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MEAN-DEVIATIONAL VERSUS INCREMENTAL MODELS IN ADAPTIVE CONTROL

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Mean-deviational and incremental formulations of the familiar adaptive control algorithms, incorporating the recursive LS estimator coupled with different optimal single-step controllers, are extensively examined when controlling various complex SISO plants governed by the ARMAX model. In the conclusive recommendations based on a number of simulation runs, we emphasize that the mean-deviational controllers can surpass the incremental ones with respect to robustness which is the most important requirement for adaptive controllers.

1. Introduction

It is necessary for the satisfactory performance of a closed-loop control system that both servo and regulatory behaviours of the closed loop are of good quality. As regards the servo behaviour, it is known that the control methods like MV, EHAC1 and EPSAC can provide error-free tracking of the constant setpoint even in their non-incremental formulations provided that the mean $E(e(t))$ of the noise $e(t)$ is zero and $T(1) = 1$, where T is the observer polynomial. On the other hand, the methods like GMV, EHAC2 and GPC suffer from having non-zero steady-state error in that case. However, it can be shown that selecting proper $T(1)$ removes the steady-state error in those methods as well. Anyway, it is in general not necessary to provide the proper servo behaviour of process controllers e.g. to insert the integration action into the loop or to handle input-output signals in a special way.

In order to fulfil the error-free regulatory behaviour requirement in the case of non-zero-mean noise it is necessary either 1) to use a more accurate (i.e. complex) model for the noise effects or 2) to introduce the integration action into the loop, or 3) to employ the incremental algorithms, or 4) to use the mean-deviational formulation.

It is known that the first method may eventually lead to numerical problems, even in the simplest case of the "offset" models (Latawiec and Chyra, 1983). Although the above methods 2 and 3 are different, in general, they lead to similar results, involving integration in a control law. The mean-deviational algorithms are "close" to the incremental ones but still they avoid the explicit introduction of the integration action into the control loop.

Now, there are three fundamental model formulations used in practical implementations of adaptive control algorithms, namely "offset-deviational", "incremental" and "mean-deviational" (Latawiec and Chyra, 1983). As regards the two latter ones that

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are the most applicable and attracted considerable interest, the incremental version is much more popular. At least three possible reasons for such vogue can be pointed out. First, the way of implementation of the incremental algorithm is straightforward, which is not quite the case for the mean-deviational version. Second, the incremental formulation directly involves the occurrence of the integration action in the control law, thus fulfilling the fundamental performance requirement of the steady-state error-free regulation. Third, initial applications of (incremental) adaptive control algorithms have originated from high control accuracy specifications rather than robustness requirements that have been raised later. The first self-tuners employing the minimum variance (MV) control law and later provided with robustness-oriented measures are one spectacular example. The mean-deviational algorithms are known to be more robust, at the cost of slower regulation, in comparsion with the incremental ones.

The incremental models have been extensively applied in a number of practically oriented projects on adaptive control as well as in commercially available adaptive controllers (Åström and Wittenmark, 1989; Åström and Hägglund, 1990; Clarke and Mohtadi, 1989; Krämer and Unbehauen, 1991). In this paper, an attempt is made to review mean-deviational and incremental formulations and re-evaluate their usefulness in robust adaptive control schemes.

Adaptive controllers considered here are a combination of the standard adaptive LS estimator and either the generalized minimum variance (GMV) controller (Clarke and Gawthrop, 1979) or one of the extended horizon adaptive controllers (EHAC) (Latawiec, 1991). Having introduced the mean-deviational and incremental models we outline the control algorithms and review our simulation experiments. In the comparison of the two formulations we tackle many various aspects. Whilst not claiming the clear superiority of any model in specific applications, we indicate that the mean-deviational formulation might have been underestimated, especially in the robust control environment.

2. Models and Parameter Estimation

Consider a discrete-time SISO plant governed by the general model

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + e(k) + \frac{C(q^{-1})}{D(q^{-1})}e(t) \tag{1}$$

where $u(t)$, $y(t)$ are the input and the output, respectively, at time t, $d \geq 1$ is the time delay, A, B, C and D are the polynomials in backward shift operator of order na, nb, nc and nd, respectively, and $e(t)$ is uncorrelated noise of not necessarily zero mean.

It is well-known that the polynomial C is in practice unlikely to be effectively estimated, so it is used rather as a design, observer polynomial $T(q^{-1})$ instead of $(C = T)$. Likewise, the polynomial D, often set as equal to $\Delta = 1 - q^{-1}$, is usually used to make the controller act in an incremental way.

