Int. J. Appl. Math. Comput. Sci., 2002, Vol.12, No.2, 277–283

CONFIDENCE AND SELF-CONFIDENCE: PERCEIVED AND REAL

DAVID W. PEARSON*

* EURISE (Roanne Research Group), Jean Monnet University, Saint-Etienne
I.U.T. de Roanne, 20, avenue de Paris, 42334 Roanne, France
e-mail: david.pearson@univ-st-etienne.fr

The problem of modelling the dynamics of confidence levels between two individuals is investigated. A model, based on a master equation approach, is developed and presented. An important feature of the model is that self-confidence is modelled along with its interaction with confidence towards others. Simulation results are presented.

Keywords: quantitative sociodynamics, confidence, self-confidence

1. Introduction

There are many situations in which social interactions and emerging social phenomena play important and, sometimes, fundamental roles. In our particular case we are studying networks of small to medium enterprises. In these enterprise networks each individual enterprise keeps its own identity but works in close collaboration with the others in order to produce something that the enterprises working alone would not be able to produce. The very fact that the enterprises are small to medium (mostly small in practice) means that there are very few employees, i.e. ranging from 1 to 10 in most cases. Because of this, when we try to model such a network, we are obliged to take into account the human factors. For example, two company heads who have a close affinity are more likely to achieve a successful collaboration even when their respective commercial fields are different.

In developing mathematical models of these situations it is therefore necessary to take into consideration these social phenomena during the modelling process. This is where the relatively new scientific discipline of quantitative sociodynamics comes along. The basic objective of quantitative sociodynamics is to use tools issuing from "hard sciences" such as mathematics, computer science and physics, and "soft sciences" such as psychology and sociology, in order to model and simulate social phenomena (Conte *et al.*, 1997; Gilbert and Doran, 1994; Hegselmann *et al.*, 1996; Helbing, 1995; Liebrand *et al.*, 1998; Weidlich and Haag, 1983).

We have already looked at the problem of modelling confidence levels and their dynamical evolutions for enterprise networks (Pearson *et al.*, 2001) and the real and perceived confidence level problem (Pearson and Boudarel, 2001). In this article we push this modelling approach a little further in proposing how an individual might modify his or her estimations of what others think of him/her.

2. Confidence and Self-Confidence

In Fig. 1 we illustrate the situation that we are trying to model in a mathematical way. The two nodes represent two individuals. Each arc directed from one node to another is meant to represent the level of confidence of one of the individuals towards the other. Clearly, an arc leaving and entering the same node represents the level of self-confidence that an individual has in himself/herself.

We make a further (important for us) distinction. The continuous arcs represent the real or known levels of confidence and the dotted arcs represent perceived levels of confidence. For example, looking from the "world view" of Individual 1, there is a known level of confidence in Individual 2 and a known level of self-confidence. However, what Individual 2 thinks of Individual 1 and Individual 2's own level of self-confidence can only be perceived, or estimated, by Individual 1.

We introduce the following variables and notation: Let x_{ij} denote the *real confidence level* of Individual j towards Individual i and $\hat{x}_{ij}$ denote the *perceived confidence level* of Individual j towards Individual i. We assume that each individual knows what he/she thinks of him/herself and what he/she thinks of the other, but only perceives the confidence levels of the other. We thus define

$$x_1 = \begin{bmatrix} x_{11} \\ x_{21} \\ \hat{x}_{12} \\ \hat{x}_{22} \end{bmatrix}, \quad x_2 = \begin{bmatrix} \hat{x}_{11} \\ \hat{x}_{21} \\ x_{12} \\ x_{22} \end{bmatrix}.$$

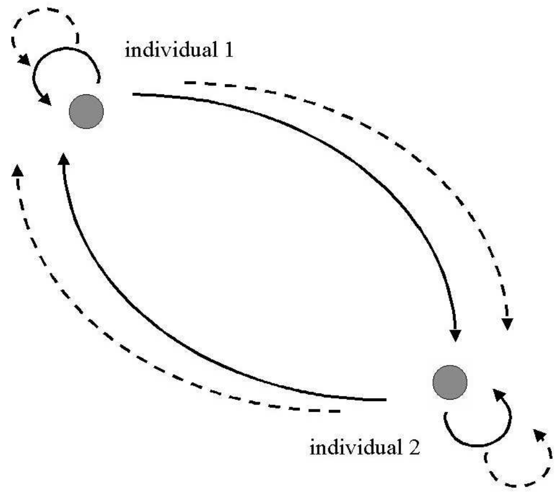

Fig. 1. Relationship between two individuals.

