MULTIPLE TIME SCALE AND PARAMETER ESTIMATION FOR STOCHASTIC DISTRIBUTED PARAMETER SYSTEMS

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The main idea of this work is to use combined experience and complementary work to jointly investigate the combination of separation/duality concepts and singular perturbation methods in order to develop new algorithms for the simultaneous estimation of stochastic distributed parameter process state and model parameters when both can experience multiple and different time domains. Besides the development of general theoretical concepts, specific process applications are considered both computationally and experimentally.

1. Introduction

Sophisticated technological processes based on nonstationary diffusion or heat/mass exchange phenomena which are affected by stochastic disturbances are widely found in numerous industries. To control these processes efficiently, one needs to have information concerning the fundamental physical laws and the important model parameters.

However, to obtain complete information about a distributed parameter system (DPS) is rather difficult because of random error in pointwise measurements and other uncertainties which are difficult to take into account. Indeed, some DPS parameter measurements are currently impossible. Thus, the only way to obtain complete process information is to construct a state and parameter estimation. This estimation is usually based on Kalman filtering techniques (Sage *et al.*, 1976).

In some works (for example Azhogin *et al.*, 1986) the state estimation problem is solved under parametric uncertainty conditions based on the

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nonstrict separation principle. According to this principle, one may use parametric identification and the suboptimal Kalman filter algorithms in turn as an iterative procedure. The nonstrict separation principle is based on the structural separability of state estimation and identification problems. However, it does not account for the parametric dependence that exists for these problems. This leads to state and parameter estimation error. Another source of error in these algorithms (Zgurovsky, 1988) is the necessity to approximate the covariance matrix in the suboptimal Kalman filter algorithm. In addition, these algorithms require the process to satisfy stationary conditions which substantially decrease their practical value.

The use of quasi-steady state and perturbation analysis for state variable identification has been investigated, too. The works of Clough (1975) and Ramirez and Clough (1976) showed that classical distributed parameter identification algorithms were not effective in estimating the fast dynamic response of composition and temperature profiles down a fixed- bed catalytic reactor for the production of styrene monomer and the slow coking of the catalyst activity. The styrene system is described by Clough and Ramirez (1976). A new steady-state distributed parameter filter was developed for composition and temperature identification and a dynamic Kalman filter was used to estimate the catalyst activity. The use of these two different time scale filters resulted in an efficient and stable identification scheme.

Gaafar and Ramirez (1985), Ramirez and Gaafar (1985) and Gaafar (1987) showed the usefulness of using perturbation analysis techniques introduced by Kokotovic and colleagues (Sannuti and Kokotovic, 1969, Chow and Kokotovic, 1976 , Kokotovic *et al.*, 1976) for control and identification problems. Gaafar and Ramirez (1985) extended the analysis to multiple time domains and applied it to the fixed-bed catalytic reactor problem.

A large number of physical processes in nature are based on the phenomena of a heat and mass diffusion and convection. The intensity of each mechanism depends upon numerous conditions and these mechanisms can change independently each with a different time scale. Therefore, it is very important to extend the singular perturbation approach for lumped parameter system to distributed parameter systems of the diffusive and convective type.

We perform this extension on the basis of combining the singular perturbation approach with the separation/duality principle for distributed parameter systems.

2. The Development of Joint Singular Perturbation and Separation Duality (SPSD) Approach

In terms of both singular perturbation and separation/duality approaches, the mathematical description of the stochastic diffusion and convection processes is:

$$\frac{\partial X(t,\mathbf{z})}{\partial t} = \mathcal{N}(X,t,\mathbf{z}) - W(t,\mathbf{z}) - \sum_{j=1}^{K} \delta_j(\mathbf{z}) U_j(t), \tag{1}$$

$$\mathcal{N}(X,t,\mathbf{z}) = \sum_{i=1}^{2} \frac{\partial}{\partial z_i} \left[\, b(X) k(t,\mathbf{z}) \tfrac{\partial X}{\partial z_i} \, \right] - \sum_{i=1}^{2} c(t,\mathbf{z}) \frac{\partial X}{\partial z_i} - d(t,\mathbf{z}) X \tag{2}$$

$$\xi_1 \frac{\partial k(t,\mathbf{z})}{\partial t} = f_1[X, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})], \tag{3}$$

$$\xi_2 \frac{\partial c(t,\mathbf{z})}{\partial t} = f_2[X, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})], \tag{4}$$

$$\xi_3 \frac{\partial d(t,\mathbf{z})}{\partial t} = f_3[X, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})]. \tag{5}$$

The space $\sum = \Omega \times [0, t_f]$; $\Omega \in \mathbb{R}^2$ is a two-dimensional spatial domain with boundary $\partial\Omega$. In equations (1)–(7) t is time; $\mathbf{z} = [z_1, z_2]$ is the space coordinate vector; $X(\cdot), W(\cdot)$ and $U_j(\cdot)$ denote the state variable, the stochastic disturbance and the control variable at the point $z^j, U_j(t) = U(t, z^j)$, corresponding; $b(\cdot), c(\cdot)$ and $d(\cdot)$ are parameters of the process; $\mathcal{N}(\cdot)$ is the non-linear operator describing the process. The initial and boundary conditions are given by

$$X(0,\mathbf{z}) = X_0(\mathbf{z}), \tag{6}$$

$$\frac{\partial X(t,\mathbf{z})}{\partial n} = \tau(t,\mathbf{z})[X(t,\mathbf{z}) - X_{ext}(t,\mathbf{z})] + W_b(t,\mathbf{z}), \ \mathbf{z} \in \partial\Omega. \tag{7}$$

We suppose that the diffusion coefficient $b(X)$, the convection coefficient $c(t,z)$ and forcing coefficient $d(t,z)$ change independently of each other in different time scales ξ_1, ξ_2 and ξ_3, respectively.

