix}\\
X_{2_i}-L^4_{h_i}+R^{2T}_{k_i}A^T_{j_i}+R^{5T}_{k_i}(A_{j_i}-A_{r_i})^T & X_{3_i}-L^5_{h_i}+R^{5T}_{k_i}A^T_{r_i} & \begin{pmatrix}-L^6_{h_i}+R^{2T}_{k_i}C^T_{j_i}\\ +R^{5T}_{k_i}(C_{j_i}-C_{r_i})^T-R^{8T}_{j_ik_i}\end{pmatrix}\\
-L^7_{h_ih_i}+R^{3T}_{k_i}A^T_{j_i}+R^{6T}_{k_i}(A_{j_i}-A_{r_i})^T & -L^8_{h_ih_i}+R^{6T}_{k_i}A^T_{r_i} & \begin{pmatrix}H_{j_ik_i}-L^9_{h_ih_i}+R^{3T}_{k_i}C^T_{j_i}\\ +R^{6T}_{k_i}(C_{j_i}-C_{r_i})^T-R^{9T}_{j_ik_i}\end{pmatrix}
\end{bmatrix},
$$

$$
\ominus^3_{j_ik_i}=\begin{bmatrix} B^{wT}_{j_i} & 0 & 0\\ 0 & B^T_{r_i} & 0\\ 0 & 0 & B^{\nu T}_{h_i}\\ 0 & 0 & H_{j_ik_i}\end{bmatrix},\quad
\ominus^4_{j_ik_i}=\begin{bmatrix} -\gamma_i^2 I & 0 & 0 & 0\\ 0 & -\gamma_i^2 I & 0 & 0\\ 0 & 0 & -\gamma_i^2 I & 0\\ 0 & 0 & 0 & -I\end{bmatrix}.
$$

by a spring.

4.1. Numerical example with interconnected systems having the same order.

Consider a set of 3 interconnected TS systems S_1, S_2 and S_3, each having 2 fuzzy rules and given by

- *Subsystem* S_1:

$$A_{1_1}=\begin{bmatrix}-11 & 0.1\\ 1 & -12\end{bmatrix},\quad A_{2_1}=\begin{bmatrix}b & 0.2\\ 1 & -10\end{bmatrix},$$

$$B_{1_1}=\begin{bmatrix}0.2\\ 1.2\end{bmatrix},\quad B_{2_1}=\begin{bmatrix}0.1\\ 1.2\end{bmatrix},$$

$$B^w_{1_1}=\begin{bmatrix}-0.1\\ 0.1\end{bmatrix},\quad B^w_{2_1}=\begin{bmatrix}0.1\\ -0.1\end{bmatrix},$$

$$F^2_{1_1}=\begin{bmatrix}1 & 0.01\\ 0.1 & 1\end{bmatrix},$$

$$F^3_{1_1}=F^3_{2_1}=\begin{bmatrix}0 & 0\\ -0.5b+0.7a & 0\end{bmatrix},$$

$$F^2_{2_1}=\begin{bmatrix}1 & 0.01\\ 0.3 & 1\end{bmatrix},$$

$$C_{1_1}=\begin{bmatrix}4 & 0\end{bmatrix},\quad C_{2_1}=\begin{bmatrix}2.65 & 0\end{bmatrix};$$

- *Subsystem* S_2:

$$A_{1_2}=\begin{bmatrix}-10.1 & 0.5\\ 0 & -13\end{bmatrix},\quad A_{2_2}=\begin{bmatrix}5a-6b & 0.5\\ 0 & -2b+2.8a\end{bmatrix},$$

$$B_{1_2}=\begin{bmatrix}0.3\\ 0.1\end{bmatrix},\quad B_{2_2}=\begin{bmatrix}2\\ 0.1\end{bmatrix},$$

$$B^w_{1_2}=\begin{bmatrix}0.2\\ -1.2\end{bmatrix},\quad B^w_{2_2}=\begin{bmatrix}0.3\\ 0.45\end{bmatrix},$$

