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SELF-TUNING GENERALIZED PREDICTIVE CONTROL WITH INPUT CONSTRAINTS

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The handling of various input constraints in the self-tuning generalized predictive control (STGPC) problem of ARIMAX/ARMAX systems is considered. The methods based on the Lagrange multipliers and Lemke's algorithm are used to solve the constrained optimization problem. A self-tuning controller is implemented in an indirect way, and the considered constraints imposed on the control input signal are of the rate, amplitude and energy types. A comparative simulation study of self-tuning control system behaviour is given with respect to the design parameters and constraints. The stability of a closed-loop control system is analyzed and the computational loads of both the methods are compared.

Keywords: generalized predictive control, constraints, self-tuning, ARIMAX/ARMAX systems

1. Introduction

Predictive control is popular in academic research and industry for its simplicity and succesful industrial applications.

Constraints of different kind are ubiquitous in control engineering applications, therefore the way of handling them in control system design is an important question. However, this does not often happen in the design of control algorithms proposed in the literature. Disregarding constraints or imposing them on the control signal in a heuristic way may cause performance deterioration or even instability, especially in adaptive control of unstable systems.

Taking constraints into account in the design stage inherently leads to the solution of a constrained optimization problem. It is well-known that quadratic programming (QP) techniques can be applied to solve miscelaneous types of predictive control problems under constraints.

The generalized predictive control (GPC) considered in this paper is perhaps, apart from the dynamic matrix control (DMC), the most succesful representative amongst predictive control proposals. The application of the QP to solve the GPC problem is widely used, see, e.g. the comments given in (Kothare *et al.*, 1996; Rossiter and Kouvaritakis, 1993). The constrained GPC was also discussed in (Camacho,

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1993; Camacho and Bordons, 1995), where the QP problem was transformed into the so-called *Linear Complementarity Problem* (LCP) which, in turn, was solved using Lemke's algorithm. This reduced the amount of computations as compared to the QP. Another attempt to reduce the computational burden was presented in (Tsang and Clarke, 1988), where the Lagrange multipliers (LM) were used to handle separately rate and amplitude constraints. As an alternative to the QP, a method based on a modification of Lawson's weighted least-squares algorithm was proposed in (Rossiter and Kouvaritakis, 1993). An algorithm for selecting the control weighting so that unconstrained GPC over the control horizon satisfies the rate and amplitude constraints was derived in (Lee *et al.*, 1997). Another approach to solve the constrained optimization problem involves *Linear Matrix Inequalities* (LMI) (Kothare *et al.*, 1996). In (Soufian *et al.*, 1997), an interesting approach based on the dynamic programming was proposed to solve the constrained model predictive control. The desaturating approach to adaptive receding-horizon predictive control in the case of simultaneous amplitude and rate constraints is presented in (De Nicolao *et al.*, 1996).

In recent years some research has been done on the predictive control of stochastic systems containing parametric uncertainties. Here, as in all adaptive systems, it is necessary to combine both the facets of the adaptive controller, i.e. identification and control algorithms, in order to obtain a proper interplay between them resulting in a robust performance of the adaptive controller.

In this paper, the constrained STGPC for a discrete-time stochastic system of ARIMAX/ARMAX structure with unknown but constant parameters is considered. For the indirect adaptive controller considered here, the controller parameters are tuned on the basis of system parameter estimates along with the *Certainty Equivalence Principle*, and the rate, amplitude and energy constraints are assumed to be imposed on the control input. To solve the constrained optimization problem, the concepts proposed in (Camacho, 1993; Tsang and Clarke, 1988) are adopted for simulation comparison and computational analysis.

Investigation of closed-loop stability and performance properties for the STGPC with input constraints is difficult and analytically unfeasible, especially for unstable open-loop systems. Consequently, the objective of this paper is to present a simulation-based comparison of these properties with respect to control design parameters and constraints. A comparison of computational loads for both the methods is also drawn. Stable, unstable and non-minimum phase systems of second order are taken for the simulation study.