Clearly, x_1 corresponds to Individual 1's world view and x_2 corresponds to Individual 2's world view. Our problem is to model the dynamics of x_1 and x_2, and for this we make use of the master equation approach, which is essentially probabilistic (Helbing, 1995; Weidlich and Haag, 1983).

We begin by assigning a scale to confidence levels. For mathematical simplicity, we choose a scale symmetric about the origin, going from $-N$ to N, where N is some positive integer. In this way $-N$ represents absolute non-confidence, 0 neutral confidence and N absolute confidence. There are thus $2N+1$ points on the scale.

We introduce the variables $x_{ij}^n(t)$ standing for the probabilities that Individual j has confidence level n towards Individual i at time t where $-N \leq n \leq N$ and $t \geq 0$, with $\hat{x}_{ij}^n(t)$ corresponding to the perceived probabilities. Hereafter we suppress direct reference to the time variable t and simply write x_{ij}^n etc., it being understood that these are always considered to be dynamical quantities. For each arc in Fig. 1 we therefore have a discrete random variable with a probability distribution defined by the variables $x_{ij}^n(t)$, and we model the dynamical evolution of this distribution.

We do not imagine that an individual will be aware of a probability distribution concerning his/her self-confidence or confidence in the other individual. We imagine rather that what is known or perceived is the mean value of the variable. For this reason we associate the mean values of the variables with the arcs in Fig. 1 and define

$$x_{ij} = \sum_{n=-N}^{N} n x_{ij}^n. \tag{1}$$

The set of differential equations for the probabilities is defined by the following formula (Helbing, 1995; Weidlich and Haag, 1983):

$$\begin{aligned}\dot{x}_{ij}^n &= w_{ij}\big(n|n-1\big)x_{ij}^{n-1} + w_{ij}\big(n|n+1\big)x_{ij}^{n-1}\\ &\quad - w_{ij}\big(n-1|n\big)x_{ij}^n - w_{ij}\big(n+1|n\big)x_{ij}^n,\end{aligned}$$

where e.g., $w_{ij}(n|n-1)$ represents the transition probability from $n-1$ to n in a unit time period, and $w_{ij}(n|n-1)x_{ij}^{n-1}$ is the probability flux from value $n-1$ to value n. We define the following functions:

$$u_{ij}(n) = w_{ij}\big(n+1|n\big), \quad v_{ij}(n) = w_{ij}\big(n-1|n\big).$$

The above can then be written as

$$\begin{aligned}\dot{x}_{ij}^n &= u_{ij}(n-1)x_{ij}^{n-1} + v_{ij}(n+1)x_{ij}^{n+1}\\ &\quad - \big(u_{ij}(n) + v_{ij}(n)\big)x_{ij}^n.\end{aligned} \tag{2}$$

We note that the probability fluxes are from neighbouring values to neighbouring values in the above. In other words, we limit fluxes from value n to $n-1$ and $n+1$ and not $n-2$ or $n+2$, etc. This is to avoid abrupt transitions. We note, however, that we do not entirely exclude this possibility and we will further investigate this in our future work.

The system of equations (2) is valid for each variable x_{ij}, $i, j \in \{1, 2\}$, and for $-N \leq n \leq N$ there are therefore $8(2N+1)$ differential equations in total. To preserve the logic and ensure that there is no flux downwards from $-N$ to $-N-1$ and none upwards from N to $N+1$, the following boundary conditions are imposed:

$$u_{ij}(N)x_{ij}^N = v_{ij}(-N)x_{ij}^{-N} = 0.$$

The modelling exercise is simplified to that of finding suitable forms for the functions $u_{ij}(n)$ and v_{ij} in (2). In (Pearson and Boudarel, 2001) we presented our preliminary ideas for these functions. In this paper we begin with these ideas as a base and add some improvements. We begin with the variable x_{11} and propose the following form for $u_{11}(n)$:

$$\begin{aligned}u_{11}(n) &= \alpha\lambda e^{-\beta(a_{11}-n-1)^2}\\ &\quad + (1-\alpha)\mu e^{-\gamma(\hat{x}_{12}-n-1)^2} f(\hat{x}_{22}).\end{aligned} \tag{3}$$