2.1. The Incremental Formulation

Let $C = T$ and $D = 1 - q^{-1}$. Now, eqn. (1) can be rewritten as

$$A'(q^{-1})y(t) = q^{-d}B(q^{-1})\Delta u(t) + T(q^{-1})e(t) \tag{2}$$

with

$$A'(q^{-1}) = (1 - q^{-1})A(q^{-1}), \quad \Delta u(t) = u(t) - u(t-1) \tag{3}$$

which will be referred to as the incremental (INC) model.

Note that $u(t)$ and $y(t)$ can be taken either as the absolute (or full-value) input and output signals $U(t)$ and $Y(t)$, respectively, or as the deviations of the absolute input/output signals from their fixed operating point's reference values U_0, Y_{ref}.

2.2. The Mean-Deviational Formulation

Let $C = T$, $D = 1$. Now, eqn. (1) can be rewritten as

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + T(q^{-1})e(t) \tag{4}$$

with

$$u(t) = U(t) - U_m(t), \quad y(t) = Y(t) - Y_{\text{ref}} \tag{5}$$

where $U_m(t)$ is the mean value of $U(t)$ and Y_{ref} is the absolute output reference value. Equations (4) and (5) will be referred to as the mean-deviational (MED) model.

The main problem in the implementation of the MED model is the computation of the mean $U_m(t)$. The simplest way to update continuously the mean $U_m(t)$ is to use the first-order low-pass filter

$$U_m(t) = \alpha U_m(t-1) + (1-\alpha)U(t) \tag{6}$$

with α normally close to 1, but switched to a lower value in the case of detection of load or setpoint changes. Note that α can be tuned independently of the estimator's forgetting factor, thus providing the separation of handling the rapid load-type or setpoint changes from that of slow process variations.

2.3. Parameter Estimator

The unknown parameters a'_j or a_j and b_j of the above models are estimated by the familiar recursive least-squares procedure, provided with the UD factorization option, the exponential forgetting adaptation mechanism and blow-up countermeasures. In the sequel, we use the p, P and ρ denotations for the parameter vector, the covariance and the forgetting factor, respectively.

3. Optimum Control Rules

The single-step GMV and EHAC control laws are examined here, not only for their simplicity but also for a variety of process dynamics that various versions of EHAC can effectively control.

3.1. GMV Control

The GMV control of the linear system desribed by one of the two model equations quoted is optimal in the sense of the performance index (Clarke and Gawthrop, 1979)

$$\min_{u(t)} E\{[y(k+d) - y_{\mathrm{ref}}]^2 + \beta u^2(t)\} \tag{7}$$

where y_{ref} is the reference output and $\beta \geq 0$ is the control weighting constant.

The GMV control law is given as

$$u(t) = \frac{y_{\mathrm{ref}} - H(q^{-1})y(t)}{B(q^{-1})F(q^{-1}) + \frac{\beta}{b_0}} \tag{8}$$

where the polynomials F and H of order $d-1$ and $na-1$, respectively, are determined from the polynomial identity

$$T(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-d}H(q^{-1}) \tag{9}$$

3.2. EHAC Methods

In EHAC1, the objective is to design the control $u(t)$ so as to minimize $E\{y(t+k) - y_{\mathrm{ref}}\}$ under the assumption $u(t) = u(t+1) = \dots = u(t+k-d)$ and one of the two model equation constraints, where the prediction horizon $k > d$.

Introduce the polynomial identity

$$T(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-k}H(q^{-1}) \tag{10}$$

where the polynomials F and H of order $k-1$ and $na-1$, respectively, can be determined given T, A and $k \geq d$ ($k = d$ being the MV control). The EHAC1 control law is as follows (Latawiec, 1991)

$$u(t) = \frac{1}{G'(1)}\left[y_{\mathrm{ref}} - H(q^{-1})y(t) - q^{-1}G''(q^{-1})u(t)\right] \tag{11}$$

where

$$G(q^{-1}) = F(q^{-1})B(q^{-1}) = G'(q^{-1}) + G''(q^{-1})q^{-k+d-1} \tag{12}$$

with

$$\begin{aligned} G'(q^{-1}) &= g_0 + g_1 q^{-1} + \dots + g_{k-d}q^{-k+d} \\ G''(q^{-1}) &= g''_1 + g''_2 q^{-1} + \dots + g''_{nb+d-1}q^{-nb-d+2} \end{aligned} \tag{13}$$

Note that for EHAC1 we have

$$G'(1) = G'_1(1) = \sum_{i=0}^{k-d} g_i \tag{14}$$

In EHACr, the control $u(t)$ is determined so as, together with $u(t+1), u(t+2), ..., u(t+k-d)$, to minimize the performance index $E\{y(t+k) - y_{\text{ref}}\}$ subject to the input "energy" constraint

$$\min_{u(t),...,u(t+k-d)} \sum_{i=0}^{k-d} u^r(t+i) \tag{15}$$

where $r \geq 2$ is an even number.