2. Standard GPC

The standard unconstrained GPC problem of ARIMAX/ARMAX systems will be first shortly characterized. An ARMAX model is given by

$$A(q^{-1})y_t = q^{-1}B(q^{-1})u_t + C(q^{-1})e_t, \tag{1}$$

where A, B and C are polynomials in the backward shift operator q^{-1}, i.e.

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na},$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_{nb} q^{-nb},$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{nc} q^{-nc},$$

while y_t is the output, u_t denotes the control input, and e_t is assumed to be a zero-mean white noise with variance σ_e^2.

In some applications it is more preferable to use the ARIMAX model given by

$$A(q^{-1}) y_t = q^{-1} B(q^{-1}) u_t + \frac{C(q^{-1})}{\Delta} e_t, \tag{2}$$

where $\Delta = 1 - q^{-1}$.

The GPC cost function is taken in the form

$$J(N_y, N_u, q_u) = E\left[\sum_{i=1}^{N_y} (y_{t+i} - r_{t+i})^2 + q_u \sum_{i=1}^{N_u} u_{t+i-1}^{*2}\right], \tag{3}$$

where the weight $q_u \geq 0$ and the horizons N_y, N_u are basic design parameters of GPC. The notation u_t^* signifies u_t for positional control based on an ARMAX model or Δu_t for incremental control when an ARIMAX model is assumed. Usually, a choice $N_u \leq N_y$ is made, which means $u_{t+i}^* = 0$ for $i \geq N_u$.

The goal of the GPC is to force the output y_t to follow the reference signal r_t taking into account the control effort. It is well-known that (3) can be expressed as

$$J(N_y, N_u, q_u) = (G\bar{u} + f - r)^T (G\bar{u} + f - r) + q_u \bar{u}^T \bar{u}, \tag{4}$$

where the matrix G is composed of the impulse response coefficients, $\{g_i\}$, of the control channel $B/A\Delta$ in the case of the ARIMAX model

$$G = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_{N_y-1} & g_{N_y-2} & \cdots & g_{N_y-N_u} \end{bmatrix},$$

and the vectors f and r are

$$f = \left(f_{t+1}, \cdots, f_{t+N_y}\right)^T,$$

$$r = \left(r_{t+1}, \cdots, r_{t+N_y}\right)^T.$$

The unconstrained optimal control is then (Bitmead *et al.*, 1990; Camacho and Bordons, 1995; Tsang and Clarke, 1988)

$$\bar{u}^o = \left(G^T G + q_u I\right)^{-1} G^T (r - f), \tag{5}$$

where

$$\bar{u}^o = \left(\Delta u_t^o, \ldots, \Delta u_{t+N_u-1}^o\right)^T, \tag{6}$$

and the free response $f_{t+i} = \hat{y}_{t+i/t}$, where $\hat{y}_{t+i/t}$ is the prediction of y_{t+i}, $i = 1, \dots, N_y$, assuming that $u_{t+i} = 0$, $i = 0, \dots, N_y - 1$. The first element of the sequence (6), i.e. Δu_t^o, is applied to the system. Then the optimization procedure starts again at the next time instant $t + 1$ with the current data. In the non-adaptive case, this means that in the control law (5) only the vectors f and r should be updated. However, in the self-tuning implementation the system parameters $\theta = (a_1, \dots, a_{na}, b_0, \dots, b_{nb}, c_1, \dots, c_{nc})^T$ should be updated as well.

The GPC controller for an ARMAX system needs some modifications to the two polynomial partitions in the standard derivation (Bitmead *et al.*, 1990). In the first identity associated with the prediction partition, the polynomial A must be used, not $A\Delta$ as in the ARIMAX case. In the second identity concerning the control variables separation, the polynomials $G_i = \sum_{j=0}^{i-1} g_j q^{-j}$ appearing in the ARIMAX derivation must be replaced by $G_i\Delta$ for the ARMAX case. Thus, in the GPC for an ARMAX system, the coefficients g_i are the impulse response parameters of the transfer function B/A.