The restrictions on the model parameters are:

$$b_l \le b(X) \le b_u, \quad k_l \le k(t,z) \le k_u,$$

$$c_l \leq x(t,z) \leq c_u, \quad d_l \leq d(t,z) \leq d_u, \tag{8}$$

$$\tau_l \leq \tau(t,z) \leq \tau_u, \quad -g_l \leq \partial k(t,z)/\partial z \leq g_u.$$

Let's define a cost functional to develop the time recursive state and parameter estimation algorithms:

$$I(\widehat{X},P) = \int_{t_0}^{t_f} \left\{ \int_{\Omega} P(t,\mathbf{z},\mathbf{r}) d\mathbf{z} d\mathbf{r} + \sum_{i=1}^{N} [\widehat{Y}(t,\mathbf{z}^i) - \widehat{X}(t,\mathbf{z}^i)]^2 \right\} dt. \tag{9}$$

The state estimate $X(t,\mathbf{z})$ and differential sensitivity function $P(t,\mathbf{z},\mathbf{r})$ are defined from the Kalman filter equations as:

$$\frac{\partial \widehat{X}(t,\mathbf{z})}{\partial t} = \mathcal{N}(X,t,\mathbf{z})\widehat{X}(t,\mathbf{z}) - \sum_{j=1}^{K} \delta_j(\mathbf{z}) U_j(t) -$$

$$- \sum_{i=1}^{N} (P(t,\mathbf{z},\mathbf{r}^i) H(t,\mathbf{r}^i)/R(t,\mathbf{z}) [Y(t,\mathbf{z}^i) - H(t,\mathbf{z}^i)\widehat{X}(t,\mathbf{z}^i)], \tag{10}$$

$$\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial t} = \sum_{i=1}^{2} \frac{\partial}{\partial z_i} \left[\frac{\partial b(X)}{\partial X} \Big|_{X=\widehat{X}} k(t,z) \frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i} \right] -$$

$$- \sum_{i=1}^{2} c(t,\mathbf{z}) \frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i} - d(t,\mathbf{z}) P(t,\mathbf{z},\mathbf{r}) + \sum_{i=1}^{2} \frac{\partial}{\partial r_i} \left[\frac{\partial b(\widehat{X})}{\partial \widehat{X}} k(t,\mathbf{r}) \frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial r_i} \right] -$$

$$- \sum_{i=1}^{2} c(t,\mathbf{r}) \frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial r_i} - d(t,\mathbf{r}) P(t,\mathbf{z},\mathbf{r}) + \psi(t,\mathbf{z},\mathbf{r}) + \tag{11}$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} [H(t,\mathbf{z}^i)/R(t,\mathbf{z})][Y(t,\mathbf{z}^i) - H(t,\mathbf{z}^i)\widehat{X}(t,\mathbf{z}^i)] P(t,\mathbf{z},\mathbf{r}^i) P(t,\mathbf{z}^j,\mathbf{r}),$$

$$\xi_1 \frac{\partial k(t,\mathbf{z})}{\partial t} = f_1[\widehat{X}, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})], \tag{12}$$

$$\xi_2 \frac{\partial c(t,\mathbf{z})}{\partial t} = f_2[\widehat{X}, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})], \tag{13}$$

$$\xi_3 \frac{\partial d(t,\mathbf{z})}{\partial t} = f_3[\widehat{X}, k(t,\mathbf{z}), c(t,\mathbf{z}), d(t,\mathbf{z})]. \tag{14}$$

The initial and boundary conditions are given by

$$\widehat{X}(0,\mathbf{z}) = \widehat{X}_0(\mathbf{z}), \quad P(0,\mathbf{z},\mathbf{r}) = P_0(\mathbf{z},\mathbf{r}),$$

$$b(\widehat{X})|_{t=0} = \psi_b^0(\mathbf{z}); \quad c(0,z) = \psi_c^0(\mathbf{z}), \quad d(0,z) = \psi_d^0(\mathbf{z}). \tag{15}$$

$$\frac{\partial \widehat{X}(t,\mathbf{z})}{\partial n} = \tau(t,\mathbf{z})[\widehat{X}(t,\mathbf{z}) - X_{ext}(t,\mathbf{z})],$$

$$\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial n} = k(t,\mathbf{z})b(\widehat{X})/R(t,\mathbf{z}) = \tau(t,\mathbf{z})P(t,\mathbf{z},\mathbf{r}), \ \mathbf{z} \in \partial\Omega \tag{16}$$

$$\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial \gamma} = k(t,\mathbf{r})b(\widehat{X})/R(t,\mathbf{r}) = \tau(t,\mathbf{z})P(t,\mathbf{z},\mathbf{r}), \ \mathbf{r} \in \partial\Omega.$$