The meaning of each term and parameter in (3) is as follows:

- the first term represents Individual 1's own fixed level of self-confidence,
- the second term represents the potential change in Individual 1's self-confidence brought about by what Individual 1 perceives as Individual 2's confidence in Individual 1,

- $\alpha \in [0,1]$ is a weighting factor where a value close to 1 represents a self-centred individual and a value close to 0 represents an easily influenced individual,
- $a_{11} \in \{-N, -N+1, \dots, N\}$ is Individual 1's intrinsic self-confidence,
- β and γ define the spread of the Gaussian functions where elevated values represent a slim possibility of influence and small values lead to a stronger possibility of influence and thus change,
- $\lambda > 0$ and $\mu > 0$ are scaling parameters,
- $f : [-N, N] \to [0,1]$ is a function relating to the perceived self-confidence of Individual 2.

The objective of the function f is to attenuate the influence of Individual 2 if this individual has little self-confidence or too much self-confidence. For numerical simplicity, we propose a form for this function as shown in Fig. 2, where the function parameters n_1, n_2, f_1 and f_2 are also shown.

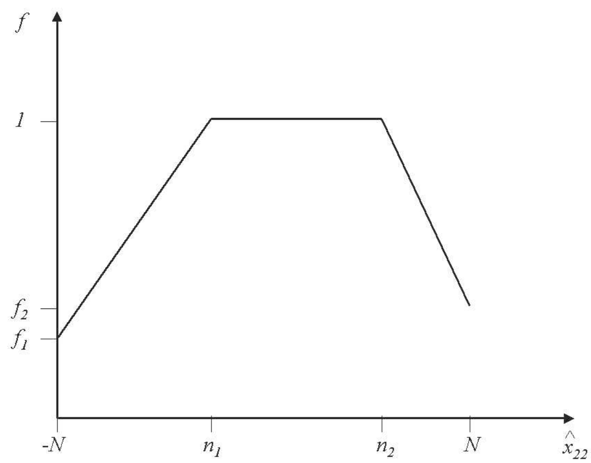

Fig. 2. The attenuating function.

The form of function u_{21} is similar and based on the principle that an individual is more likely to have confidence in another individual if he/she thinks that the individual has confidence in him/her (Le Cardinal *et al.*, 1997):

$$u_{21}(n) = \alpha\lambda e^{-\beta(a_{21}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(\hat{x}_{12}-n-1)^2}. \tag{4}$$

The two terms and the parameters in (4) play the same roles as the corresponding ones in (3) and so we do not repeat them here. We simply remark that the parameters α, β, γ, λ and μ do not necessarily have the same values in (3) and (4). Here, and in the remainder of the paper, we avoid writing α_{ij} etc. in order to simplify the notation.

We have not multiplied the second term in (4) by the attenuating function f because we postulate that an individual is more likely to have confidence in another individual if he/she believes that the confidence is reciprocal, regardless of the self-confidence of the other individual. We are in fact looking through this idea at present.

Continuing in the same order, we come to the differential equations for the first of the perceived variables $\hat{x}_{12}$. In (Pearson and Boudarel, 2001) we based the function $\hat{u}_{12}$ simply on the fixed idea, i.e. on the corresponding first term in (4). In this paper we propose a modified approach to the dynamics of the perceived variables. The approach is very simple; we imagine that the dynamics is based on the fixed idea of the individual when the difference between the real and perceived values is less than some threshold value $|x_{12} - \hat{x}_{12}| < \epsilon$. However, when this difference becomes greater than the threshold value, a second term comes into play as in (3) and (4). The form of $\hat{u}_{12}$ is therefore as follows:

$$\hat{u}_{12}(n) = \begin{cases} \lambda e^{-\beta(\hat{a}_{12}-n-1)^2} & \text{if } |x_{12} - \hat{x}_{12}| < \epsilon, \\ \alpha\lambda e^{-\beta(\hat{a}_{12}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(x_{12}-n-1)^2} & \text{otherwise.} \end{cases} \tag{5}$$