The EHACr control law is identical to that of eqn. (11), with a different value of the coefficient $G'(1)$ (Latawiec, 1991)

$$G'(1) = G'_r(1) = \frac{1}{\sqrt[r-1]{g_{k-d}}} \sum_{i=0}^{k-d} \sqrt[r-1]{g_i^r} \tag{16}$$

For $r = 2, 4, ...$ we obtain the consecutive versions EHAC2, EHAC4, etc. For $r \to \infty$ we have the "EHACinf" controller, for which

$$G'(1) = G'_\infty(1) = \text{sgn}(g_{k-d}) \sum_{i=0}^{k-d} |g_i| \tag{17}$$

Note that for $k = d$ EHAC1 and all the EHACr controls are equivalent and identical to the MV control.

Although the discussed EHAC controllers differ, in general, just by one parameter from each other, they demonstrate entirely different behaviours both in transient and steady states. Also, the simple EHAC controllers can provide effective control for a variety of complex plants.

3.3. Steady-State Regulation

It can be easily shown (Latawiec and Chyra, 1983; Latawiec, 1991) that, with $T(1) = 1$ and zero-mean noise, the output mean value for the GMV and EHAC control systems is equal to

$$y_m = \frac{y_{\text{ref}}}{1 + \frac{\gamma}{K}} \tag{18}$$

where $\gamma = \beta/b_0$ for GMV, $\gamma = G'_r(1) - G'_1(1)$ for EHAC controls, and $K = B(1)/A(1)$.

Note that for the MED formulation we obtain the steady-state error-free control, no mattter what T or γ are.

3.4. Control Specifications

Specifications for y_{ref} and the absolute control variable $U(t)$ are as follows:

- $y_{\text{ref}} = 0$ for the MED model,
- $y_{\text{ref}} = 0$ or $y_{\text{ref}} = Y_{\text{ref}}$ (if, respectively, deviational or absolute process variables are used) for the INC model,

- $U(t) = U' + u(t)$, where $U' = U_m(t)$, for the MED model,
- $U' = U(t-1)$ for the INC model.

Wherever appropriate in all the above equations, substitutions of $\Delta u(t)$ for $u(t)$ and A' for A are required for the INC model.

4. Simulation Experiments

We have performed hundreds of simulation runs and only selected representative examples could be listed below. Let the system be actually governed by eqn. (1) with $D(q^{-1}) = 1$ and Gaussian $\mathcal{N}(0, \lambda)$ noise $e(t)$, the mean of which is offset to 2 for $t > 150$. Let $na = nb = nc = 2$, $d = 1$, $U_0 = 10$, $Y_{\text{ref}} = 40$, $b_1 = b_{10}$ for $t \leq 200$ and $b_1 = b_{10}[1 + 0.5 \sin(0.0128(t - 200))]$ for $t > 200$, $\hat{p}(0) = 0$, $P(0) = 10^5 I$

In order to pursue the drifting phenomena in the plant, we make the controllers switch the exponential forgetting constants from $\rho_1 = 0.99$ to $\rho_2 = 0.90$ and from $\alpha_1 = 0.995$ to $\alpha_2 = 0.85$ (return preferably gradual) if $\text{sgn}[U(t) - U_m(t)]$ remained constant thrughout, say, seven sampling intervals.

In the incremental and the deviational model eqns. (2) and (4), respectively, we assume $T = 1$ so as not to obscure the mainstream of our considerations.

Example 1. (The system originally non-minimum phase) The following parameters have been chosen: $a_1 = 0.8$, $a_2 = 0.1$, $b_0 = 0.75$, $b_{10} = 0.65$, $b_2 = -0.2$, $\lambda = 0.2$, $c_1 = c_2 = 0$, GMV control, $\beta = 0.1$, switching of ρ and α not included. The performance of the control system can be seen in Fig. 1 where the values

$$\sigma_y^2 = \frac{1}{50} \sum_{t=1}^{50} \left[y(t) - y_{\text{ref}} \right]^2, \quad \sigma_u^2 = \frac{1}{50} \sum_{t=1}^{50} u^2(t)$$

have been plotted every 50 sampling instants. (Parameter estimates have also been monitored for analysis purposes). Quite similar results have been obtained for the EHAC2 controller with $k = 2$.