$$F^1_{1_2}=\begin{bmatrix}0.5 & 0\\ 0.01 & 0\end{bmatrix},\quad F^3_{1_2}=\begin{bmatrix}0.5 & -0.01\\ 0.2 & 0.3\end{bmatrix},$$

$$F^1_{2_2}=\begin{bmatrix}1 & 0.1\\ 0.8a-0.96b & 0\end{bmatrix},\quad F^3_{2_2}=\begin{bmatrix}0 & -0.01\\ 0.3 & 0.3\end{bmatrix},$$

$$C_{1_2}=\begin{bmatrix}2 & 0.1\end{bmatrix},\quad C_{2_2}=\begin{bmatrix}0.6 & 0\end{bmatrix};$$

• *Subsystem* S_3:

$$A_{1_3} = \begin{bmatrix} a & 0.2 \\ 0.1 & -20 \end{bmatrix}, \quad A_{2_3} = \begin{bmatrix} -21 & 0.5 \\ 0.5 & a \end{bmatrix},$$

$$B_{1_3} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad B_{2_3} = \begin{bmatrix} 0.8 \\ 1.5 \end{bmatrix},$$

$$B^w_{1_3} = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, \quad B^w_{2_3} = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix},$$

$$F^1_{1_3} = \begin{bmatrix} 0 & -0.01 \\ 0 & 0 \end{bmatrix}, \quad F^2_{1_3} = \begin{bmatrix} 0 & 0.6 \\ -0.01 & 0.3 \end{bmatrix},$$

$$F^1_{2_3} = \begin{bmatrix} 0 & 0.02 \\ 0 & 0 \end{bmatrix}, \quad F^2_{2_3} = \begin{bmatrix} 0 & 0.6 \\ -0.01 & 0.7 \end{bmatrix},$$

$$C_{1_3} = \begin{bmatrix} 3 & 0.1 \end{bmatrix}, \quad C_{2_1} = \begin{bmatrix} 3.45 & 0.2 \end{bmatrix},$$

where a and b are two scalar parameters dedicated to checking the feasibility fields of the LMI-based conditions.

To deal with output-feedback tracking control, consider the following reference models to specify the desired trajectories of each subsystem S_i $(i = 1, 2, 3)$:

• *reference model for Subsystem* S_1:

$$A_{r_1} = \begin{bmatrix} -30.1 & 0 \\ 0 & -32.1 \end{bmatrix}, \quad B_{r1} = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix}, \quad C_{r1} = \begin{bmatrix} 1 & -3 \end{bmatrix};$$

• *reference model for Subsystem* S_2:

$$A_{r_2} = \begin{bmatrix} -33 & 0 \\ 0 & -30.1 \end{bmatrix}, \quad B_{r2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad C_{r2} = \begin{bmatrix} 1 & 1 \end{bmatrix};$$

• *reference model for Subsystem* S_3:

$$A_{r_3} = \begin{bmatrix} -32.1 & 0 \\ 0.1 & -33 \end{bmatrix}, \quad B_{r3} = \begin{bmatrix} 2 \\ 0.3 \end{bmatrix}, \quad C_{r3} = \begin{bmatrix} -3 & 1 \end{bmatrix}.$$

Figure 2 shows the feasibility fields obtained from Theorems 1 and 2, as well as the ones obtained from a corollary by Wang and Tong (2006) and Theorem 7 by Liu *et al.* (2014). To do so, the feasibility of the LMI conditions considered were checked for $a = \begin{bmatrix} -15 & 15 \end{bmatrix}$ and $b = \begin{bmatrix} -5 & 5 \end{bmatrix}$ with unit step. For this example, over 341 points were tested for each LMI conditions considered, the solutions obtained from the corollary of Wang and Tong (2006) (33 solutions, 9.6%) are included in those obtained by taking account of both Theorem 7 by Liu *et al.* (2014) (114 solutions, 33.34%) and Theorem 1 (148 solutions, 43%). Note that, for some points (11 in total), the proposed conditions failed to provide solutions when Theorem 7 by Liu *et al.* (2014) succeeded. However, over the whole tested area, Theorem 2 produced 322 solutions (94%), i.e., over twice all the previous conditions, which indicates that the proposed results allow us to significantly enhance conservatism reduction when no solution exists in relation to previous ones.

