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# A FAULT ESTIMATION AND FAULT–TOLERANT CONTROL BASED SLIDING MODE OBSERVER FOR LPV DESCRIPTOR SYSTEMS WITH TIME DELAY

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This paper considers the problem of fault-tolerant control (FTC) and fault reconstruction of actuator faults for linear parameter varying (LPV) descriptor systems with time delay. A polytopic sliding mode observer (PSMO) is synthesized to achieve simultaneous reconstruction of LPV polytopic descriptor system states and actuator faults. Exploiting the reconstructed actuator faults and state estimates, a fault-tolerant controller is designed to compensate the impact of actuator faults on system performance by stabilizing the closed-loop LPV delayed descriptor system. Besides, the controller and PSMO gains are obtained throughout the resolution of linear matrix inequalities (LMIs) using convex optimization techniques. The developed PSMO could force the output estimation error to converge to zero in a finite time when the actuators faults are bounded through the reinjection of the output estimation error via a nonlinear switching term. Simulation results applied to a given numerical system are presented to highlight the superiority and effectiveness of the proposed approach.

**Keywords:** fault estimation, fault-tolerant control, LPV descriptor systems, sliding mode observer, time delay, LMIs.

## 1. Introduction

Fault-tolerant control is known to be a control technique combining diagnosis and control methods for the sake of better stability and performance of a given system even if faults raise. Fault diagnosis and isolation (FDI) on the one hand and FTC on the other play, all together, a significant role within any numerical system. In fact, FDI provides sufficient information through filters, observers, or residual generation to clearly identify the faulty sensor, the actuator or any internal faults that could occur within a given system (Chandra *et al.*, 2016). In their literature survey, Lopez-Estrada *et al.* (2019) present various methodologies for observer synthesis and fault-related strategies for convex systems under different representations: Takagi–Sugeno fuzzy models, linear parameter varying (LPV) and quasi-LPV systems, whereas the purpose of the FTC is to maintain sufficient security and stability even if the system is faulty.

With the above in mind, fault tolerant control has attracted much attention. Li and Wang (2020) investigated the fault tolerant tracking consensus problem for a class of leader-follower multi-agent systems. Stefanovski and Juricic (2020) proposed a new formulation of the FTC problem based on fault estimation in presence of disturbances. Ben Zina and Chaabane (2019) presented a method for designing a robust fault tolerant tracking control technique for nonlinear uncertain systems described by Takagi–Sugeno fuzzy models with unmeasurable premise variables subject to sensor faults.

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Rabaoui *et al.* (2018; 2020) used new adaptive proportional-integral (PI) and PID control algorithms in order to ensure trajectory tracking despite the presence of actuator faults and unknown inputs for LPV systems based on an adaptive polytopic observer. The previously listed literature and even other works were mainly based on regular systems. This is to say, there are no algebraic relations between system variables.

Furthermore, problems related to FTC of descriptor systems, also called implicit systems or singular systems or differential-algebraic equations (DAEs), have been widely studied. In fact, Su *et al.* (2018) studied the problem of optimal fault tolerant control for a class of descriptor time-varying systems with nonlinear input. Jia *et al.* (2019) designed a fault-tolerant preview controller for a class of impulse controllable continuous time descriptor systems with sensor faults.

Generally speaking, LPV descriptor systems could approximate a highly complicated nonlinear singular system using the polytopic representation. The purpose of this approach is to represent the descriptor system as an interpolation of simple local models (Hamdi *et al.*, 2012). The LPV descriptor system formulation can have powerful analysis and design properties and is a suitable way of representing real systems (Halalchi *et al.*, 2011). However, few studies have been concerned with LPV approaches to the joint problems of state reconstruction and fault diagnosis (FD) or fault estimation for descriptor systems. In the work of Hamdi *et al.* (2019) the problem of fault detection and reconstruction of actuator faults for linear parameter varying descriptor systems is considered. The results of Hamdi *et al.* (2012) are extended by Shi and Patton (2014) to investigate an active fault tolerant control scheme based on a proportional derivative extended state observer for an LPV descriptor system.

Then, again, time-delay is another factor that can degrade system performance; it is a built-in feature in many engineering systems (Briat, 2015). The presence of time-delay, along with faults, could easily drive the system to the instability state. Therefore, investigating FTC design of time-delay systems has great practical and theoretical significance. Concerning observer and FD design, very few works have been devoted to descriptor LPV systems with time delay. Hassanabadi *et al.* (2016) studied the problem of a robust fault detection based on an unknown input observer for LPV singular delayed systems in the presence of disturbances and actuator faults. Hamdi *et al.* (2018) presented the problem of robust fault detection of delayed descriptor LPV systems including disturbances and actuator faults. This presented FD method is based on comparing the online real system behavior with the results of estimation obtained with time delays by an adaptive polytopic observer. Accordingly, several FTC approaches have been developed for time-delay regular systems (Chen and Saif, 2006). But few works are focusing on FTC of polytopic LPV and time-delay descriptor systems. The problem of robust fault tolerant control for a class of singular systems subject to both time-varying state-dependent nonlinear perturbation and actuator saturation was investigated by Zhiqiang *et al.* (2010). Observer based fault reconstruction and fault-tolerant control for Takagi–Sugeno fuzzy descriptor systems subject to time delays and external disturbances were also studied (Jia *et al.*, 2015).

The sliding mode observer is employed in situations including state estimation and fault detection since it is insensitive to matched uncertainties, nonlinearity, or disturbances. The main advantage of using sliding mode observers over their linear counterparts is that, while in sliding, they are insensitive to the matched unknown inputs, they can be used to reconstruct disturbance and faults. The reconstruction of these parameters have found useful applications in fault diagnosis and fault tolerant control.

Ben Brahim *et al.* (2015) considered robust reconstruction of simultaneous actuator and sensor faults for a class of uncertain Takagi–Sugeno nonlinear systems with immeasurable premise variables. Shahnazi and Zhao (2016) suggested a new method based on the adaptive fuzzy proportional-derivative sliding mode observer to estimate sensor faults and states simultaneously for a class of uncertain nonlinear systems. Alwi *et al.* (2012) proposed a new LPV based sliding mode observer scheme for fault reconstruction. Chen *et al.* (2019) developed a linear variable parameter SMO to estimate the state vector and sensor faults, which can ensure asymptotic stability of the state estimation errors. By using the SMO, Gomez-Penate *et al.* (2019) present an FDI method for stability analysis of the Takagi–Sugeno systems with immeasurable premise variables. This method is robust to disturbances, sensor noise, and uncertainty on the premise variables. A sliding mode fuzzy observer for disturbance and actuator fault estimation in the presence of bounded uncertainties was designed for fuzzy systems (Gerland *et al.*, 2010). However, few results have been reported to design a sliding mode observer for the descriptor case. Chan *et al.* (2019) proposed two sliding mode observers in a cascade to reconstruct a fault for a class of non-infinitely observable descriptor systems. Ooi *et al.* (2017; 2015) developed a sliding mode observer for a class of infinitely unobservable descriptor systems used for state and fault estimation.