For the system (1)–(9), we need to define estimates of the state $\widehat{X}^*(t,\mathbf{z})$ and spatially distributed parameters $\widehat{b}^*(X), \widehat{c}^*(t,\mathbf{z}), \widehat{k}^*(t,\mathbf{z}), \widehat{d}^*(t,\mathbf{z}), \widehat{\tau}^*(t,\mathbf{z})$ under the condition that $\widehat{X}(t,\mathbf{z}), \widehat{b}(X), \widehat{c}(t,\mathbf{z}), \widehat{d}(t,\mathbf{z})$ change in different time scales, for which the inequality $I(\widehat{X}^*, \widehat{c}^*, \widehat{k}^*, \widehat{c}^*, \widehat{d}^*, \widehat{\tau}^*) < I(\widehat{X}, \widehat{\phi}, \widehat{b}, \widehat{k}, \widehat{c}, \widehat{d}, \widehat{\tau})$ should be valid for all admissible values of states and parameters.

The solution of the above problem is sought on the basis of combining the singular perturbation technique and duality method of joint filtering.

For transforming the above minimization problem under the restrictions (10), (11) to an unconditional minimization problem, we introduce the Lagrange function:

$$L\{\cdot\} = I(t,X,P) + \int_{t_0}^{t_f} \int_{\Omega} \left\{ \left[\frac{\partial \widehat{X}(t,\mathbf{z})}{\partial t} - \mathcal{N}(X,t,\mathbf{z})\widehat{X}(t,\mathbf{z}) + \right.\right.$$

$$+ \sum_{j=1}^{K} \delta_j(\mathbf{z}) U_j(t) + \sum_{i=1}^{N} P(t,\mathbf{z},\mathbf{z}^i) H(t,\mathbf{z}^i)/R(t,\mathbf{z})[\widehat{Y}(t,\mathbf{z}^i) -$$

$$-H(t,\mathbf{z}^i)\widehat{X}(t,\mathbf{z}^i)]\Big]\,\phi(t,\mathbf{z})+\left[\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial t}-\right.$$

$$-\sum_{i=1}^{2}\frac{\partial}{\partial z_i}\left[\frac{\partial b(\widehat{X})}{\partial\widehat{X}}k(t,\mathbf{z})\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i}\right]+\sum_{i=1}^{2}c(t,\mathbf{z})\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i}-$$

$$-d(t,\mathbf{z})P(t,\mathbf{z},\mathbf{r})-\sum_{i=1}^{2}\frac{\partial}{\partial r_i}\left[\frac{\partial b(\widehat{X})}{\partial\widehat{X}}k(t,\mathbf{z})\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial r_i}\right]+$$

$$+\sum_{i+1}^{2}c(t,\mathbf{r})\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial r_i}+d(t,\mathbf{r})P(t,\mathbf{z},\mathbf{r})-\sum_{i=1}^{N}\sum_{j=1}^{N}\Big\{H(t,\mathbf{z}^i)/R(t,\mathbf{z})[Y(t,\mathbf{z}^i)-$$

$$-H(t,\mathbf{z}^i)\widehat{X}(t,\mathbf{z}^i)]P(t,\mathbf{z},\mathbf{r}^i)P(t,\mathbf{z}^j,\mathbf{r})]\Big\}\,Q[(t,\mathbf{z},\mathbf{r})+$$

$$+\left[\xi_1\frac{\partial k(t,\mathbf{z})}{\partial t}-f_1[\widehat{X},b(\widehat{X}),k(t,\mathbf{z}),c(t,\mathbf{z}),d(t,\mathbf{z})]\right]F_1(t,\mathbf{z})+$$

$$+\left[\xi_2\frac{\partial c(t,\mathbf{z})}{\partial t}-f_2[\widehat{X},b(\widehat{X}),k(t,\mathbf{z}),c(t,\mathbf{z}),d(t,\mathbf{z})]\right]F_2(t,\mathbf{z})+$$

$$+\left[\xi_3\frac{\partial d(t,\mathbf{z})}{\partial t}-f_3[\widehat{X},b(\widehat{X}),k(t,\mathbf{z}),c(t,\mathbf{z}),d(t,\mathbf{z})]\right]F_3\,(t,\mathbf{z})\}\,d\mathbf{z}d\mathbf{r}dt,$$

where $\phi(t,\mathbf{z}),Q(t,\mathbf{r},\mathbf{z}),F_1(t,\mathbf{z}),F_2(t,\mathbf{z}),F_3(t,\mathbf{z})$ are conjugate functions, which are defined by the co-state equations:

$$\frac{\partial\phi(t,\mathbf{z})}{\partial t}=\sum_{i=1}^{2}\frac{\partial}{\partial z_i}\left[b(\widehat{X})k(t,\mathbf{z})\frac{\partial\phi(t,\mathbf{z})}{\partial z_i}\right]+c(t,\mathbf{z})\sum_{i=1}^{2}\frac{\partial\phi(t,\mathbf{z})}{\partial z_i}-$$

$$-d(t,\mathbf{z})\phi(t,\mathbf{z})+2\sum_{i=1}^{N}[\widehat{Y}_i(t)-\widehat{X}(t,\mathbf{z})]\delta(\mathbf{z}-\mathbf{z}^i)- \qquad (17)$$

$$-\sum_{i=1}^{N} P(t,\mathbf{z},\mathbf{z}^i)H(t,\mathbf{z})\phi(t,\mathbf{z})\delta(\mathbf{z}-\mathbf{z}^i)-$$

$$-2\int_{\Omega_r}\sum_{i=1}^{N}\sum_{j=1}^{N}[H(t,\mathbf{z})/R(t,\mathbf{z})]\times[H(t,\mathbf{z})k(t,\mathbf{z},\mathbf{r})\delta(\mathbf{z}-\mathbf{z}^i)]\times$$

$$\times P((t,\mathbf{z},\mathbf{r}^i)P(t,\mathbf{z}^j,\mathbf{r}))d\mathbf{r},$$

$$\frac{\partial Q(t,\mathbf{z},\mathbf{r})}{\partial t}=\sum_{i=1}^{2}\frac{\partial}{\partial z_i}\left[\frac{\partial b(\widehat{X})}{\partial\widehat{X}}k(t,\mathbf{z})\frac{\partial Q(t,\mathbf{z},\mathbf{r})}{\partial z_i}\right]+$$

$$+c(t,\mathbf{z})\sum_{i=1}^{2}\frac{\partial Q(t,\mathbf{z},\mathbf{r})}{\partial z_i}-d(t,\mathbf{z})Q(t,\mathbf{z},\mathbf{r})+\sum_{i=1}^{2}\frac{\partial}{\partial r_i}\left[\frac{\partial b(\widehat{X})}{\partial\widehat{X}}\times\right.$$

$$\left.\times\, k(t,\mathbf{z})\frac{\partial Q(t,\mathbf{z},\mathbf{r})}{\partial r_i}\right]+c(t,\mathbf{z})\sum_{i=1}^{2}\frac{\partial Q(t,\mathbf{z},\mathbf{r})}{\partial r_i}-d(t,\mathbf{r})Q(t,\mathbf{z},\mathbf{r})+ \qquad (18)$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N}[H(t,\mathbf{z}^i)/R(t,\mathbf{z})][\widehat{Y}_i(t)-H(t,\mathbf{z})\widehat{X}(t,\mathbf{z})\delta(\mathbf{z}-\mathbf{z}^i)]P(t,\mathbf{z},\mathbf{r}^i)\times$$

$$\times[Q(t,\mathbf{z},\mathbf{r})\delta(\mathbf{z}-\mathbf{z}^j)]-\sum_{i=1}^{N}[H(t,\mathbf{z}^i)/R(t,\mathbf{z})][\widehat{Y}_i(t)-$$

$$-H(t,\mathbf{z})\widehat{X}(t,\mathbf{z})\delta(\mathbf{z}-\mathbf{z}^i)]\phi(t,\mathbf{z})\delta(\mathbf{z}-\mathbf{z}^i)-I,$$

and the co-state equations which define conjugate functions $F_1(t,\mathbf{z}), F_2(t,\mathbf{z})$, $F_3(t,\mathbf{z})$ can be derived from the following expressions if a special form of $f_1(\cdot), f_2(\cdot), f_3(\cdot)$ is known

$$\int_{t_Q}^{t_k}\int_{\Omega}\left\{-\xi_1\frac{\partial F_1}{\partial t}\delta k\right\}d\mathbf{z}d\mathbf{r}dt=\int_{t_Q}^{t_k}\int_{\Omega}\left\{f_1(\cdot)+\delta f_1(\cdot)\right\}F_1 d\mathbf{z}d\mathbf{r}dt- \tag{19}$$

$$-\int_{t_Q}^{t_k}\int_{\Omega}f_1(\cdot)F_1 d\mathbf{z}d\mathbf{r}dt,$$

$$\int_{t_Q}^{t_k}\int_{\Omega}\left\{-\xi_2\frac{\partial F_2}{\partial t}\delta c\right\}d\mathbf{z}d\mathbf{r}dt=\int_{t_Q}^{t_k}\int_{\Omega}\left\{f_2(\cdot)+\delta f_2(\cdot)\right\}F_2 d\mathbf{z}d\mathbf{r}dt- \tag{20}$$

$$-\int_{t_Q}^{t_k}\int_{\Omega}f_2(\cdot)F_2 d\mathbf{z}d\mathbf{r}dt,$$

$$\int_{t_Q}^{t_k}\int_{\Omega}\left\{-\xi_3\frac{\partial F_3}{\partial t}\delta d\right\}d\mathbf{z}d\mathbf{r}dt=\int_{t_Q}^{t_k}\int_{\Omega}\left\{f_3(\cdot)+\delta f_3(\cdot)\right\}F_3 d\mathbf{z}d\mathbf{r}dt- \tag{21}$$

$$-\int_{t_Q}^{t_k}\int_{\Omega}f_3(\cdot)F_3 d\mathbf{z}d\mathbf{r}dt.$$

The final conditions for these equations are

$$\phi(t_f,\mathbf{z})=Q(t_f,\mathbf{z})=F_1(t_f,\mathbf{z})=F_2(t_f,\mathbf{z})=F_3(t_f,\mathbf{z})=0.$$