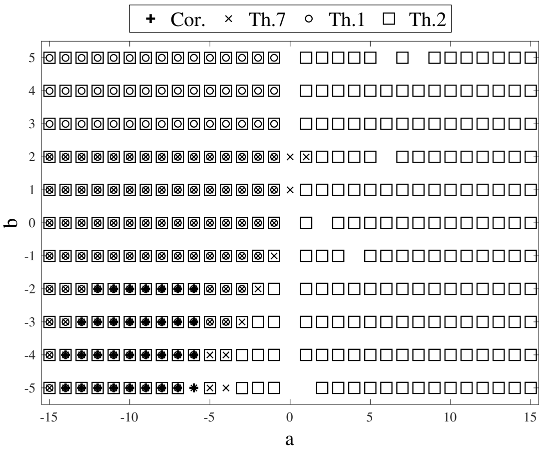

Fig. 2. Feasibility domains (Section 4).

4.2. Numerical example with interconnected systems having different orders. In the previous example, we considered three TS subsystems with the same dimension (second-order systems) and the same number of rules (two for each subsystems). However, the problem is further compounded when the orders of subsystems η_i and/or when the numbers of their respective fuzzy rules m_i are different. To test this case, consider a set of 2 interconnected TS systems S_1 $(\eta_1 = 2, m_1 = 4)$ and S_2 $(\eta_2 = 4, m_2 = 2)$ given by

• *Subsystem* S_1:

$$A_{1_1} = \begin{bmatrix} -11 & 0.1 \\ 0.08a - 0.04b & -12 \end{bmatrix}, \quad A_{3_1} = \begin{bmatrix} -11 & 0.1 \\ 0.1 & -10 \end{bmatrix},$$

$$A_{2_1} = \begin{bmatrix} b & 0.2 \\ 0.04a - 0.04b & -10 \end{bmatrix}, \quad A_{4_1} = \begin{bmatrix} b & 0.2 \\ 0.1 & -12 \end{bmatrix},$$

$$B_{1_1} = B_{2_1} = \begin{bmatrix} 0.2 \\ 1.2 \end{bmatrix}, \quad B_{3_1} = B_{4_1} = \begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix},$$

$$B^w_{1_1} = B^w_{2_1} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \quad B^w_{3_1} = B^w_{4_1} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},$$

$$F^2_{1_1} = F^2_{2_1} = \begin{bmatrix} 1 & 0.01 & 0 & 0 \\ 0.1 & 1 & -0.5b + 0.7a & 0 \end{bmatrix},$$

$$F^2_{3_1} = F^2_{4_1} = \begin{bmatrix} 1 & 0.01 & 0 & 0 \\ 0.3 & 1 & -0.5b + 0.7a & 0 \end{bmatrix},$$

$$C_{1_1} = C_{2_1} = \begin{bmatrix} 4 & 0 \end{bmatrix}, \quad C_{3_1} = C_{4_1} = \begin{bmatrix} 2.65 & 0 \end{bmatrix};$$

• *Subsystem* S_2:

$$A_{1_2} = \begin{bmatrix} -10.1 & 0.5 & 0.5 & -0.01 \\ 0 & -13 & 0.2 & 0.3 \\ 0 & 0.6 & a & 0.2 \\ -0.01 & 0.3 & 0.1 & -20 \end{bmatrix},$$