The main contribution in this paper is to address the problem of robust fault estimation and fault tolerant control of delayed LPV descriptor systems including disturbances. The system under consideration also includes actuator faults. After converting the descriptor system to a polytopic representation, a polytopic SMO is constructed for simultaneous state and fault estimation of the system. The fault estimate is then used to construct a fault-tolerant controller, which stabilizes the closed-loop system. A sufficient condition which guarantees the stability of the error reconstruction dynamics and the existence of a polytopic SMO is provided. To the best of our knowledge, the strategy based on the use of SMO schemes for fault tolerant control of LPV delayed descriptor systems presented here is innovative and interesting. Such a method is much less mature and has some fundamentally different advantages.

The remainder of this paper is organized as follows. The system description and the problem statement are given in Section 2. Then, the polytopic sliding mode observer is proposed and analyzed, and the problem of FTC design for polytopic LPV delayed descriptor systems is formulated in Section 3. Finally and before concluding, simulation results are provided to demonstrate the design effectiveness in Section 4.

## 2. Problem formulation and preliminaries

Let us introduce a time delay LPV descriptor system, affected by additive actuator faults and unknown inputs, whose state space representation is given as follows:

$$\begin{cases} E\dot{x}(t) = A(\theta(t))x(t) + A_d(\theta(t))x(t-\tau_d) \\ \qquad\qquad + B(\theta(t))u(t) + F(\theta(t))f(t) + Rd(t), \\ y(t) = Cx(t), \\ x(\xi) = \phi(\xi), \end{cases} \tag{1}$$

where $\phi(\xi)$ is the initial state $\forall\ \xi \in [-\tau_d, 0]$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ $(p \le n)$ is the input vector, $y(t) \in \mathbb{R}^m$ is the output vector, $d(t) \in \mathbb{R}^q$ is the disturbance of finite energy, $f(t) \in \mathbb{R}^k$ is the actuator fault vector and $\theta(t)$ is a varying parameter vector. The time delay $\tau_d$ is a time-varying differentiable function that satisfies

$$0 \le \tau_d \le \tau, \quad 0 \le \dot{\tau}_d \le \beta < 1, \tag{2}$$

$E$ is a singular matrix with constant parameters satisfying $\operatorname{rank}(E) = r < n$. It is assumed that all parameters $\theta_i(t), i = 1, \dots, l$ are bounded, measurable as in the work of Rodrigues *et al.* (2013), and their values remain in the domain of an hypercube such that (Fen, 1995)

$$\theta(t) \in \Gamma = \{\theta_i \mid \underline{\theta}_i \le \theta_i(t) \le \overline{\theta}_i\}, \quad \forall t \ge 0, \tag{3}$$

where $\underline{\theta}_i$ and $\overline{\theta}_i$ represent respectively the minimum and maximum values of $\theta_i(t)$. These matrices $A(\theta(t)), A_d(\theta(t)), B(\theta(t)), F(\theta(t))$ and $R$ of the LPV descriptor system (1) are with an affine parameter dependence of $\theta(t)$.

The LPV descriptor system (1) with bounded parameters can be represented by a polytopic form where the vertices $S_i$ of the polytope (Rodrigues *et al.*, 2015) are defined such that $S_i = \begin{bmatrix} A_i & A_{di} & B_i & F_i & R & C \end{bmatrix}, \forall i \in [1, \dots, h]$ where $h = 2^l$. The polytopic coordinates are denoted by $\rho(\theta(t))$ and vary within the convex set $\Omega$:

$$\Omega = \Big\{\rho(\theta(t)) \in \mathbb{R}^h, \rho_i(\theta(t)) \ge 0 \\ \text{and} \, \forall i = 1, \dots, h, \sum_{i=1}^{h} \rho_i(\theta(t)) = 1\Big\}. \tag{4}$$

Consequently, the system (1) can be rewritten through a polytopic LPV descriptor representation:

$$\begin{cases} E\dot{x}(t) = \sum\limits_{i=1}^{h} \rho_i(\theta(t)) \left[A_i x(t) + B_i u(t)\right. \\ \qquad\qquad \left. + A_{di}x(t-\tau_d) + F_i f(t) + Rd(t)\right], \\ y(t) = Cx(t), \end{cases} \tag{5}$$

where $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times p}$, $F_i \in \mathbb{R}^{n\times p}$, $R \in \mathbb{R}^{n\times q}$ and $C \in \mathbb{R}^{m\times n}$ are time invariant matrices defined for the $i$-th vertex of the polytope.

The concern of this paper is to seek a control law to ensure the closed-loop stability of the system (5) as well as actuator fault estimation. For this purpose, state and fault estimates will be used to minimize fault influence on system stability.

## 3. Design and analysis of active fault tolerant control for a delayed LPV descriptor system

In this section, a polytopic sliding mode observer for a delayed LPV descriptor system will be designed. The convergence of the unknown input observer will be analyzed. For instance, the following structural assumptions are required for the design of the observer:

**Assumption 1.** (*Yeu* et al.*, 2005*) We assume that

$$\operatorname{rank}(CF_i) = \operatorname{rank}(F_i) = p$$

and $p + q \le m, \forall i = 1, \dots, h$.

**Assumption 2.** (*Dai, 1989*) The matrix triple $(E, A_i, C)$ is R-observable, for all $i = 1, \dots, h$, i.e.,

$$\operatorname{rank} \begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n, \quad \forall s \in \mathcal{C}, \tag{6}$$

where $\mathcal{C}$ denotes the complex plane.

**Assumption 3.** (*Dai, 1989*) The matrix triple $(E, A_i, C)$ is impulse-observable, for all $i = 1, \dots, h$, i.e.,

$$\operatorname{rank} \begin{bmatrix} E & A_i \\ 0 & E \\ 0 & C \end{bmatrix} = n + \operatorname{rank}(E). \tag{7}$$

**Assumption 4.** (*Rodrigues* et al.*, 2015*) The fault $f(t)$ satisfies $\|f(t)\| \le \alpha_1$, where $\alpha_1$ is a non-negative real number.