Our justification for this is that an individual, under normal circumstances, will adopt a certain type of behaviour towards another individual if he/she has confidence in that individual. This behaviour is fairly predictable within certain limits or tolerance. Hence, if we put forward a hypothesis that the individual has a certain level of confidence that he/she does not have, then we should be able to detect the difference between the assumed level and the real level when this difference becomes sufficiently large, i.e. greater than the tolerance threshold. For example, in real life situations we could imagine detecting such differences at meetings where an individual may express himself/herself in a way contrary to our assumption or may vote against us.

We have chosen to model this correction term in a discontinuous fashion rather than continuously because we believe that the fact that there is a measurable difference between a hypothesised behaviour and a real behaviour would actually produce a discontinuous response in an individual.

To complete the sequence, we construct $\hat{u}_{22}$ in the same way:

$$\hat{u}_{22}(n) = \begin{cases} \lambda e^{-\beta(\hat{a}_{22}-n-1)^2} & \text{if } |x_{22} - \hat{x}_{22}| < \epsilon, \\ \alpha\lambda e^{-\beta(\hat{a}_{22}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(x_{12}-n-1)^2} & \text{otherwise,} \end{cases} \tag{6}$$

The functions v_{11}, v_{21}, $\hat{v}_{12}$ and $\hat{v}_{22}$ are developed from (3), (4), (5) and (6) by replacing $-n-1$ by $-n+1$ in each corresponding formula. What is more, by symmetry we can present the functions for $\hat{u}_{11}$, $\hat{u}_{21}$, u_{12} and u_{22} by removing hats in (3), (4), (5) and (6) and putting hats where there are not any:

$$\hat{u}_{11}(n) = \begin{cases} \lambda e^{-\beta(\hat{a}_{11}-n-1)^2} & \text{if } |x_{11} - \hat{x}_{11}| < \epsilon, \\ \alpha\lambda e^{-\beta(\hat{a}_{11}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(x_{11}-n-1)^2} & \text{otherwise}, \end{cases} \tag{7}$$

$$\hat{u}_{21}(n) = \begin{cases} \lambda e^{-\beta(\hat{a}_{21}-n-1)^2} & \text{if } |x_{21} - \hat{x}_{21}| < \epsilon \\ \alpha\lambda e^{-\beta(\hat{a}_{21}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(x_{21}-n-1)^2} & \text{otherwise}, \end{cases} \tag{8}$$

$$u_{12}(n) = \alpha\lambda e^{-\beta(a_{12}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(\hat{x}_{21}-n-1)^2}, \tag{9}$$

$$u_{22}(n) = \alpha\lambda e^{-\beta(a_{22}-n-1)^2} + (1-\alpha)\mu e^{-\gamma(\hat{x}_{21}-n-1)^2} f(\hat{x}_{11}). \tag{10}$$

As above, the functions $\hat{v}_{11}$, $\hat{v}_{21}$, v_{12} and v_{22} are developed from (7)–(10) by replacing $-n-1$ by $-n+1$ in each formula.

Our model was programmed using a standard high performance numerical calculation software package and run on an ordinary Pentium III laptop PC. We tried using various integration methods (both stiff and non-stiff integrators), but found no perceptible differences in the results. For this reason, we normally use Runge-Kutta or predictor-corrector routines supplied with the software to integrate the differential equations, with a slight preference given to the predictor-corrector method simply because it runs a bit faster.

3. Some Simulation Results

In this section we present some simulation results for a particular pair interaction like in Fig. 1. In the first instance we need to choose a value for the scale parameter N. Previous simulation studies have shown that there are no appreciable numerical difficulties encountered when we have $1 \leq N \leq 10$ (Pearson and Boudarel, 2001), and for this particular simulation we chose $N = 5$.