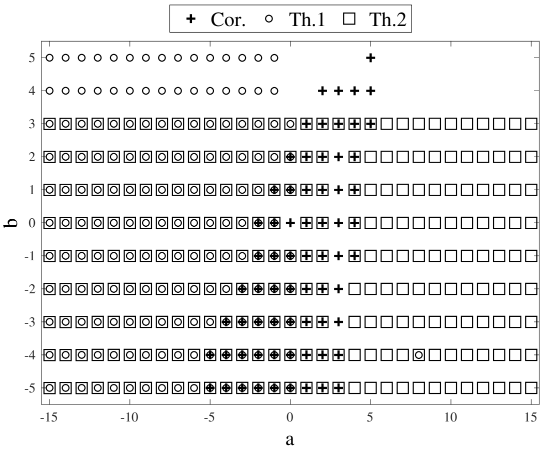

Fig. 3. Feasibility domains (Section 4.2).

$$A_{2_2} = \begin{bmatrix} 4b-7a & 0.5 & 0 & -0.01 \\ 0 & -b+1.4a & 0.3 & 0.3 \\ 0 & 0.6 & -21 & 0.5 \\ -0.01 & 0.7 & 0.5 & a \end{bmatrix},$$

$$B_{1_2} = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0 \\ 0 & 1 \\ 0 & 0.5 \end{bmatrix}, \quad B_{2_2} = \begin{bmatrix} 2 & 0 \\ 0.1 & 0 \\ 0 & 0.8 \\ 0 & 1.5 \end{bmatrix},$$

$$B^w_{1_2} = \begin{bmatrix} 0.2 & 0 \\ -1.2 & 0 \\ 0 & 0.1 \\ 0 & 0.4 \end{bmatrix}, \quad B^w_{2_2} = \begin{bmatrix} 0.3 & 0 \\ 0.45 & 0 \\ 0 & -0.1 \\ 0 & 0.5 \end{bmatrix},$$

$$F^1_{1_2} = \begin{bmatrix} 0.5 & 0 \\ 0.01 & 0 \\ 0 & -0.01 \\ 0 & 0 \end{bmatrix}, \quad F^2_{1_2} = \begin{bmatrix} 1 & 0.1 \\ 0.8-0.96b & 0 \\ 0 & 0.02 \\ 0 & 0 \end{bmatrix},$$

$$C_{1_2} = \begin{bmatrix} 2 & 0.1 & 0 & 0 \\ 0 & 0 & 3 & 0.1 \end{bmatrix}, \quad C_{2_2} = \begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0 & 3.45 & 0.2 \end{bmatrix},$$

where a and b are two scalar parameters dedicated to check the feasibility fields of the LMI-based conditions. To deal with their output tracking control, consider the following reference model to specify the desired trajectories of each subsystem:

- *reference model for Subsystem S_1*:

$$A_{r_1} = \begin{bmatrix} -30.1 & 0 \\ 0 & -32.1 \end{bmatrix}, \quad B_{r1} = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix}, \quad C_{r1} = \begin{bmatrix} 1 & -3 \end{bmatrix};$$

- *reference model for Subsystem S_2*:

$$A_{r_2} = \begin{bmatrix} -33 & 0.2 & 0.1 & -0.33 \\ 0 & -30.1 & 0 & 0 \\ 0 & 0 & -32.1 & 0 \\ -0.33 & 0.2 & 0.1 & -33 \end{bmatrix},$$

$$B_{r2} = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0 \\ 0 & 2 \\ 0 & 0.3 \end{bmatrix}, \quad C_r = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}.$$

Note that the conditions proposed by Liu *et al.* (2014) do not apply in this case since they are only available when the order of the interconnected TS subsystems is the same. Thus, Fig. 3 shows the feasibility fields obtained from Theorems 1 and 2, as well as the ones obtained from corollary by Wang and Tong (2006) with the same (a, b) area as in the previous example. For this example, over 341 points tested for each LMI condition considered, the results obtained from the corollary by Wang and Tong (2006) provide 68 solutions (i.e., 20% of the tested points), the ones from Theorem 1 provide 174 solutions (51%), mostly included in the ones of Theorem 2, which provide 272 solutions (79.8%). Once more, note that for few points (12 in total) the proposed conditions failed to provide solutions when the corollary by Wang and Tong (2006) succeeded. However, over the whole tested area, Theorem 2 provides a bigger feasibility field (more than 1.56 times larger than the ones obtained from the previous results), which indicates again that the proposed results allows us to enhance conservatism reduction when no solution exists in relation on previous ones.