**3.1. LPV sliding mode observer design.** Consider the following polytopic sliding mode observer (PSMO) with time delay:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \left[ N_i z(t) + N_{di} z(t-\tau_d) \right. \\ \qquad \left. + G_i u(t) + L_i y(t) + L_{di} y(t-\tau_d) + \Upsilon \vartheta(t) \right], \\ \hat{x}(t) = z(t) + T_2 y(t). \end{cases} \tag{8}$$

with the initial condition $z(\xi) = \phi(\xi)$, $\forall\ \xi \in [-\tau_d, 0]$, where $z(t) \in \mathbb{R}^n$ is the observer state vector, $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state vector and $\vartheta(t)$ represents a discontinuous switched component to induce a sliding motion (Edwards *et al.*, 2000). The gain matrix $\Upsilon \in \mathbb{R}^{n\times m}$ is the feedforward injection map. $N_i$, $N_{di}$, $G_i$, $L_i$, $L_{di}$ and $T_2$ are unknown matrices of appropriate dimensions to be determined. The feedforward compensation signal $\vartheta(t)$ is a discontinuous function such that (Yeu *et al.*, 2005)

$$\vartheta(t) = -\psi \frac{e_y(t)}{\|e_y(t)\|}, \quad e_y(t) \neq 0, \tag{9}$$

where $\psi$ is a positive real number and $e_y(t)$ is the output estimation error. The term $\vartheta(t))$ is designed to be discontinuous with respect to the sliding surface $S = \{e_y : Ce_x = 0\}$ to force the trajectories of $e_y(t)$ onto $S$ in finite time.

Let us define the following state estimation error $e_x(t)$ from (5) and (8):

$$e_x(t) = x(t) - \hat{x}(t) = (I_n - T_2 C)x(t) - z(t). \tag{10}$$

Moreover, for

$$\operatorname{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n$$

there exist nonsingular matrices $T_1 \in \mathbb{R}^{n\times n}$ and $T_2 \in \mathbb{R}^{n\times m}$ such that (Hamdi *et al.*, 2019)

$$T_1 E + T_2 C = I_n. \tag{11}$$

Consequently, the error dynamics are given by

$$\dot{e}_x(t) = T_1 E \dot{x}(t) - \dot{z}(t). \tag{12}$$

Then, the above error equation (12) is differentiable for $t > 0$, and the time-derivative satisfies the following differential-delay equation:

$$\begin{aligned} \dot{e}_x(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) \Big[ & N_i e_x(t) + N_{di} e_x(t-\tau_d) \\ & + (T_1 A_i - L_i C - N_i T_1 E)x(t) + T_1 R d(t) \\ & + (T_1 A_{di} - L_{di} C - N_{di} T_1 E)x(t-\tau_d) \\ & + (T_1 B_i - G_i)u(t) + T_1 F_i f(t) - \Upsilon\vartheta(t) \Big] \end{aligned} \tag{13}$$

if the following conditions are satisfied $\forall\, i = 1, \dots, h$:

$$T_1 A_i - L_i C - N_i T_1 E = 0, \tag{14}$$

$$T_1 A_{di} - L_{di} C - N_{di} T_1 E = 0, \tag{15}$$

$$T_1 B_i - G_i = 0, \tag{16}$$

$$T_1 R = 0. \tag{17}$$

The substitution of (11) into (14) and (15) yields

$$N_i = T_1 A_i + K_i C, \tag{18}$$

$$N_{di} = T_1 A_{di} + K_{di} C, \tag{19}$$

where

$$K_i = N_i T_2 - L_i, \tag{20}$$

$$K_{di} = N_{di} T_2 - L_{di}. \tag{21}$$

Then, estimation error dynamics can be minimized as

$$\begin{aligned} \dot{e}_x(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) [& N_i e_x(t) + N_{di} e_x(t-\tau_d) \\ & + T_1 F_i f(t) - \Upsilon\vartheta(t)]. \end{aligned} \tag{22}$$

Thereby, the dynamics of the output reconstruction error $e_y(t)$ are

$$\begin{aligned} \dot{e}_y(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) [& C N_i e_y(t) + C T_1 F_i f(t) \\ & + C N_{di} e_y(t-\tau_d) - C\Upsilon\vartheta(t)]. \end{aligned} \tag{23}$$

Up to this level, to design gain matrices $T_1$ and $T_2$ such that the constraints (11) and (17) are simultaneously satisfied, an augmented matrix equation composed of these tow constraints is as follows:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} E & R \\ C & 0 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix}. \tag{24}$$

A solution $\begin{bmatrix} T_1 & T_2 \end{bmatrix}$ exists if (Rodrigues *et al.*, 2015)

$$\operatorname{rank} \begin{bmatrix} E & R \\ C & 0 \end{bmatrix} = n + \operatorname{rank}(R) \tag{25}$$

Then, a particular solution of (24) using the generalized inverse matrix denoted by $(\cdot)^+$ is given by

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} E & R \\ C & 0 \end{bmatrix}^+. \tag{26}$$

Therefore, the equation of ideal sliding mode may be obtained from conditions such as $e_y(t) = 0$, $e_y(t-\tau_d) = 0$ and $\dot{e}_y(t) = 0$, $\dot{e}_y(t-\tau_d) = 0$. In (23), $\vartheta(t)$ acts as an input signal. Therefore, the virtual equivalent feedforward signal (Yeu *et al.*, 2005) may be described for each $C\Upsilon \neq 0$ as

$$\vartheta(t) = \sum_{i=1}^{h} \rho_i(\theta(t)) (C\Upsilon)^{-1} [C T_1 F_i f(t)]. \tag{27}$$

Consider the feedforward compensation signal (9), where $\psi$ is a positive real number with

$$\psi \geq \frac{\alpha_1}{C\Upsilon}\|CT_1\|\,\|\sum_{i=1}^{h}\rho_i(\theta(t))F_i\|. \quad (28)$$

Up to this level, it is worth determining the matrix $\Upsilon$ and the signal $\vartheta(t)$ such that the stability of the LPV sliding mode observer with the time delay (8) is preserved. To study stability, $\Upsilon$ and $\vartheta(t)$ can be interpreted as the control input distribution and the input of the reconstruction error system, respectively.