The INC model proves better steady-state behaviour and lower sensitivity to changes in D; also, its tracking the time-varying process conditions is quicker.

It has been found that although the general behaviour of the scheme based on the INC model is much better (except for the very beginning where it is slower convergent), it is not possible even to approximately reconstruct the true parameters a_j, b_j from the estimates $\hat{a}'_j, \hat{b}_j$ even if $\hat{b}_0$ is fixed to be equal to b_0. On the contrary, the estimates $\hat{a}_j$, $\hat{b}_j$ are "close" to the true parameters in the control scheme based on the MED model. That feature is important in practice. As a matter of fact, for open-loop stable processes (as in the majority of industrial plants) we can easily introduce the open-loop stability constraint into the LS estimator, e.g.

$$1 + \hat{a}_1 + \hat{a}_2 > 0, \;\; 1 - \hat{a}_1 + \hat{a}_2 > 0, \;\; 1 - \hat{a}_1 > 0$$

The insertion of the above supervising measure to the estimator is only desirable if the estimates are "close" to the true parameters.

In the second example, we insert the above open-loop stability constraint into the estimator. It is interesting that the LS estimation algorithm is not formally modified to include the constraint along the constrained minimization procedure. The estimates are simply not accepted when outside the open-loop stability region and the estimation scheme based on the MED model has appeared to be insensitive to such a corruption (this is not the case for the "offset" models that we have also examined elsewhere). In the second example we also introduce switching ρ and α for the MED model. (Switching ρ for the INC model is meaningless).

Example 2. (The strongly disturbed, originally non-minimum phase system) We adopt the following parameters: $a_1 = -0.8$, $a_2 = 0.1$, $b_0 = 0.75$, $b_{10} = 0.65$, $b_2 = -0.2$, $\lambda = 0.8$, $c_1 = -0.5$, $c_2 = 0.2$, the estimate $\hat{b}_0$ is fixed to be the extremely underestimated value of 0.1, EHACinf control, $k = 2$ (the performance for the GMV control with $q = 0.4$ is similar).

The excellent robust performance can be observed for the MED model (Fig. 2). However, the performance for the INC model is also surprisingly correct, considering the lack of any robustifying measures the MED model is provided with.

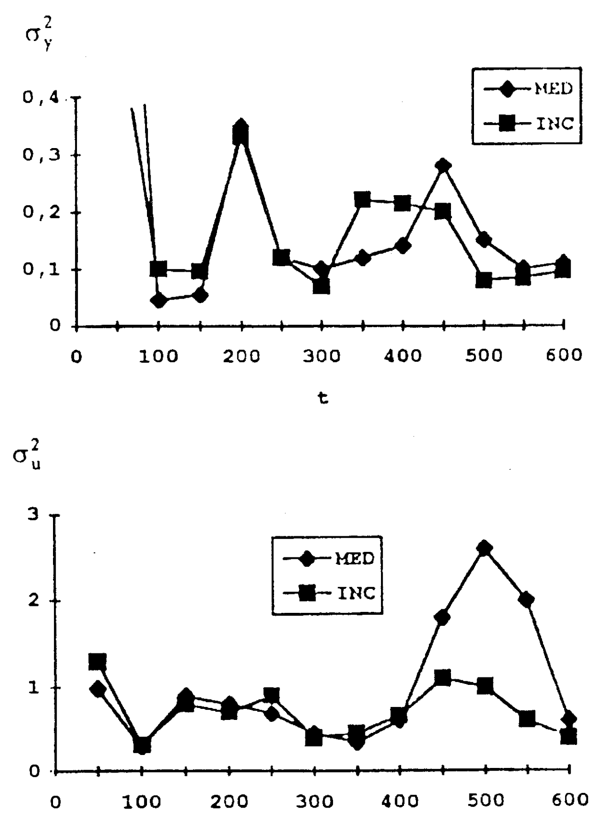

Fig. 1. Performance of control objectives, Example 1.