The necessary conditions for implementation of the state and parameter estimation algorithms may be given by

$$\frac{\partial L(\cdot)}{\partial\widehat{b}(\widehat{X})}=k(t,\mathbf{z})\sum_{i=1}^{2}\frac{\partial\widehat{X}(t,\mathbf{z})}{\partial z_i}\frac{\partial\phi(t,\mathbf{z})}{\partial z_i}+\sum_{j=1}^{3}\frac{1}{\xi_j}\frac{\partial f_j[\cdot]}{\partial\widehat{b}(\widehat{X})}F_j(t,\mathbf{z}), \tag{22}$$

$$\frac{\partial L(\cdot)}{\partial\widehat{k}(t,\mathbf{z})}=b(\widehat{X})\sum_{i=1}^{2}\frac{\partial\widehat{X}(t,\mathbf{z})}{\partial z_i}\frac{\partial\phi(t,\mathbf{z})}{\partial z_i}+\int_{\Omega_r}\frac{\partial b(\widehat{X})}{\partial\widehat{X}}\sum_{i=1}^{2}\frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i}\times \tag{23}$$

$$\times\frac{Q(t,\mathbf{z},\mathbf{r})}{\partial z_i}d\mathbf{r}+\sum_{j=1}^{3}\frac{1}{\xi_j}\frac{\partial f_j[\cdot]}{\partial\widehat{k}(t,\mathbf{z})}F_j(t,\mathbf{z}),$$

$$\frac{\partial L(\cdot)}{\partial c(t,\mathbf{z})} = \sum_{i=1}^{2} \frac{\partial \widehat{X}(t,\mathbf{z})}{\partial z_i}\phi(t,\mathbf{z}) + \int_{\Omega_r} \sum_{i=1}^{2} \frac{\partial P(t,\mathbf{z},\mathbf{r})}{\partial z_i} Q(t,\mathbf{z},\mathbf{r})d\mathbf{r} + \sum_{j=1}^{3} \frac{1}{\xi_j} \frac{\partial f_j[\cdot]}{\partial \widehat{c}(t,\mathbf{z})} F_j(t,\mathbf{z}), \tag{24}$$

$$\frac{\partial L(\cdot)}{\partial \widehat{d}(t,\mathbf{z})} = \widehat{X}(t,\mathbf{z})\phi(t,\mathbf{z}) + \int_{\Omega_r} P(t,\mathbf{z},\mathbf{r})Q(t,\mathbf{z},\mathbf{r})d\mathbf{r} + \sum_{j=1}^{3} \frac{1}{\xi_j} \frac{\partial f_j[\cdot]}{\partial \widehat{d}(t,\mathbf{z})} F_j(t,\mathbf{z}), \tag{25}$$

$$\frac{\partial L(\cdot)}{\partial \widehat{r}(t,\mathbf{z})} = -[\widehat{X}(t,\mathbf{z}) - X_{ext}(t,\mathbf{z})]\psi(t,\mathbf{z}) - \int_{\partial\Omega_r} P(t,\mathbf{z},\mathbf{r})Q(t,\mathbf{z},\mathbf{r})d\mathbf{z}. \tag{26}$$

For implementation of the state and parameter estimation algorithm, we follow (Zgurovsky, 1988). It is first necessary to define the time scales ξ_1, ξ_2 and ξ_3 for the parameters $b(X), c(t,\mathbf{z}), d(t,\mathbf{z})$ and for the state X. Then to order these time scales, we solve the algorithm for the fast time domain first, and for the slow time domain next.

Remark. A prior determination of the time scales is a special case of this procedure.

For defining the time scales post priori, we can use the second derivatives as an estimate of the state and parameter rate of change

$$\left|\frac{\partial^2 L(\cdot)}{\partial \widehat{k}^2(t,\mathbf{z})}\right| = \left|\sum_{j=1}^{3} \frac{\partial^2 f_j[\cdot]}{\partial \widehat{k}^2(t,\mathbf{z})} F_j(t,\mathbf{z})\right|,$$

$$\left|\frac{\partial^2 L(\cdot)}{\partial \widehat{c}^2(t,\mathbf{z})}\right| = \left|\sum_{j=1}^{3} \frac{\partial^2 f_j[\cdot]}{\partial \widehat{c}^2(t,\mathbf{z})} F_j(t,\mathbf{z})\right|, \tag{27}$$

$$\left|\frac{\partial^2 L(\cdot)}{\partial \widehat{d}^2(t,\mathbf{z})}\right| = \left|\sum_{j=1}^{3} \frac{\partial^2 f_j[\cdot]}{\partial \widehat{d}^2(t,\mathbf{z})} F_j(t,\mathbf{z})\right|.$$

We can then order the time scales and perform the state and parameter estimation procedure as described in (Zgurovsky, 1988) for the fast time domain first, and for the slow time domain last.