For the natural confidence level trends of the two individuals we chose the following values:

$$a = \begin{bmatrix} a_{11} \\ a_{21} \\ \hat{a}_{12} \\ \hat{a}_{22} \\ \hat{a}_{11} \\ \hat{a}_{21} \\ a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} N-1 \\ -N+1 \\ -N+1 \\ N-1 \\ N-1 \\ N-1 \\ N-1 \\ 0 \end{bmatrix},$$

which can be interpreted in the following way:

- Individual 1 has a high level of self-confidence,
- Individual 1 does not have any confidence in Individual 2,
- Individual 1 thinks that individual 2 does not have confidence in Individual 1,
- Individual 1 thinks that Individual 2 has a high level of self-confidence,
- Individual 2 thinks that Individual 1 has a high level of self-confidence,
- Individual 2 thinks that Individual 1 has confidence in Individual 2,
- Individual 2 has confidence in Individual 1,
- Individual 2 has neutral self-confidence.

For all the simulations we chose the values $\beta = \gamma = 0.1$ for all of the equations, and for the parameters of the function f in Fig. 2 we chose $n_1 = 1$, $n_2 = 4$, $f_1 = 0.5$ and $f_2 = 0.5$ for both individuals. Furthermore, we set $\lambda = \mu = 2$ in all the equations. These values were fixed for all the simulations simply because we wished to illustrate the effects of the other parameters in this article.

In the first simulation we wanted to simulate a fairly neutral situation, where each individual takes into consideration in equal amounts his/her own opinion and the perceived opinions of the other and does not react to measured events. Thus we set $\alpha = 0.5$ and $\epsilon = 2N + 1$ for each individual. The results for this first simulation can be observed in Fig. 6 as seen from Individual 1's point of view and Fig. 7 as seen from Individual 2's point of view. In these and the following figures, when the world is seen from Individual 1's point of view, the real confidence levels correspond to the two images to the left of the figure and the perceived confidence levels are the two images to the right of the figure. When the world view of Individual 2 is represented, the situation is reverse, with the

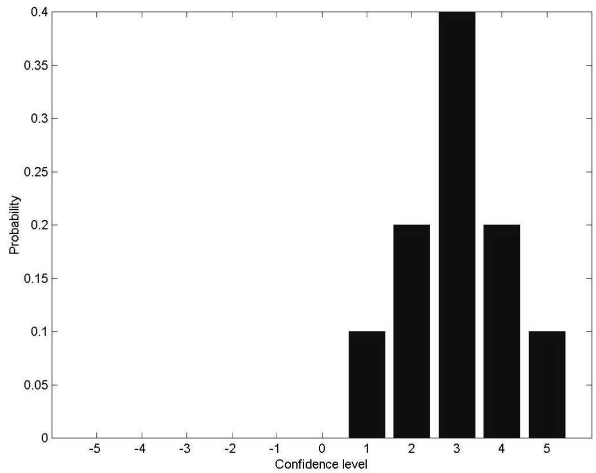

Fig. 3. Initial distribution tending towards confidence.

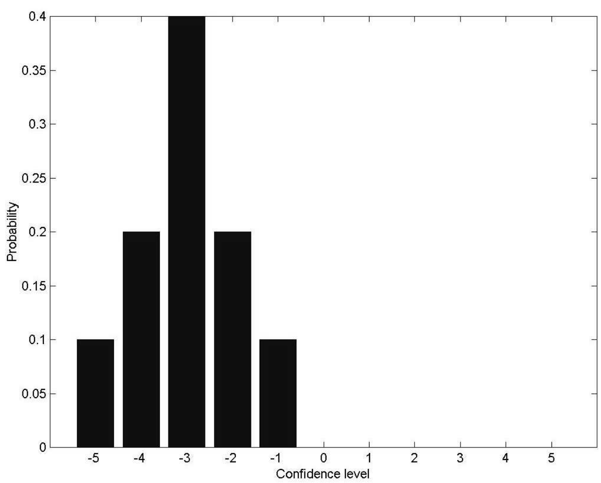

Fig. 4. Initial distribution tending towards non-confidence.