4.3. Interconnected inverted pendulums. Consider two inverted pendulums connected by a spring depicted in Fig. 4. Each pendulum S_i $(i = 1, 2)$ is driven by an input torque u_i.

Assuming that both the pendulums consist of a point mass m_i affixed to the end of a massless rigid body rod, the i-th pendulum dynamics equation is given by

$$J_i \ddot{\theta}_i = m_i g l_i \sin\theta_i + k a^2(\theta_i - \theta_\alpha) - d_i \dot{\theta}_i + u_i + w_i, \quad (26)$$

where $\alpha = 1, 2$, $\alpha \neq i$, θ_i are the pendulums angular positions [rad] with respect to the erect position, w_i are external disturbances applied to each pendulum, with the parameters given in Table 1.

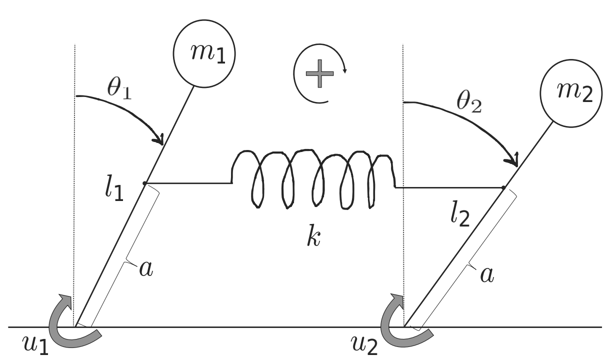

Fig. 4. Inverted pendulums connected by a spring.

Denote by $x_i = \begin{bmatrix}\theta_i & \dot{\theta}_i\end{bmatrix}^T$ the state vector of each pendulum, and assume that only the position θ_i is measured with noise ν_i, the dynamical model (26) can be rewritten as

$$\begin{cases} \dot{x}_i = \begin{bmatrix} 0 & 1 \\ \dfrac{m_i g l_i}{J_i}\frac{\sin\theta_i}{\theta_i} + \dfrac{ka^2}{J_i} & -\dfrac{d_i}{J_i} \end{bmatrix} x_i \\ \quad + \begin{bmatrix} 0 \\ \dfrac{1}{J_i} \end{bmatrix} u_i + \begin{bmatrix} 0 \\ \dfrac{1}{J_i} \end{bmatrix} w_i \\ \quad + \begin{bmatrix} 0 & 0 \\ -\dfrac{ka^2}{J_i} & 0 \end{bmatrix} x_\alpha, \\ y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} x_i + \nu_i. \end{cases} \tag{27}$$

Note that each subsystem contains one bounded nonlinear term $\mathrm{sin}_c\,\theta_i = \sin(\theta_i)/\theta_i \in [-\rho, 1]$, with $\rho \approx 0.217$, only depending on the outputs $y_i = \theta_i$. Thus, applying the sector nonlinear approach (Tanaka and Ohtake, 2001), the whole system can be represented as two interconnected TS subsystems (1), each having two rules, with

$$A_{1_i} = \begin{bmatrix} 0 & 1 \\ \dfrac{-m_i g l_i \rho + ka^2}{J_i} & -\dfrac{d_i}{J_i} \end{bmatrix},$$

$$A_{2_i} = \begin{bmatrix} 0 & 1 \\ \dfrac{m_i g l_i + ka^2}{J_i} & -\dfrac{d_i}{J_i} \end{bmatrix},$$