3. Examples of multiple time scale state and parameter identification

3.1. Nuclear reaction with iodine–xenone poisoning (Yemelianov, 1976)

$$\frac{1}{V}\frac{\partial\phi(t,\mathbf{z})}{\partial t} = D(\mathbf{z})\frac{\partial^2\phi(t,\mathbf{z})}{\partial z_1^2} + D(\mathbf{z})\frac{\partial^2\phi(t,\mathbf{z})}{\partial z_2^2} + [KF(\mathbf{z}) - \sum(\mathbf{z}) -$$

$$- \gamma X_e(t,\mathbf{z})]\phi(t,\mathbf{z}) - U(t,\mathbf{z})\phi(t,\mathbf{z}) + W\phi(t,\mathbf{z}), \tag{28}$$

$$\xi_1\frac{\partial X_e(t,\mathbf{z})}{\partial t} = Y_{Xe}F(\mathbf{z})\phi(t,\mathbf{z}) + \lambda_I I(t,\mathbf{z}) - \lambda_{Xe}X_e(t,\mathbf{z}) - \delta X_e(t,\mathbf{z})\phi(t,\mathbf{z}), \tag{29}$$

$$\xi_2\frac{\partial I(t,\mathbf{z})}{\partial t} = Y_I F(\mathbf{z})\phi(t,\mathbf{z}) - \lambda_I I(t,\mathbf{z}). \tag{30}$$

The initial and boundary conditions are given by

$$\phi(0,\mathbf{z}) = \phi_0(\mathbf{z}), \quad I(0,\mathbf{z}) = I_0(\mathbf{z}), \quad X_e(0,\mathbf{z}) = X_{e0}(\mathbf{z}),$$

$$\frac{\partial\phi(t,\mathbf{z})}{\partial\mathbf{z}} = \phi_1(t,\mathbf{z}), \ \frac{\partial I(t,\mathbf{z})}{\partial\mathbf{z}} = \phi_2(t,\mathbf{z}), \ \frac{\partial X_e(t,\mathbf{z})}{\partial\mathbf{z}} = \phi_3(t,\mathbf{z}), \ \mathbf{z}\in\partial\Omega. \tag{31}$$

Where $\phi(t,\mathbf{z})$ is the density of neutron flow in the reactor, $F(t,\mathbf{z})$ is the cross section of the uranium nucleus; $\sum(\mathbf{z})$ is absorbtion cross section; $U(t,\mathbf{z})$ is additional absorbtion (control action); $I(t,\mathbf{z})$ is iodine concentration; $X_e(t,\mathbf{z})$ is xenon concentration; γ is the constant of neutron absorbtion by xenon; Y_I, Y_{X_e} are the constants of separation for iodine and xenon accordingly; λ_I, λ_{X_e} are the constants of β–disintegration iodine and xenon accordingly; $\phi_1(t,\mathbf{z})$, $\phi_2(t,\mathbf{z})$, $\phi_3(t,\mathbf{z})$ are the functions which determine $\phi(t,\mathbf{z})$, $I(t,\mathbf{z})$, $X_e(t,\mathbf{z})$ on boundary $\partial\Omega$ accordingly; ξ_1, ξ_2 are time scales of $I(t,\mathbf{z})$ and $X_e(t,\mathbf{z})$ changing accordingly.

The problem is to estimate the $\phi(t,\mathbf{z}), I(t,\mathbf{z})$ and $X_e(t,\mathbf{z})$, which change in different time scales.

3.2. The Process of Water Flooding for an Oil Reservoir (Chavent, Cohen, 1977)

$$M(z)\frac{\partial s(t,z)}{\partial t} - \frac{\partial}{\partial z}\left[k(s)\alpha(s)\frac{\partial s(t,z)}{\partial z}\right] - V_1(t)\frac{\partial}{\partial z}[b(s)] = 0, \qquad (32)$$

$$\xi_1 \frac{\partial \alpha(s)}{\partial t} = f_1[\alpha(s), b(s)], \qquad (33)$$

$$\xi_2 \frac{\partial b(s)}{\partial t} = f_2[\alpha(s), C(s)]. \qquad (34)$$

The initial and boundary conditions are described by

$$s(t,z)\,|_{t=0} = s_0(z), \quad s(t,z)\,|_{z=0} = s_M(t), \qquad (35)$$

$$s(t,z)\Big|_{z=z_f} = s_m(t) + [s_M(t) - s_m(t)]\,Y(t-t_0). \qquad (36)$$

Where $s(t,z)$ is water saturation of the porous media; $V(t)$ is velocity of water flow; $k(s)$ is the permeability of the porous media; $\alpha(s)$ is the saturation function which defines the intensity of the diffusion process; $b(s)$ is the saturation function which defines the intensity of the conviction process; $s_m(t)$, $s_M(t)$ are the minimum and maximum values of saturation, respectively; $Y(t)$ is a step function at time instant $t = t_0$; ξ_1 and ξ_2 are time scales of $\alpha(s)$ and $b(s)$ changing accordingly.