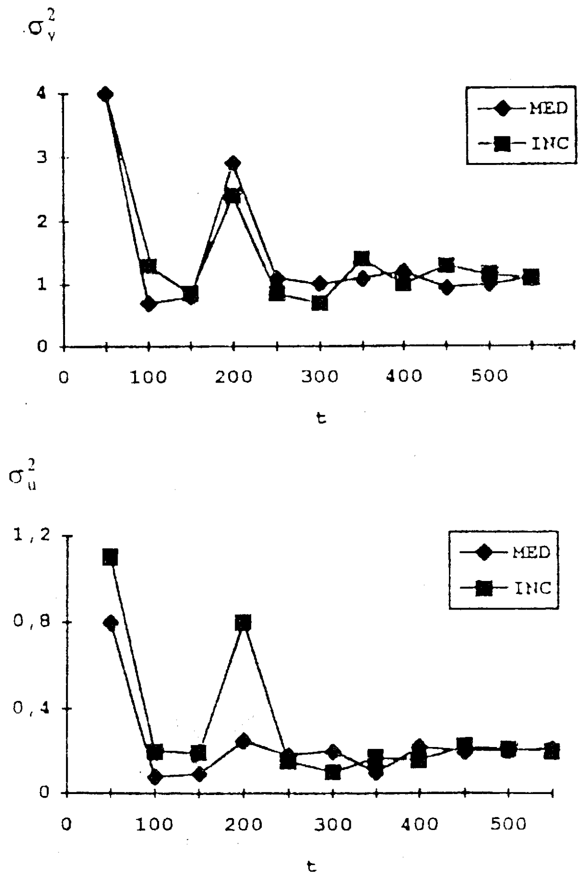

Fig. 2. Performance of control objectives, Example 2.

5. Discussion

Making use of the INC model leads to the occurrence of the integration action in a control law. However,the injection of the *unit* pole into the closed loop is meaningless as it is cancelled by the identical zero. Right the same holds for the MED model, with *almost unit* pole/zero cancellation (at $q = \alpha$). Anyway, both formulations are "close" to each other and both provide steady-state error-free servo and regulatory controls.

Whilst effectively dealing with low-frequency disturbances, in particular load-type changes, incremental controllers suffer from three main disadvantages, jeopardizing robustness of the control system:

- They are sensitive to high-frequency disturbances both in the process and the input-output measurements. Therefore, careful prefiltering of the process variables and, possibly, thorough design of the observer polynomial may be necessary. Sensitivity to disturbances is even more detrimental under adaptive control, where the "smooth" operation of the recursive estimator usually necessitates special precautions.

- They fail to provide "integrity" to the closed-loop system. A feedback system possesses integrity if it remains stable in the face of switching off of control loop(s) by actuator or sensor failures. For a SISO control system, integrity is ensured provided both the open-loop and the closed-loop systems are stable. Integral/incremental controllers violate this condition.

- They are not able to separate slow time-varying effects in the process from rapid load-type changes in the disturbance. The latter changes, detected reliably by incremental controllers, happen unfortunately to enter the adaptive parameter estimator as well. This causes the estimator to overact undesirably as if the process parameters were changing themselves which might have not been the case at all. As was mentioned before, the mean-deviational controller can be arranged so as to separate slow process variations from rapid setpoint or load-type disturbance changes.

With the above drawbacks, the incremental formulation still possesses one essential advantage, namely the simplicity of implementation. Owing to this feature, the INC model can outplay the MED one, especially in a low-frequency noise control environment.

The more complex implementation of the mean-deviational formulation may bring a margin of arbitrariness. First, diverse algorithms for the control mean calculation can be employed, with some parameter(s) to be set depending on the process and/or disturbance properties. In particular, the coefficient α of the first-order exponential filter has to be determined. Second, additional service in some "difficult" situations is often desirable. An example is switching of ρ and/or α coefficients. Also, certain robustifying measures like testing the open-loop stability (if applicable) can be welcome. All these efforts can yield robust adaptive control for processes corrupted with strong and wide- band disturbances, outperforming what could have been done with the incremental formulation.

Now, mean-deviational control algorithms can be made more robust than the incremental ones at the price of more complex implementation and slower regulation for load-type changes in process disturbances.

6. Conclusion

We have examined adaptive control schemes combining adaptive LS estimation and either the GMV or EHAC control laws, based on the incremental and mean- deviational models.

The straightforward implemented INC model has proved to be effective in many advanced process control applications. Its usefulness has been confirmed here, in particular for quality regulation under low-frequency disturbances. However, our recommendation is to avoid, if possible, the injection of signal increments into the adaptive control loop in the case of high-level, high-frequency disturbances.

The implementation of the MED model is more complex than for INC. However, the mean-deviational controllers can outperform the incremental ones with respect to robustness which is the most important requirement for adaptive controllers.

Unfortunately, the clear-cut assignment of the above models to specific processes and/or disturbances is not possible. Apparently, both formulations should be used in advanced process control engineering and the MED model could be recommended in more sophisticated adaptive control problems.

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