$$B_{1_i} = B_{2_i} = B^w_{1_i} = B^w_{2_i} = \begin{bmatrix} 0 \\ 1 \\ \overline{J_i} \end{bmatrix}, \quad B^\nu_{1_i} = B^\nu_{2_i} = 1,$$

$$F^2_{1_i} = F^2_{2_i} = \begin{bmatrix} 0 & 0 \\ -\dfrac{ka^2}{J_i} & 0 \end{bmatrix}, \quad C_{1_i} = C_{2_i} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and the membership functions

$$h_{1_i}(y_i) = \frac{1 - \mathrm{sin}_c\, y_i}{1-\rho}, \quad h_{2_i}(y_i) = \frac{\mathrm{sin}_c\, y_i - \rho}{1-\rho}.$$

To specify the desired output trajectories of each subsystem, the reference models (2) are considered with

$$A_{r_i} = \begin{bmatrix} -100 & 0 \\ 0 & -100 \end{bmatrix}, \quad B_{r_i} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}, \quad C_{r_i} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

which consist of a first-order-low pass filter described in the frequency domain as $y_i(s)/r_i(s) = 1/(1+\kappa s)$ with the time constant $\kappa = 1/100$ [s] and a unit gain in the filter passband; see the work of Seddiki *et al.* (2010) for other examples detailing how to design reference models in the T–S model-based trajectory tracking framework.

By solving the conditions proposed in Theorem 2 with the MATLAB LMI Toolbox, the decentralized non-PDC controller gains, given in Table 2, are designed. Note that, with the parameters presented in Table 1, Theorem 1 as well as the conditions proposed by Liu *et al.* (2014) as well as Wang and Tong (2006) failed to produce any result. This confirm the interest in the design procedure proposed in Theorem 2.

In the sequel, a closed-loop simulation of the interconnected inverted pendulums is proposed with the initial states $x_1(0) = \begin{bmatrix}0.1 & 0\end{bmatrix}^T$, $x_2(0) = \begin{bmatrix}-0.1 & 0\end{bmatrix}^T$ and $x_{r_1}(0) = x_{r_2}(0) = \begin{bmatrix}0 & 0\end{bmatrix}^T$ and the following reference signals for output tracking:

$$r_1(t) = \begin{cases} 0 & \text{for } 0 \le t < 24 \text{ and} \\ & \quad 96 \le t < 120, \\ -\frac{\pi}{4}\sin\left(\frac{\pi}{5}t\right) & \text{for } 24 \le t < 48, \\ -\frac{\pi}{6}\sin\left(\frac{\pi}{5}t\right) & \text{for } 48 \le t < 72, \\ -\frac{\pi}{4}\sin\left(\frac{\pi}{5}t\right) & \text{for } 72 \le t < 96, \end{cases}$$

$$r_2(t) = \begin{cases} 0 & \text{for } 0 \le t < 20 \text{ and} \\ & \quad 100 \le t < 120, \\ \frac{\pi}{4}\sin\left(\frac{\pi}{3}t\right) & \text{for } 20 \le t < 40, \\ \frac{\pi}{6}\sin\left(\frac{\pi}{9}t\right) & \text{for } 40 \le t < 80, \\ \frac{\pi}{4}\sin\left(\frac{\pi}{3}t\right) & \text{for } 80 \le t < 100. \end{cases}$$

First, a simulation is performed without external disturbances and measurement noise. The results are shown in Figs. 5 and 6. Figure 5 exhibits the output trajectories and the desired references signals while Fig. 6 shows the evolution of the output tracking error and the control signal, for each pendulum.

Then, the robustness of the proposed control scheme against external disturbances and measurement noise is evaluated by applying resistive torques as an external disturbance on each pendulum during short periods and a measurement noise set as a Gaussian white noise with power 0.02. As shown in Figs. 7–8, a disturbance of 10 Nm is applied to Pendulum 1 between 110 s and 114 s. Then, for the second pendulum, a resistive torque of -10 Nm is applied during the time interval running from 60 s to 70 s. In these cases, a week deviation of output trajectories, when the designed controllers attenuate the disturbances, can be observed in Figs. 7 and 8.