### 3.1.1. Stability analysis.

**Theorem 1.** *Given scalars $\tau > 0$, $\sigma$ $\gamma$ and $0 \leq \beta \leq 1$, the exponential stability of the estimation error system (22) for any time-varying delay $\tau_d$ such that $0 \leq \tau_d \leq \tau$, $0 \leq \dot{\tau}_d \leq \beta < 1$ is guaranteed in the mean square sense, if there exist symmetric positive definite matrices $Q, X$ and matrices $W_i = QK_i$, $W_{di} = QK_{di}$ such that, $\forall\, i \in [1, \ldots, h]$, the following LMI holds:*

$$\begin{bmatrix} \Delta_i & * & * & * \\ (T_1F_i)^TQ & -\frac{1}{\beta}I_p & * & * \\ N_{di}^TQ & 0 & X_{\tau d} & * \\ -\Upsilon^TQ & 0 & 0 & -I_m \end{bmatrix} < \sigma I, \quad (29)$$

*where*

$$\Delta_i = (T_1A_i)^TQ + Q(T_1A_i) + (W_iC)^T + (W_iC) + \gamma^2 I_n,$$

$$N_{di}^TQ = (T_1A_{di})^TQ + C^TW_{di}^T.$$

*Proof.* Let $V(e_x(t))$ be a Lyapunov−Krasovskii functional of the following form:

$$V(e_x(t)) = e_x^T(t)Qe_x(t) + \int_{t-\tau_d}^{t} e_x^T(s)Xe_x(s)\,\mathrm{d}s, \quad (30)$$

where $Q > 0$ and $X > 0$. By differentiating $V(e_x(t))$ along the trajectory of Eqn. (22), we obtain

$$\begin{aligned}\dot{V}(e_x(t), \vartheta(t))X &= \sum_{i=1}^{h}\rho_i(\theta(t))\{e_x^T(t)[e_x^T(t-\tau_d)N_{di}^TQe_x(t) \\ &\quad + N_i^TQ + QN_i + X]e_x(t) \\ &\quad - (1-\dot{\tau}_d)e_x^T(t-\tau_d)Xe_x(t-\tau_d) \\ &\quad + e_x^T(t)QN_{di}e_x(t-\tau_d) \\ &\quad + 2e_x^T(t)QT_1F_if(t) - 2e_x^T(t)Q\Upsilon\vartheta(t)\}.\end{aligned} \quad (31)$$

Using the fact that $\dot{\tau}_d \leq \beta < 1$ and writing $X_{\tau_d} = (1-\beta)X$, we obtain

$$\begin{aligned}\dot{V}(e(t)) \leq &\sum_{i=1}^{h}\rho_i(\theta(t))\{e_x^T(t-\tau_d)N_{di}^TQe_x(t) \\ &+ e_x^T(t)[+N_i^TQ + QN_i + X]e_x(t) \\ &+ e_x^T(t)QN_{di}e_x(t-\tau_d) \\ &- e_x^T(t-\tau_d)X_{\tau_d}e_x(t-\tau_d) \\ &+ 2e_x^T(t)QT_1F_if(t) - 2e_x^T(t)Q\Upsilon\vartheta(t)\}.\end{aligned} \quad (32)$$

If $\|f(t)\| \leq \alpha_1$, then

$$\begin{aligned}&\dot{V}(e(t)) \\ &\leq \sum_{i=1}^{h}\rho_i(\theta(t))\{e_x^T(t-\tau_d)N_{di}^TQe_x(t) \\ &\quad + e_x^T(t)[N_i^TQ + QN_i + X]e_x(t) \\ &\quad + e_x^T(t)QN_{di}e_x(t-\tau_d) + 2\alpha_1\left\|e_x^T(t)QT_1F_i\right\| \\ &\quad - e_x^T(t-\tau_d)X_{\tau_d}e_x(t-\tau_d) - 2e_x^T(t)Q\Upsilon\vartheta(t)\}.\end{aligned} \quad (33)$$

For any positive scalar $\sigma$, the following inequality could be written:

$$2\alpha_1\left\|e^T(t)Q(T_1F_i)\right\| \leq \sigma^{-1}\alpha_1^2\left\|e^T(t)Q(T_1F_i)\right\|^2 + \sigma.$$

It follows that

$$\begin{aligned}\dot{V}(e(t)) \leq &\sum_{i=1}^{h}\rho_i(\theta(t))\{e_x^T(t-\tau_d)N_{di}^TQe_x(t) \\ &+ e_x^T(t)[N_i^TQ + QN_i + X]e_x(t) \\ &- \vartheta^T(t)\Upsilon^TQe_x(t) \\ &+ e_x^T(t)QN_{di}e_x(t-\tau_d) \\ &+ \sigma - e_x^T(t)Q\Upsilon\vartheta(t) \\ &- e_x^T(t-\tau_d)X_{\tau_d}e_x(t-\tau_d) \\ &+ \sigma^{-1}\alpha_1^2\left\|e_x^T(t)QT_1F_i\right\|^2\}.\end{aligned} \quad (34)$$

By taking $\beta = \sigma^{-1}\alpha_1^2$, we can obtain

$$\begin{aligned}\dot{V}(e(t)) \leq &\sum_{i=1}^{h}\rho_i(\theta(t))\{e_x^T(t-\tau_d)N_{di}^TQe_x(t) \\ &+ e_x^T(t)[N_i^TQ + QN_i + X]e_x(t) \\ &- e_x^T(t)Q\Upsilon\vartheta(t) \\ &+ e_x^T(t)QN_{di}e_x(t-\tau_d) \\ &- e_x^T(t-\tau_d)X_{\tau_d}e_x(t-\tau_d) \\ &- \vartheta^T(t)\Upsilon^TQe_x(t) + \sigma \\ &+ \beta e_x^T(t)QT_1F_i(T_1F_i)^TQe_x(t)\}.\end{aligned} \quad (35)$$