The problem is to estimate the state $s(t,z)$ and parameters $\alpha(s)$, $b(s)$, which change in different time scales.

3.3. Styrene Reactor (Gaafar, 1987)

Due to multiple time scales of the styrene reactor's concentration wave and thermal wave and the catalyst deactivation, SPSD approach for this plant is preferable.

The mathematical representation of the dynamic behaviour of the distributed parameter styrene reactor is (Gaafar, 1987):

$$\gamma V\frac{\partial c_i}{\partial t} + \frac{\partial c_i}{\partial z} = -\alpha r_i, \quad i = 1,2,...,10 \qquad (37)$$

$$((1-\gamma)\rho_c C_{p_c} + \gamma\rho C_p)\frac{\partial T_k}{\partial t} = -\rho C_p q\frac{\partial T_k}{\partial z} - \sum_{j=1}^{N} R_j \Delta H_j; \quad k = 1,2,...,6 \qquad (38)$$

$$\frac{\partial \alpha}{\partial t} = \frac{\beta(SOR)e^{-t/\tau 1} - \alpha}{\tau_2}. \tag{39}$$

Note that the concentrations c_i change fast (per 2 sec), the temperature of reactor T_k changes moderately (per 20 min) and the activity α changes slowly (per 30 min for steam regeneration (steam) and per l month for coking).

Defining

$$\Theta\tau = t, \quad \xi_1 = ((1-\gamma)\rho_c C_{P_c} + \gamma \rho C_p)/\tau, \quad \xi_2 = \gamma V/\tau,$$

where τ is the time scale for reactor, the system (18) can be rewritten in terms of SPSD approach:

$$\frac{\partial \alpha}{\partial \Theta} = \frac{\beta(SOR)e^{-\Theta/\tau_1} - \alpha}{\tau_2},$$

$$\xi_1 = \frac{\partial T_k}{\partial \Theta} = -\rho C_p q \frac{\partial T_k}{\partial z} - \sum_{j=1}^{N} R_j \Delta H_j, \quad k = 1,2,...,6 \tag{40}$$

$$\xi_2 \frac{\partial c_i}{\partial \Theta} = -\frac{\partial c_i}{\partial z} - \alpha \tau_i, \quad i = 1,2,...,10$$

Thus, the problem is to estimate the catalyst activity parameter α, and the states of concentrations $C_i(t,z)$ and temperature $T_k(t,z)$ each of which change in different time scales.

3.4. Atmosphere Pollution Processes

The atmospheric pollution processes take place due to both diffusion and air convection. The intensity of each mechanism depends upon numerous conditions and can be changed independently with different time scales. Suppose that dangerous components $q(t,\mathbf{z}), \mathbf{z} = (z_1, z_2, z_3)$ are spread over the region Ω with the boundary surface $\partial\Omega$ consisting of the cross sectional surface of a cylinder, its bottom $\partial\Omega_0$ at $z_3 = 0$ and the top $\partial\Omega_h$ at $z_3 = h$, where h is a height of the cylinder. The spread of pollution over the atmosphere may be described by the stochastic partial differential equation having the form

$$\frac{\partial q(t,\mathbf{z})}{\partial t} + (\mathbf{u}, \text{grad } q(t,\mathbf{z})) - \sum_{i=1}^{3} \frac{\partial}{\partial z_i}\left(\gamma_i(t,\mathbf{z})\frac{\partial q(t,\mathbf{z})}{\partial z_i}\right) +$$

$$+\alpha q(t,\mathbf{z}) = f(t,\mathbf{z}) + w(t,\mathbf{z}), \quad \mathbf{z} \in \Omega, \quad t \in (0,t_f),$$

$$\xi_1 = \frac{\partial_\gamma(t,\mathbf{z})}{\partial t} = f_1[u,\gamma(t,\mathbf{z})], \tag{41}$$

$$\xi_2 = \frac{\partial u}{\partial t} = f_2[u,\gamma(t,\mathbf{z})],$$

with boundary conditions

$$q(t,\mathbf{z}) = 0, \quad \mathbf{z} \in \partial\Omega, \quad \frac{\partial q(t,\mathbf{z})}{\partial z_3} = \alpha_b q(t,\mathbf{z}), \tag{42}$$

$$\mathbf{z} \in \partial\Omega_0, \quad t \in (0,t_f), \quad \frac{\partial q(t,z)}{\partial z_3} = 0, \quad \mathbf{z} \in \partial\Omega,$$

and initial conditions

$$q(t,\mathbf{z})\,|_{t=0} = q_0(\mathbf{z}). \tag{43}$$

For point-wise sources of pollution (of chimney type), the function $f(t,\mathbf{z})$ may be presented in the form

$$f(t,\mathbf{z}) = \sum_{i=1}^{M} u_i(t,\mathbf{z})\delta(\mathbf{z}-\mathbf{z}^i) = \sum_{i=1}^{M} u_i(t), \tag{44}$$

where $u_i(t)$ is the intensity of the *i-th* source of emission; M is a total number of pollution sources; $\gamma(t,\mathbf{z})$ is the vector of diffusion coefficients; $\mathbf{u}$ is the vector of flow velocities; α is the coefficient of species absorbtion by the atmosphere; α_b is the coefficient of species interaction with the surface $\partial\Omega_h$ at $z_3 = 0$; $w(t,\mathbf{z})$ is random disturbance, which describes the stochastic nature of meteorological conditions.