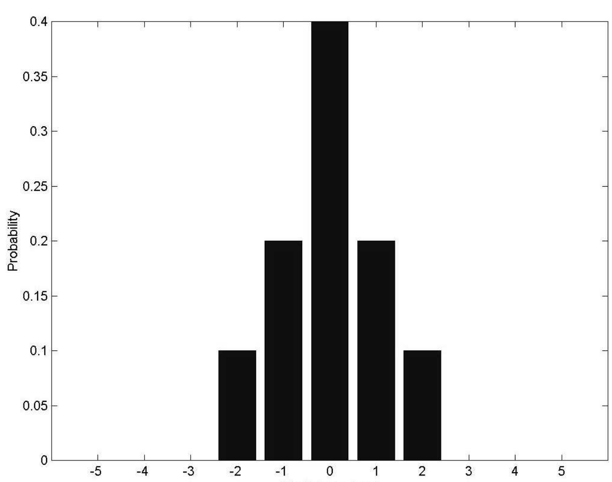

Fig. 5. Initial distribution tending towards neutral confidence.

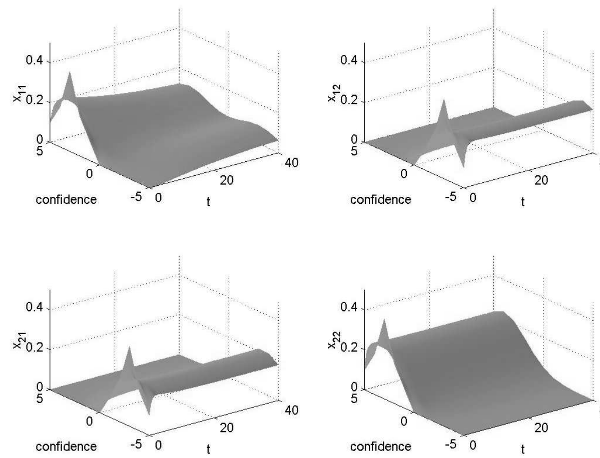

Fig. 6. Confidence level evolution of Individual 1 towards Individual 2: the first simulation.

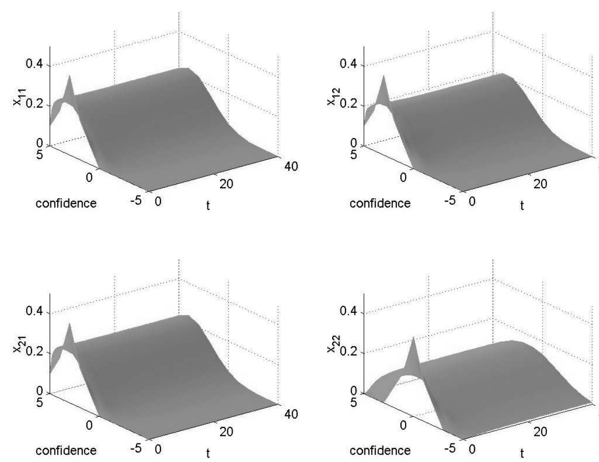

Fig. 7. Confidence level evolution of Individual 2 towards Individual 1: the first simulation.

perceived confidence levels corresponding to the images to the left, and the real confidence levels corresponding to the images to the right of the figure.

The axes on the figures are self-explanatory. The x-axis corresponds to the time variable ($0 \leq t \leq 40$ for each simulation), the y-axis corresponds to the confidence level ($-N \leq n \leq N$) and the corresponding probabilities are plotted on the z-axis ($0 \leq x_{ij}^n \leq 1$). We are thus able to visualize the temporal evolution of the distributions of the discrete variables x_{ij}.

In this first simulation we see that the distributions stabilize out fairly quickly. We remark the noticeable differences between the real and perceived confidence levels.