For $||\vartheta(t)|| \leq \gamma\, ||e_x(t)||$, the state estimation error converges asymptotically to zero and the gain from $\vartheta(t)$ to $e_x(t)$ is bounded by $\gamma$ if

$$\dot{V}(e_x(t)) - \vartheta^T(t)\vartheta(t) + \gamma^2 e_x^T(t)e_x(t) < 0.$$

Then

$$\begin{aligned}
&\dot{V}(e(t)) \\
&\leq \sum_{i=1}^{h} \rho_i(\theta(t))\{e_x^T(t-\tau_d)N_{di}^T Q e_x(t) \\
&\quad + e_x^T(t)[N_i^T Q + QN_i + X]e_x(t) - \vartheta^T(t)\vartheta(t) \\
&\quad + e_x^T(t)QN_{di}e_x(t-\tau_d) - \vartheta^T(t)\Upsilon^T Q e_x(t) \\
&\quad - e_x^T(t-\tau_d)X_{\tau_d}e_x(t-\tau_d) + \sigma - e_x^T(t)Q\Upsilon\vartheta(t) \\
&\quad + \beta e_x^T(t)QT_1F_i(T_1F_i)^T Q e_x(t) \\
&\quad + \gamma^2 e_x^T(t)e_x(t)\} < 0.
\end{aligned} \tag{36}$$

The previous inequality can be written as $\forall\, i = 1, \dots, h$:

$$\dot{V}(e_x(t)) < \sum_{i=1}^{h} \rho_i(\theta(t))\bar{e}^T(t)\Xi_i\bar{e}(t) + \sigma\} < 0, \tag{37}$$

with

$$\bar{e}(t) = \begin{bmatrix} e_x(t) \\ e(t-\tau_d) \\ \vartheta(t) \end{bmatrix},$$

$$\Xi_i = \begin{bmatrix} \Pi_i & * & * \\ N_{di}^T Q & X_{\tau d} & 0 \\ -\Upsilon^T Q & 0 & -I_m \end{bmatrix},$$

where

$$\begin{aligned}
\Pi_i = {} & N_i^T Q + Q(N_i) + X \\
& + \beta Q(T_1F_i)(T_1F_i)^T Q + \gamma^2 I_n.
\end{aligned}$$

Then, for $\sum_{i=1}^{h} \rho_i(\theta(t)) = 1$ and $\rho_i(\theta(t)) \geq 0$, $\dot{V}(e_x(t)) < 0$ if $\forall\, i \in [1, \dots, h]$

$$\Xi_i < -\sigma I. \tag{38}$$

To obtain an equivalent constraint LMI, we assume $W_i = QK_i$ and $W_{di} = QK_{di}$. Based on (11), (14), (15) and by using the Schur complement, the inequality (38) becomes

$$\begin{bmatrix} \Delta_i & * & * & * \\ (T_1F_i)^T Q & -\frac{1}{\beta}I_p & * & * \\ N_{di}^T Q & 0 & X_{\tau d} & * \\ -\Upsilon^T Q & 0 & 0 & -I_m \end{bmatrix} < -\sigma I, \tag{39}$$

where

$$\begin{aligned}
\Delta_i = {} & (T_1A_i)^T Q + Q(T_1A_i) + (W_iC)^T \\
& + (W_iC) + \gamma^2 I_n
\end{aligned}$$

$$N_{di}^T Q = (T_1A_{di})^T Q + C^T W_{di}^T,$$

which means that $e_x(t)$ and $e(t-\tau_d)$ converge to zeros according to the Lyapunov–Krasovskii stability theory in the presence of a fault and signal $\vartheta(t)$. By choosing $\Upsilon = Q^{-1}C^T$ (Yeu *et al.*, 2005), the inequality (39) being linear in $Q$ and $W_i$, it can be solved via a numerical approach within the LMI framework. ∎

**3.1.2. Actuator fault reconstruction.** Estimating actuator faults is based on the equivalent output injection concept. Then, for the sliding motion, $e_y(t) = 0$ and $\dot{e}_y(t) = 0$ hold and under the stability condition of the error dynamics, Eqn. (23) can be rewritten as

$$0 = \sum_{i=1}^{h} \rho_i(\theta(t))\big[CT_1F_if(t) - C\Upsilon\vartheta(t)\big], \tag{40}$$

which is equivalent to

$$\sum_{i=1}^{h} \rho_i(\theta(t))\big[CT_1F_if(t)\big] = \sum_{i=1}^{h} \rho_i(\theta(t))\big[C\Upsilon\vartheta(t)\big]. \tag{41}$$

An alternative approach (Edwards *et al.*, 2000) is to replace the discontinuous component $\vartheta(t)$ in (9) by the continuous approximation

$$\vartheta_\delta(t) = -\psi\frac{e_y(t)}{\|e_y(t)\| + \delta}, \tag{42}$$

where $\delta$ is a small positive scalar chosen to remove chattering in the sliding motion. By substituting (28) in (42) we obtain

$$\vartheta_\delta(t) \approx -\sum_{i=1}^{h} \rho_i(\theta(t))\alpha_1\frac{\|T_1\|\,\|F_i\|}{\|\Upsilon\|}\frac{e_y(t)}{\|e_y(t)\| + \delta}. \tag{43}$$

Since $\mathrm{rank}(CT_1F_i) = p$, it follows from (41) that actuator faults can be reconstructed by

$$\hat{f}(t) \approx \sum_{i=1}^{h} \rho_i(\theta(t))(CT_1F_i)^+ C\Upsilon\vartheta_\delta(t). \tag{44}$$

**3.2. Design of fault tolerant controllers based on the PSMO with time delay.** In this part, fault tolerant controllers (FTCs) will be synthesized by using reconstructed fault information provided by the PSMO with time delay such that the closed-loop polytopic descriptor time delay system (5) is stable. Foremost, the following assumption is made.

**Assumption 5.** We assume that

$$\mathrm{rank}(B_iF_i) = \mathrm{rank}(B_i),\ \forall i = 1, \dots, h.$$

Assumption 5 provides controllable conditions for the described control systems, and Assumption 4 guarantees the actuator fault $f(t)$ to be constrained in a given compensation range.

**Lemma 1.** (Tabatabaeipour and Bak, 2013) *Under Assumption 5, there exist a matrix* $B_i^{\ddagger} \in \mathbb{R}^{p\times n}$ *such that*

$$(I - B_i B_i^{\ddagger})F_i = 0. \tag{45}$$

Afterwards, in order to compensate for the fault effect on the delayed polytopic descriptor system (1) such that the faulty system is as close as possible to the pre-fault system, a state-feedback based FTC law is constructed as follows:

$$u(t) = u_n(t) + u_f(t), \tag{46}$$

where $u_n(t)$ is the state-feedback controller in a fault-free condition, and $u_f(t)$ is an additional controller, which is used to compensate the effect of the fault. Moreover,

$$u_n(t) = -\sum_{i=1}^{h} \rho_i(\theta(t))\Gamma_i \hat{x}(t), \tag{47}$$

$$u_f(t) = -\sum_{i=1}^{h} \rho_i(\theta(t)) B_i^{\ddagger} F_i \hat{f}(t), \tag{48}$$

where $\Gamma_i \in \mathbb{R}^{p\times n}, \quad \forall i = 1,\dots,h$ are the feedback gains matrices to be found.