In the simplest case, one may reduce the concentration pollution forecast problem to the simulation problem under the condition that all model parameters $\mathbf{u}, \gamma, \alpha, \alpha_b$ and $u_i(t), q_0(\mathbf{z})$ are known. Since those parameters depend on weather conditions, their exact description cannot be obtained on the basis of a priori data only, especially for the diffusion and convection coefficients. Thus, the problem is to estimate the state $q(t,\mathbf{z})$ and parameters $\mathbf{u}$, $\gamma(t,\mathbf{z})$, which change in different time scales.

4. Conclusions

Two main approaches to estimation of both state variables and model parameters of stochastic distributed parameters systems are considered. The first approach which is based upon the singular perturbation theory was developed and carefully investigated for multiple time scale state estimation of lumped parameter stochastic systems. The investigations showed that the singular perturbed algorithms are computationally more efficient than the complete Kalman estimator algorithms. The singular perturbation technique increases in usefulness as the stiffness, the number of stiff equations increases, or the length of time the filter is allowed to proceed increases. The second approach is based on the separation/duality theory. It was developed for state and parameter estimation of distributed parameter stochastic systems.

The singular perturbation and separation/duality approaches are combined to develop a new joint method for multiple time scale state and parameter estimation for lumped/distributed parameter systems. Four examples of possible applications of joint method SPSD were presented. The concept was developed only theoretically. It is therefore very important to continue experimental and computational investigations of the SPSD-method and use it for practical applications.

References

Azhogin, V.V., Zgurovsky, M.Z., and Novikov A.N., (1986): *Parametric identification of filtration type spatially distributed stochastic processes.* – Automatica, No.5, pp.14–22 (in Russian).

Chow, J.H. and Kokotovic, P.V., (1976): *A Decomposition of near optimum regulators with slow and fast models.*– IEEE Trans.Automat.Control, v.AC–21, p.701

Clough, D.E., (1975): *Optimization and Control of the Dehydrogenation of Ethylbenzene to Form Styrene.*– Ph.D.Dissertation, University of Colorado, Boulder.

Clough, D.E. and Ramirez, W.F., (1976): *Mathematical modeling and optimization of the dehydrogenation of ethylbenzene to form styrene.*– AIChE Journal, v.22, No.4, p.1097.

Chavent, G. and Cohen, C., (1977): *Numerical approximation and identification in 1-D parabolic degenerated non-linear diffusion and transport equations.*– Lecture Notes in Control and Inf. Sci., v.2, pp.79–92.

Gaafar, W.S. and Ramirez, W.F., (1985): *Multiple time scale state estimation.*– I.&EC Process Design and Dev, v.24, pp.1055 –1062.

Gaafar, W.S., (1987): *Optimal Estimation Algorithms for Systems with Multiple Time Modes and Delayed Measurements.*– Ph.D.Dissertation, University of Colorado, Boulder.

Kokotovic, P.V., O'Malley, R.B., Jr. and Sannuti, P., (1976): *Singular perturbation and order reduction in control theory.– An Overview.*– Automatica, No.12, p.123.

Ramirez, W.F. and Clough, D.E., (1976): *On-line estimation and identification of a non-linear distributed parameter process.*– Textes de Communications Analyze de Systems et ses Orientations Nouvelles, IRIA, p.91.

Ramirez, W.F. and Gaafar, W.S., (1985): *Distributed parameter filtering and estimation with dynamic measurements and multiple time domains.*– Proceedings of 24th Conference on DECISION AND CONTROL, Fort Lauderdale, FL (December).

Sage, A. and Melsa, I., (1976): *Estimation Theory: Control and Communication Applications.*– Moscow: Communication (in Russian).

Sannuti, O. and Kokotovic, P.V., (1969): *Near optimum design of linear systems by a singular perturbation method.*– IEEE Trans. Automat. Control, v.AC-14, No.1, p.15.

Yemelianov, N.Ga., Havrilov, N.A. and Selivestrov, B.N., (1976): *Control and Security Nuclear Energy Reactors.*– Moscow: Atomizdat (in Russian).

Zgurovsky, M.Z., (1988): *Adaptive identification of stochastic distributed parameter systems based on separation principles.*– Cybernetics and Computer Science J., v.73, pp.72–79 (in Russian).

Zgurovsky, M.Z. and Novikov, A.N., (1988): *State and parameter estimation of stochastic distributed parameter systems based on separation and duality principles.*– Avtomatica, No.4, pp.17–25 (in Russian).