Finally, note that the proposed decentralized output tracking control approach drives the subsystems to track their reference signals in the presence of measurement noise. As a matter of fact, when the sensor feedback is noisy, this lead to direct noise propagation into the control signals, which may saturate or chatter the actuators. To reduce such effects in practice, it is advised to apply low-pass filters to prevent noise propagation.

Remark 3. Table 3 summarizes the computational

Table 1. Parameters of interconnected inverted pendulums.

Parameter	Value	Designation
m_1	2.5 kg	Mass of Pendulum 1
m_2	2 kg	Mass of Pendulum 2
J_1	$2.5\,\mathrm{kg\,m^2}$	Inertia of Pendulum 1
J_2	$2\,\mathrm{kg\,m^2}$	Inertia of Pendulum 2
$l_1 = l_2$	1 m	Length of the pendulums
a	0.2 m	Distance from the pendulum to spring hinges
d_1	3.5 Nms/rad	Joint friction coefficient of Pendulum 1
d_2	4.5 Nms/rad	Joint friction coefficient of Pendulum 2
k	$8\,\mathrm{N\,m^{-1}}$	Spring stiffness coefficient
g	$9.81\,\mathrm{m\,s^{-2}}$	Acceleration of the gravity

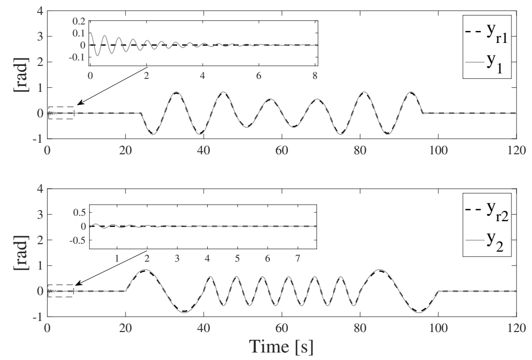

Fig. 5. Output trajectories without external disturbances.

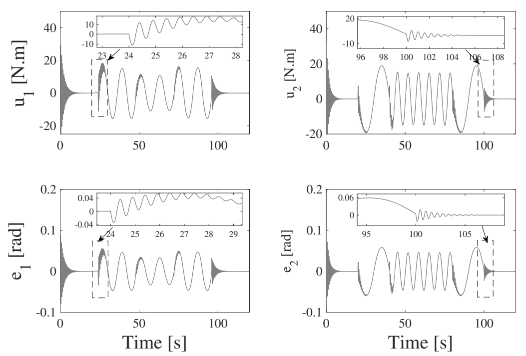

Fig. 6. Output tracking error and the control signal without external disturbances.

Table 2. Controller parameters.

1st Controller		2nd Controller	
Parameter	Value	Parameter	Value
K_{11}	658.08	K_{21}	645.09
K_{12}	650.07	K_{22}	679.07
H_{11}	2	H_{21}	1.99
H_{12}	2	H_{22}	2
γ_1	5.50	γ_2	5.49

complexity of the conditions of Theorems 1 and 2 regarding the related LMI-based results proposed by Wang and Tong (2006) as well as Liu *et al.* (2014). Note that, in the latter studies, PDC controllers are designed with quadratic Lyapunov functions instead of non-PDC ones in our paper, with extended quadratic Lyapunov functions. This explain, on the one hand, the conservatism improvement achieved by Theorems 1 and 2. On the other, note that slack decision variables were introduced to relax the conditions of Theorem 1 and 2 by applying descriptor redundancy as well as Lemma 2. This explains why our results involve more decision variables to provide a greater degree of freedom to convex optimization algorithms, and so conservatism improvement. Of course, the price to pay is an increase in the computational cost but, from our point of view, this does not constitute a big drawback since this computation is done off-line and we continuously observe growing improvements of computer capabilities.