Then, substituting (46) into (5), we obtain

$$\begin{aligned}
&E\dot{x}(t)\\
&= \sum_{i=1}^{h} \rho_i(\theta(t)) \big[A_i x(t) + A_{di}x(t-\tau_d) + F_i f(t)\\
&\quad - B_i\Gamma_i\hat{x}(t) - B_i B_i^{\ddagger} F_i \hat{f}(t) + Rd(t)\big]\\
&= \sum_{i=1}^{h} \rho_i(\theta(t)) \big[A_i x(t) + A_{di}x(t-\tau_d) - B_i\Gamma_i\hat{x}(t)\\
&\quad - F_i\hat{f}(t) + F_i f(t) + Rd(t)\big]\\
&= \sum_{i=1}^{h} \rho_i(\theta(t)) \left[(A_i - B_i\Gamma_i)x(t) + A_{di}x(t-\tau_d)\right.\\
&\quad \left.+ B_i\Gamma_i e_x(t)) + F_i e_f(t) + Rd(t)\right]\\
&= \sum_{i=1}^{h} \rho_i(\theta(t)) \left[(A_i - B_i\Gamma_i)x(t) + A_{di}x(t-\tau_d)\right.\\
&\quad \left.+ \varpi(t)\right],
\end{aligned} \tag{49}$$

where $\varpi(t) = B_i\Gamma_i e_x(t)) + F_i e_f(t) + Rd(t)$ and $e_f(t) = f(t) - \hat{f}(t)$. Here $\varpi(t)$ can be considered an external disturbance, and the boundedness of $e_x(t)$ and $e_f(t)$ can be guaranteed based on Theorem 1 and Assumption 4, respectively. Accordingly, to design feedback gain matrices $\Gamma_i \in \mathbb{R}^{p\times n}, \quad \forall i = 1,\dots,h$ such that the closed-loop LPV delayed descriptor system (49) is stable, the following theorem is provided.

**Theorem 2.** *Suppose that Theorem 1 holds. The closed-loop LPV delayed descriptor system (49) is stable if there exist positive definite symmetric matrices* $X_1$, $P_1$ *and* $Q_2$ *such that* $\forall\, i \in [1,\dots,h]$, *the following LMI holds:*

$$X_1^T E^T = E X_1 \geq 0, \tag{50}$$

$$\begin{bmatrix} \Psi_i & X_1 A_{di} \\ A_{di}^T X_1 & -X_{1\tau d} \end{bmatrix} < 0, \tag{51}$$

*where*

$$\Psi_i = X_1 A_i{}^T + A_i X_1 - U_i^T B_i{}^T - B_i U_i + Q_2 + P_1$$

*and*

$$U_i = \Gamma_i X_1$$

*Proof.* Let $V_1(x(t))$ be a Lyapunov–Krasovskii functional of the following form:

$$\begin{aligned}
V_1(x(t)) &= x^T(t)E^T X_1 e_x(t)\\
&\quad + \int_{t-\tau_d}^{t} x^T(s) Q_2 x(s)\,\mathrm{d}s,
\end{aligned} \tag{52}$$

where $Q_2 > 0$ and $X_1 > 0$. By differentiating $V_1(x(t))$ along the last equation of (49), we obtain

$$\begin{aligned}
&\dot{V}_1(x(t))\\
&= \sum_{i=1}^{h} \rho_i(\theta(t))\{x^T(t)[X_1(A_i - B_i\Gamma_i)^T\\
&\quad + (A_i - B_i\Gamma_i)X_1 + Q_2]x(t)\\
&\quad + x^T(t-\tau_d)A_{di}^T X_1 x(t)\\
&\quad + x^T(t)X_1 A_{di}x(t-\tau_d) + 2x^T(t)X_1\varpi(t)\\
&\quad - (1-\dot{\tau}_d)x^T(t-\tau_d)Q_2 x(t-\tau_d)\}.
\end{aligned} \tag{53}$$

According to the proof of Theorem 1, by using the fact that $\dot{\tau}_d \leq \beta < 1$ and by noting $X_{1\tau_d} = (1-\beta)X_1$, (53) becomes

$$\begin{aligned}
&\dot{V}_1(x(t))\\
&\leq \sum_{i=1}^{h} \rho_i(\theta(t))\{x^T(t)[X_1(A_i - B_i\Gamma_i)^T\\
&\quad + (A_i - B_i\Gamma_i)X_1 + Q_2]x(t)\\
&\quad + x^T(t-\tau_d)A_{di}^T X_1 x(t)\\
&\quad + x^T(t)X_1 A_{di}x(t-\tau_d)\\
&\quad - x^T(t-\tau_d)X_{1\tau_d}x(t-\tau_d)\\
&\quad + 2x^T(t)X_1\varpi(t)\}.
\end{aligned} \tag{54}$$

For a symmetric positive definite matrix $P_1$, we have (Zhang *et al.*, 2009)

$$2x^T(t)X_1\varpi(t) \leq x^T(t)P_1x(t) + \delta_1, \tag{55}$$

where $\delta_1 = \|\varpi(t)\|^2 \lambda_{\max}(X_1^T P_1^{-1} X_1)$. The yields

$$\begin{aligned}
&\dot{V}_1(x(t)) \\
&\quad < \sum_{i=1}^{h} \rho_i(\theta(t))\{x^T(t)[X_1(A_i - B_i\Gamma_i)^T \\
&\qquad + (A_i - B_i\Gamma_i)X_1 + Q_2]x(t) \\
&\qquad + x^T(t-\tau_d)A_{di}^T X_1 x(t) \\
&\qquad + x^T(t)X_1 A_{di} x(t-\tau_d) \\
&\qquad - x^T(t-\tau_d)X_{1\tau_d} x(t-\tau_d) \\
&\qquad + x^T(t)P_1 x(t) + \delta_1\}
\end{aligned} \tag{56}$$

which leads to the following expression:

$$\dot{V}_1(x(t)) < \sum_{i=1}^{h} \rho_i(\theta(t))\zeta^T(t)\Lambda_i\zeta(t) + \delta_1\}, \tag{57}$$

with

$$\Lambda_i = \begin{bmatrix} \Psi_i & X_1 A_{di} \\ A_{di}^T X_1 & -X_{1\tau d} \end{bmatrix},$$

$$\zeta(t) = \begin{bmatrix} x(t) \\ x(t-\tau_d \end{bmatrix},$$

where

$$\Psi_i = X_1 {A_i}^T + A_i X_1 - U_i^T {B_i}^T - B_i U_i + Q_2 + P_1$$

with $U_i = \Gamma_i X_1$.