3. GPC Subject to Constraints

Now, the GPC in the presence of input rate, amplitude and energy constraints will be examined. As already mentioned, QP techniques are reported to be computationally demanding. However, as pointed out by Tsang and Clarke (1988), for reducing the computation load the separate treating of amplitude and rate constraints can be advantageous. First, the main results of (Tsang and Clarke, 1988) are given for the case of rate and amplitude constraints issuing from the unconstrained GPC solution (5) to an ARIMAX model.

3.1. Lagrange-Multiplier (LM) Method

3.1.1. Rate Constraint

The rate of the control input is constrained in magnitude, i.e.

$$|\Delta u_t| \le du_c. \tag{7}$$

Using the LM method as proposed in (Tsang and Clarke, 1988), the constrained optimal control in the case when only one future control saturates at $+du_c$ or $-du_c$ can be found from

$$\bar{u}^c = \bar{u}^o + \left(G^T G + q_u I\right)^{-1} \lambda_j e_j, \tag{8}$$

where $e_j = (0, 0, \dots, 1, 0, \dots, 0)^T$, and unity is in the j-th position. To find the Lagrange multiplier λ_j, the following equation has to be solved for $+du_c$, say:

$$du_c = \Delta u_{t+j-1}^o + g_{jj} \lambda_j \tag{9}$$

with respect to λ_j for $j = 2, \dots N_u$, where g_{jj} is the (j, j) entry of the matrix $(G^T G + q_u I)^{-1}$. Now, the optimal constrained Δu_t^c can be found by putting λ_j back to (8).

The above procedure can be extended to the case when more than one future control saturate (i.e. for $N_u > 2$). However, as pointed out in (Tsang and Clarke, 1988), an unsolved problem is on which limits the optimum lies. A reasonable heuristic solution is based on the assumption that the constrained optimal control lies on the constraint which is violated by the unconstrained optimum. In particular, this reasoning is valid for $N_u = 2$. In the presented approach, the optimization procedure starts for λ_j which corresponds to the control most saturated with respect to the unconstrained optimum. Next, the control horizon is reduced by one and the procedure is repeated in the same way until all controls are feasible.

3.2. Amplitude Constraint

The amplitude constraint imposed on the control input is given as follows:

$$|u_t| \leq u_c. \tag{10}$$

First, it can be noted that the calculation of optimal amplitude-constrained controls for an ARIMAX is more complex than for an ARMAX model. The opposite statement can be made in the case of the rate constraint.

Again, the use of the Lagrange-multiplier method for solving the amplitude-constrained GPC for the ARIMAX model is possible; however, in the general case the involved computations do not make this approach beneficial any more. The important case $N_u = 2$ makes an exception. Consider this case following the idea of (Tsang and Clarke, 1988). If the future control is feasible, then

$$u_t^c = \mathrm{sat}[u_t^o; u_c], \tag{11}$$

where $u_t^o = u_{t-1} + \Delta u_t^o$ is the control signal obtained by the standard unconstrained algorithm.

When the future control signal violates the constraint, say $u_{t+1} = u_c$, we get

$$\Delta u_t = \Delta u_t^o - \frac{\sigma_1}{\sigma_1 + \sigma_2}\left[u^{'} - (\Delta u_t^o + \Delta u_{t+1}^o)\right], \tag{12}$$

where σ_1 and σ_2 are the sums of the first and second rows of the (2×2) matrix $(G^T G + q_u I)^{-1}$, respectively, and $u^{'} = u_c - u_{t-1}$. Note that the constrained control sequence lies on the boundary $u^{'} = \Delta u_t + \Delta u_{t+1}$. The applied control signal then follows from

$$u_t^c = \mathrm{sat}[u_{t-1} + \Delta u_t; u_c], \tag{13}$$

where Δu_t is given by (12). In case $N_u > 2$, when more than one control saturates, the procedure described earlier can be similarly applied.