In the second simulation we wanted to see the difference that measured confidence levels could make. We therefore kept the same values for $\alpha = 0.5$ but altered $\epsilon = 1$, i.e. a difference of one unit could be measured with

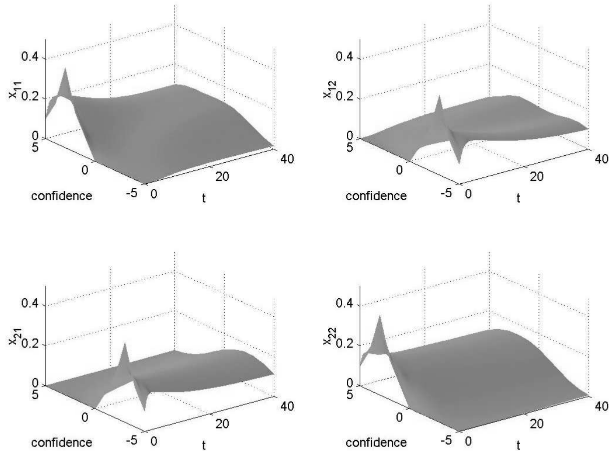

Fig. 8. Confidence level evolution of Individual 1 towards Individual 2: the second simulation.

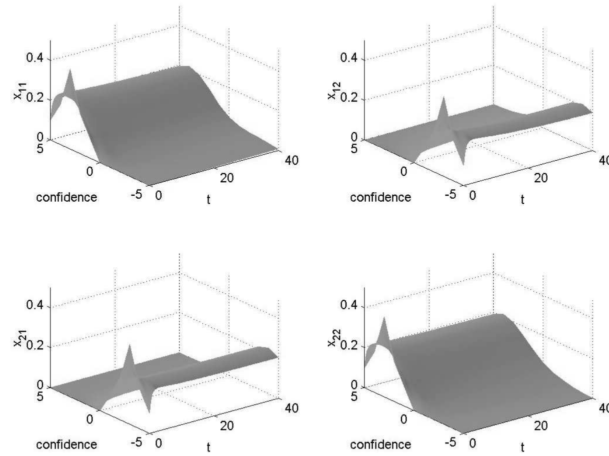

Fig. 10. Confidence level evolution of Individual 1 towards Individual 2: the third simulation.

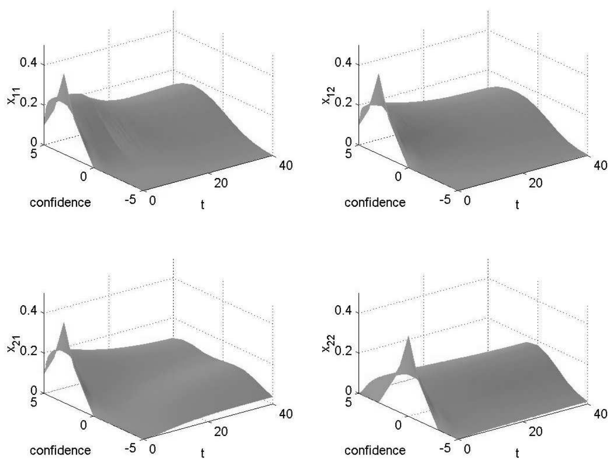

Fig. 9. Confidence level evolution of Individual 2 towards Individual 1: the second simulation.

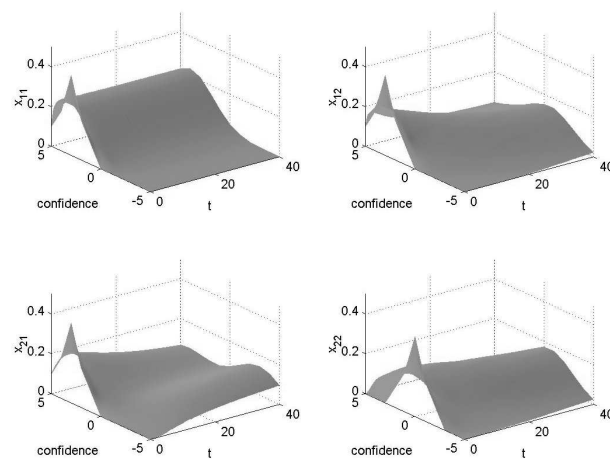

Fig. 11. Confidence level evolution of Individual 2 towards Individual 1: the third simulation.

reasonable accuracy by each individual. The results for the second simulation can be seen in Figs. 8 and 9.

We see that there is a difference between the first simulation and the second one. The detection of differences between hypothesised and real behaviours had influence on the dynamics.