If $\Lambda_i < 0$, then there exists a scalar $\varepsilon$ such that $\dot{V}_1(x(t)) < -\varepsilon\|\zeta(t)\|^2 + \delta_1$. It follows that $\dot{V}_1(x(t)) < 0$ for $\varepsilon\|\zeta(t)\|^2 > \delta_1$, which means that $\zeta(t)$ converges to a small set $\xi = \left\{\zeta(t)/\|\zeta\|^2 \le \delta_1/\varepsilon\right\}$ according to the Lyapunov–Krasovskii stability theory. Moreover, controller gain matrices can be computed as $\Gamma_i = U_i X_1^{-1}$, $\forall\, i \in [1,\dots,h]$ ∎

## 4. Simulation example

Consider the time delay system characterized by the following LPV descriptor model:

$$\begin{cases} E\dot{x}(t) = A(\theta(t))x(t) + A_d(\theta(t))x(t-\tau_d) \\ \qquad\quad + B(\theta(t))u(t) + F(\theta(t))f(t) + Rd(t), \\ y(t) = Cx(t), \end{cases} \tag{58}$$

such that

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

$$A(\theta(t)) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & -\theta_1(t)+1 \\ 1 & 1 & -2+\theta_2(t) & 0 \\ -1 & -1 & 1 & -5 \end{bmatrix},$$

$$A_d(\theta(t)) = \begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3-\theta_1(t) & 0 \\ 0 & 0 & 0 & -\theta_2(t)-2 \end{bmatrix},$$

$$F(\theta(t)) = B(\theta(t)) = \begin{bmatrix} 1 \\ 0.5+\theta_1(t) \\ 0 \\ 0 \end{bmatrix},$$

$$R = \begin{bmatrix} 0 \\ 1.2 \\ 0.8 \\ 0 \end{bmatrix},$$

where the parameter varying vector is given by $\theta(t) = \begin{bmatrix} \theta_1(t) & \theta_2(t) \end{bmatrix}^T$. These time-varying parameters vary according to

$$\theta_1(t) \in \begin{bmatrix} -0.7, & 0.5 \end{bmatrix}, \quad \theta_2(t) \in \begin{bmatrix} -1.2, & 1 \end{bmatrix}.$$

Since there are two parameters, four subsystems are defined, each of them at the vertices of the parameter space. Then, the LPV descriptor model with time delay $\tau_d$, a disturbance $d(t)$ and an additive actuator fault signal $f(t)$ is defined as follows:

$$\begin{cases} E\dot{x}(t) = \sum\limits_{i=1}^{4} \rho_i(\theta(t))\,[A_i x(t) + A_{di}x(t-\tau_d) \\ \qquad\qquad + B_i u(t) + F_i f(t) + Rd(t)], \\ y(t) = Cx(t). \end{cases} \tag{59}$$

The weighting functions $\rho_i(\theta(t))$ are defined as combinations of $\theta_j$, $\forall\, j = 1, 2$ (Rodrigues *et al.*, 2015) as follows:

$$\rho_1(\theta(t)) = \frac{\theta_1(t) - \underline{\theta}_1}{\bar{\theta}_1 - \underline{\theta}_1}\frac{\theta_2(t) - \underline{\theta}_2}{\bar{\theta}_2 - \underline{\theta}_2},$$

$$\rho_2(\theta(t)) = \frac{\theta_1(t) - \underline{\theta}_1}{\bar{\theta}_1 - \underline{\theta}_1}\frac{\bar{\theta}_2 - \theta_2(t)}{\bar{\theta}_2 - \underline{\theta}_2},$$

$$\rho_3(\theta(t)) = \frac{\bar{\theta}_1 - \theta_1(t)}{\bar{\theta}_1 - \underline{\theta}_1}\frac{\theta_2(t) - \underline{\theta}_2}{\bar{\theta}_2 - \underline{\theta}_2},$$

$$\rho_4(\theta(t)) = \frac{\bar{\theta}_1 - \theta_1(t)}{\bar{\theta}_1 - \underline{\theta}_1}\frac{\bar{\theta}_2 - \theta_2(t)}{\bar{\theta}_2 - \underline{\theta}_2}.$$

The input data $u(t)$ are depicted as shown in Fig. 1, the input $d(t)$ is a Gaussian distributed random signal with zero mean and unit variance and the additive actuator fault signals affecting the polytopic LPV descriptor system with time delay are defined by

$$f(t) = \begin{cases} -0.045t + 1.125, & \forall t \in \,[10,\, 20s]\,, \\ 0.9(1+\sin(0.1t)), & \forall t \in \,[30,\, 45s]\,. \end{cases}$$

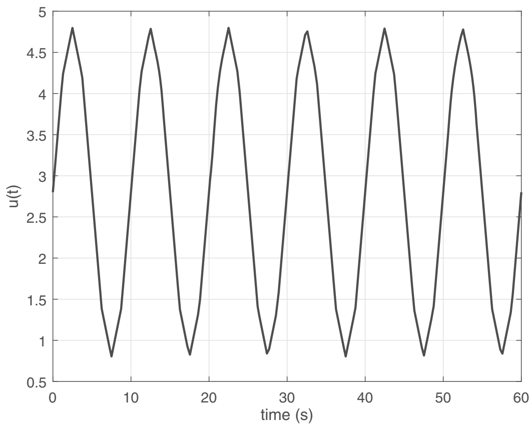


Fig. 1. Input $u(t)$.

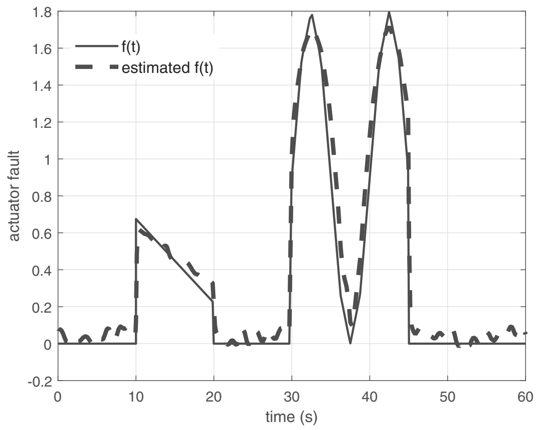


Fig. 2. Actuator fault $f(t)$ and its estimate $\hat{f}(t)$.