3.3. Simultaneous Constraints

First, both control sequences are calculated separately for each constraint. Then, in the case of an ARIMAX model, if the amplitude constraint is violated, then the smaller absolute value of Δu_t is selected as the control signal. In the case of an ARMAX model, if the rate constraint is violated, then the control signal u_t which differs from u_{t-1} less is selected and applied to the system.

3.4. Energy Constraint

Consider the GPC for an ARMAX model with the cost function (4) for $q_u = 0$,

$$J(N_y, N_u) = (G\bar{u} + f - r)^T (G\bar{u} + f - r), \tag{14}$$

under the energy constraint on the input signal

$$\bar{u}^T \bar{u} \le e_c^2. \tag{15}$$

For the minimization of (14) under the constraint (15), the Kuhn-Tucker conditions yield

$$G^T G\bar{u} + G^T (f - r) + \lambda\bar{u} = 0, \tag{16}$$

$$\lambda(\bar{u}^T \bar{u} - e_c^2) = 0, \tag{17}$$

$$\lambda \ge 0. \tag{18}$$

The optimal constrained control is then given by

$$\bar{u}^c = (G^T G + \lambda I)^{-1} G^T (r - f), \tag{19}$$

where the multiplier λ can be calculated from

$$(r - f)^T G\big(G^T G + \lambda I\big)^{-1T} \big(G^T G + \lambda I\big)^{-1} G^T (r - f) = e_c^2. \tag{20}$$

Summarizing, when the constraint (15) is fulfilled, the applied optimal control is the unconstrained optimal control $\bar{u}^o$ calculated for $q_u = 0$. Otherwise, the applied constrained control is given by (19), where the multiplier λ must be recalculated at each time step t whenever the constraint (15) is violated. It is worth noticing that $(G^T G + \lambda I)^{-1}$ can be calculated in a recursive way along with the increasing control horizon N_u. The above constrained minimization problem can also be solved iteratively with Carroll's method (Bazaraa and Shetty, 1979) using an unconstrained minimization technique for the modified cost function

$$J_m(N_y, N_u) = (G\bar{u} + f - r)^T (G\bar{u} + f - r) + r_k (e_c^2 - \bar{u}^T \bar{u})^{-1}, \tag{21}$$

where for monotonically decreasing r_k, so that $r_k \to 0$, the succsessive minimizations of (21) yield the desired constrained minimum.

3.5. The Method Based on the Solution to the Linear Complementarity Problem (LCP)

Transformation of the GPC problem into the LCP form enables the simultaneous consideration of rate and amplitude constraints of the input. In a noise-free system an amplitude constraint of the output can be additionally included. The solution method based on Lemke's algorithm (Camacho, 1993; Camacho and Bordons, 1995) can then be applied.

3.6. Handling Constraints

The input constraints (in rate and amplitude) and the output constraint (in amplitude) for an ARIMAX system can be represented as follows (Camacho, 1993):

$$R_1 \bar{u} \leq c_1, \tag{22}$$

where c_1 is the vector containing upper and lower constraints and R_1 denotes the block matrix defined by

$$c_1 = \begin{bmatrix} I_1 du_c \\ I_1 du_c \\ I_1 u_c - I_1 u_{t-1} \\ I_1 u_c + I_1 u_{t-1} \\ I_2 y_{\max} - f \\ -I_2 y_{\min} + f \end{bmatrix}, \quad R_1 = \begin{bmatrix} I \\ -I \\ T \\ -T \\ G \\ -G \end{bmatrix}, \tag{23}$$

respectively, where $I_{1(N_u \times 1)} = (1, \dots, 1)^T$, $I_{2(N_y \times 1)} = (1, \dots, 1)^T$, $T_{(N_u \times N_u)}$ is a lower triangular matrix whose non-zero entries equal 1 and I is the $N_u \times N_u$ identity matrix. Usually, the input sequence is written in the form which is further exploited in Lemke's algorithm:

$$\bar{u} = -I_1 du_c + x. \tag{24}$$

By substituting (24) into (23), the following final form of the constraints can be obtained:

$$Rx \leq c, \quad x \geq 0, \tag{25}$$

where

$$R = \begin{bmatrix} I \\ T \\ -T \\ G \\ -G \end{bmatrix} c = \begin{bmatrix} 2I_1 du_c \\ I_1 u_c + T I_1 du_c - I_1 u_{t-1} \\ I_1 u_c - T I_1 du_c + I_1 u_{t-1} \\ I_2 y_{\max} - f + G I_1 du_c \\ -I_2 y_{\min} + f - G I_1 du_c \end{bmatrix}. \tag{26}$$