In the third and final situation, we kept the value of $\epsilon = 1$ for each individual but set $\alpha = 0.9$ for Individual 1 and $\alpha = 0.2$ for Individual 2, thus simulating the situation where Individual 1 is strong headed and does not particularly take into consideration the perceived or measured confidence levels of the other, and Individual 2 lacks self-confidence and relies on perceived or measured confidence levels in order to form his/her own opinion. The results for the third simulation can be seen in Figs. 10 and 11.

From this third simulation we see that there is a considerable difference in the dynamics of $\hat{x}_{21}$ (bottom left of Fig. 11). We can assume that Individual 2 realizes that, in spite of his/her initial thoughts, Individual 1 really has no confidence in him/her.

4. Conclusion

In this paper we have presented an extension to a model developed in an earlier paper. We believe that this new improved model is a bit closer to reality. Clearly, there are still many improvements to be made and we are working on these.

We have based our model on Gaussian type functions. As stated, we are looking at other possibilities, e.g. polynomial and piecewise linear. We are also looking at other possibilities for introducing the correction terms, i.e.

a continuous correction rather than one based on a threshold. We believe, however, that the threshold correction is more realistic.

We are looking at how to measure differences between assumed and real behaviours, particularly in association with enterprise networks, where we believe some sort of questionnaire or audit procedure could reveal such differences.

Although real applications of our model will not be forthcoming in the immediate future, we can imagine how it could be applied in medium to long terms. As an example of an application we could imagine a situation where individuals need to make a decision. Clearly, in such a situation there will be certain individuals who will try to bring the others around to their way of thinking. In a lot of cases, we could imagine that the individuals who take the lead and try to convince the others that they are right will be those who have a fairly strong opinion of themselves. It would be interesting to apply our model to such a situation and fit the relevant parameters of the model to the dynamic behaviour of the individuals. The data could be in the form of notes taken by psychologists/sociologists who observe the individuals. We could even imagine the event being filmed and certain behaviours being automatically deduced.

A second application is related to the enterprise networks discussed above. In this case the self-confidence of an enterprise could be monitored by using our model. We could imagine using data such as sales, money spent on marketing, e-mails received and sent, etc. as being indicators of the state of self-confidence and confidence towards others in the network. By fitting our model to the dynamics of the network we could predict behaviour patterns and probably even suggest where the network could be improved.

Looking at further studies of such models, we believe wholeheartedly in a multidisciplinary approach. In other words, we are in contact with researchers from other disciplines such as psychology and sociology and we are slowly but surely building up an international multidisciplinary team with a view to improving our model.

Finally, we are continuing the analysis of our model, both mathematically and experimentally by simulation.

References

Conte R., Hegselmann R. and Terna P. (Eds.) (1997): *Simulating Social Phenomena*. — New York: Springer.

Gilbert N. and Doran J. (Eds.) (1994): *Simulating Societies: The Computer Simulation of Social Phenomena*. — London: UCL Press.

Hegselmann R., Mueller U. and Troitzsch K.G. (Eds.) (1996): *Modelling and Simulation in the Social Sciences from a Philosophy of Science Point of View*. — Dordrecht: Kluwer.

Helbing D. (1995): *Quantitative Sociodynamics*. — Dordrecht: Kluwer.

Le Cardinal G., Guyonnet J.-F. and Pouzoullic B. (1997): *La Dynamique de la Confiance*. — Paris: Dunod.

Liebrand W.B.G., Nowak A. and Hegselmann R. (Eds.) (1998): *Computer Modelling of Social Processes*. — Sage.

Pearson D.W., Albert P., Besombes B., Boudarel M.-R., Marcon E. and Mnemoi G. (2001): *Modelling enterprise networks: A master equation approach*. — Europ. J. Oper. Res. (in print).

Pearson D.W. and Boudarel M.-R. (2001): *Pair interactions: Real and perceived attitudes*. — J. Artif. Societ. Social Simul. (submitted).

Weidlich W. and Haag G. (1983): *Concepts and Models of a Quantitative Sociology*. — Berlin: Springer.

Received: 7 May 2001
Revised: 31 May 2002