**4.1. Delayed PSMO design.** The proposed delayed PSMO (8) is designed by solving the LMIs from Theorem 1 and the controller gain matrices $\Gamma_i$ are computed by solving the LMIs from Theorem 2. The parameter values are chosen as $\delta = 0.1, \beta = 0.5, \sigma = 0.1$. For $\Upsilon = Q^{-1}C$, the positive definite symmetric matrix $Q$ and gain matrices of the proposed observer are calculated by using the MATLAB LMI toolbox. The simulation result for the actuator fault estimation of the delayed LPV descriptor is depicted as shown in Fig. 2.

Figure 2 highlights the ability of the sliding mode polytopic observer with time delay to provide good estimation of actuator faults despite the presence of disturbance. It can be noticed that the incipient fault that occurred in the time interval $[10\,\mathrm{s}, 20\,\mathrm{s}]$ can be detected and isolated directly based on its estimated values as they start to decrease in the system. Besides, then varying actuator fault can be precisely reconstructed.

**4.2. Fault tolerant controller design.** By solving the inequalities (50) and (51) in Theorem 2 using the MATLAB LMI toolbox, the following feedback gain matrices $\Gamma_i, \quad \forall i = 1, \dots, h$ are obtained:

$$\Gamma_1 = \Gamma_2 = \begin{bmatrix} 45.23 & -24.63 & -14.66 & -14.81 \end{bmatrix},$$

$$\Gamma_3 = \Gamma_4 = \begin{bmatrix} 42.34 & -21.95 & -13.73 & -14.29 \end{bmatrix}.$$

Based on the real-time information and the reconstructed time-varying actuator fault, system output responses with the feedback FTC are shown in Figs. 3–5.

Figures 3–5 illustrate a comparison of the responses of the LPV delayed descriptor model without an actuator fault, the responses of the faulty system without FTC, and finally its responses in faulty case with a state-feedback based FTC law. Undoubtedly, the proposed strategy is robust concerning the actuator additive fault $f(t)$. It can be observed that the feedback FTC scheme provides good results in the presence of time delays and external disturbances. It achieves also more accurate reconstruction of time-varying actuator faults.

Furthermore, simulation results prove that the proposed PSMO based fault reconstruction and FTC schemes are effective for polytopic descriptor systems subject to time delay and external disturbances. Another benefit is the simplicity of gain matrices calculation based on Theorems 1 and 2, which makes it possible for the PSMO when it comes to the practical implementation. Finally and despite the presence of time delay and actuator faults which make system unstable, active FTC for LPV descriptor systems with time delay based on the PSMO proposed in this paper, allows us controlling the system safely. This presented method allows to both estimate the fault and to synthesize an FTC control law for time-delay LPV descriptor systems, which has not been done before to the best of our knowledge.

## 5. Conclusion

This paper studied the problem of fault estimation and fault tolerant control for LPV descriptor systems with time delay described by a polytopic model. The intended active fault tolerant control requires simultaneous estimations of the state and fault, obtained by a designed polytopic sliding mode observer. This observer can simultaneously reconstruct time varying actuator faults and state variables as accurately as possible. The stability analysis made with the Lyapunov–Krasovskii theory was performed in the case of a delayed polytopic sliding mode observer and feedback fault tolerant control law design. Sufficient stability conditions were given in terms of LMIs. Finally, an application to an LPV descriptor system subject to time delays and external disturbances with an additive actuator fault was presented to illustrate the effectiveness of the proposed approaches.

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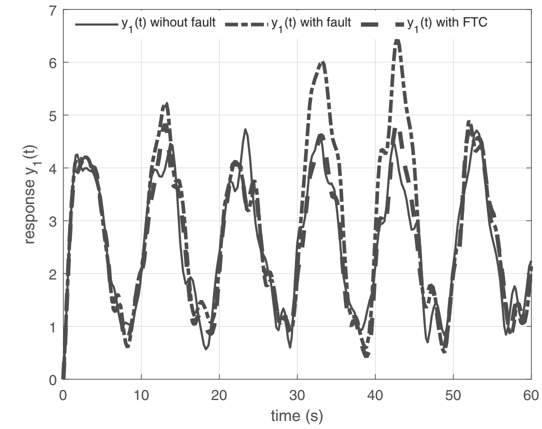


Fig. 3. Comparison of output $y_1(t)$ in the following cases: without a fault, with a fault without FTC, with a fault with FTC.

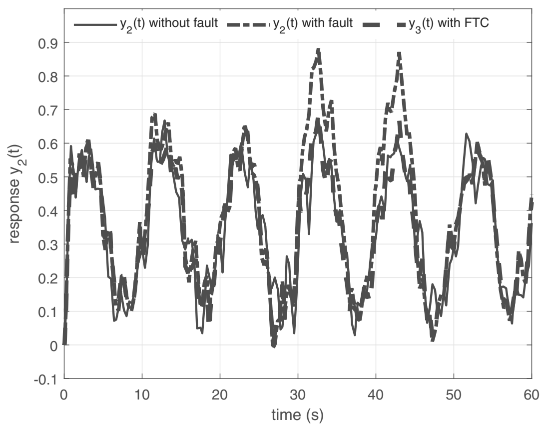


Fig. 4. Comparison of output $y_2(t)$ in the following cases: without a fault, with a fault without FTC, with a fault with FTC.

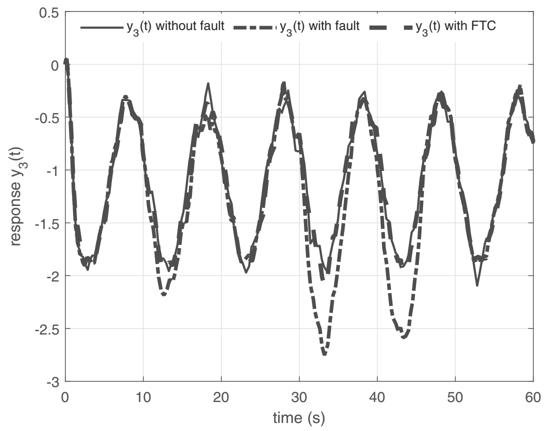


Fig. 5. Comparison of output $y_3(t)$ in the following cases: without a fault, with a fault without FTC, with a fault with FTC.

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