Remark 4. The proposed controller design methodology involves off-line resolution of a set of LMI conditions (Theorems 1 and 2). Such LMI conditions remain convex optimization problems usually solvable with well-known LMI solvers (e.g., the Matlab LMI Toolbox, YALMIP or SEDUMI), to design the controller gains (3). Note that the proposed decentralized controllers provide effective tracking control techniques for interconnected systems that are characterized by their computational efficiency and robustness. Moreover, the given decentralized non-PDC controllers (3) are easy to implement with relatively low computational complexity because they involve simple matrix manipulations. Compared with the neural network approaches proposed by Li and Tong

Table 3. Comparison of the computational complexity with previous LMI-based conditions.

Method	Variable no.	Size of LMIs
Wang and Tong, 2006	11	$(5\eta_i + \upsilon_i)^2$
Liu *et al.*, 2014	75	$(12\eta_i + \mu_i + \upsilon_i)^2$
Theorem 1	18	$(\eta_i^2 + \eta_i + 2\rho_i + \mu_i + \upsilon_i)^2$
Theorem 2	116	$(\eta_i^2 + 3\eta_i + 2\rho_i + 2\mu_i + \upsilon_i)^2$

(2017) or Qu *et al.* (2017), which are significantly harder to run on-line since they require many more computational capacities, this is obviously an advantage of the proposed off-line procedure when the controllers have to be implemented on slow processors or when they are applied to real-time decentralized control of systems with fast dynamics.

5. Conclusion

This paper dealt with the design of decentralized static output tracking controllers for a class of interconnected TS fuzzy systems subject to external bounded disturbances. To obtain LMI-based design conditions, the closed-loop dynamics of the given decentralized static output tracking control plant subject to external disturbances were expressed using a descriptor redundancy formulation, then applying the direct Lyapunov methodology with extended quadratic Lyapunov functions candidates and an H_∞ criterion. The proposed LMI conditions provide advantages in terms of conservatism regarding the related results from the literature, highlighted through numerical examples. Moreover, the effectiveness of the proposed robust decentralized static output tracking controller design was illustrated with a simulation of two inverted pendulums connected by a spring. Further works will focus on output tracking control design for a class of interconnected TS systems involving multiple time-delays, or in the presence of stochastic abrupt structural changes in the system's topology (interconnections).

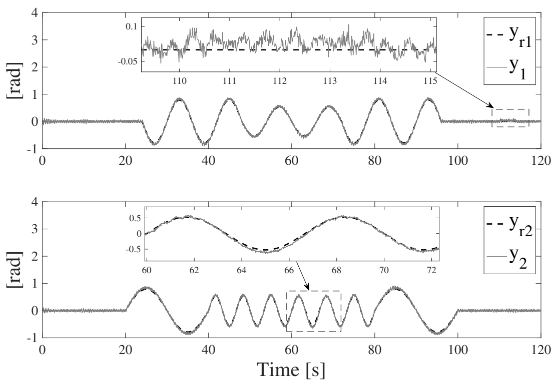

Fig. 7. Output trajectories with external disturbances and measurement noise.

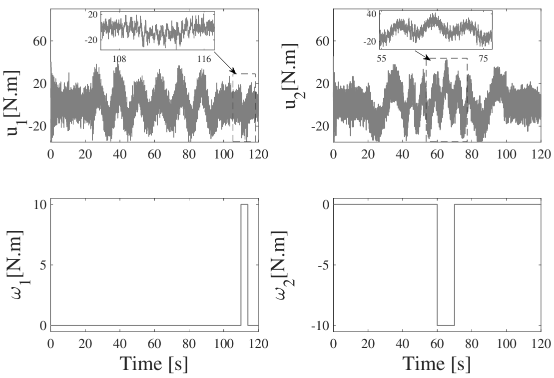

Fig. 8. Control signals and resistive torques (disturbances) with measurement noise.

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