Similar derivations can be made for an ARMAX model.

3.7. Formulation of LCP

Taking account of (24), the cost function (4) can be expressed in the form

$$J = \frac{1}{2} x^T H x + x^T a + d. \tag{27}$$

Assuming that all constraints are violated by control inputs within the horizon N_u, the Lagrange multipliers λ_1 and λ_2 are vectors. Forming the Lagrangian and differentiating it, we get (Camacho, 1993)

$$\lambda_3 + Rx = c, \qquad (28)$$

$$\lambda_2 - Hx - R^T\lambda_1 = a,$$

where λ_3 is a vector of complementary variables which change the inequality (25) into equality. The above equations take the form of the LCP

$$s - Mz = q, \qquad (29)$$

where $s = (\lambda_3^T, \lambda_2^T)^T$, $z = (\lambda_1^T, x^T)^T$ are unknown vectors while the block matrix M and the vector q, respectively given by

$$M = \begin{bmatrix} 0 & -R \\ R^T & H \end{bmatrix}, \quad q = \begin{bmatrix} c \\ a \end{bmatrix},$$

are known. The conditions $s, z \geq 0$ and $s^T z = 0$ must also be fulfilled.

The LCP can be solved in various ways, e.g. using interior point methods.

3.7.1. Lemke's Algorithm

Solution of (29) is trivial when $q \geq 0$, as it then suffices to take $s = q$ and $z = 0$. Otherwise, an additional variable $z_0 \geq 0$ is introduced to eqn. (29),

$$s - Mz - z_0 I_3 = q, \qquad (30)$$

where $I_{3[(4N_u+2N_y)\times 1]} = (1, 1, \ldots, 1)^T$. To solve (30) Lemke's algorithm can be applied (Camacho, 1993; Camacho and Bordons, 1995).

4. Closed-Loop Stability Analysis

It is well-known that in general, GPC does not guarantee closed-loop stability even in a non-adaptive case. However, by a proper choice of design parameters the stability for the unconstrained non-adaptive GPC can be achieved. In this case the closed-loop characteristic polynomial in z-domain when the open-loop system is of the ARIMAX type has the form (Bitmead *et al.*, 1990)

$$A_c = A\Delta + \sum_{i=1}^{N_y} \alpha_i (B - G_i A\Delta) z^{i-1}, \qquad (31)$$

where G_i are polynomials forming the matrix G, i.e. the coefficients of G_i correspond to the i-th row of G. The poles z_i are involved functions of design parameters N_y, N_u, q_u. Some simulation plots for $\max_i |z_i|$ as a function of design parameters are given in Section 6. The analysis of the closed-loop stability for the GPC with input constraints is much more complicated, and an analytical representation like in the unconstrained case (31) seems unfeasible. Moreover, the stability of the constrained GPC depends on initial conditions and the reference signal, and in the adaptive case additionally on the initial parameter estimates. The stability issue is especially crucial for unstable open-loop systems. In this case the global closed-loop stability of constrained STGPC cannot be assured; however, in some conditions a local stability can be achieved. Some stability results obtained through simulations are presented in Section 6.

5. Self-Tuning Control

The self-tuning controller proposed here is a certainty-equivalence controller where the unknown system parameters $\theta = (a_1, \dots, a_{na}, b_0, \dots, b_{nb}, c_1, \dots, c_{nc})^T$ are estimated on-line by means of the RELS method and next used for tuning the GPC controller. From (5) it follows that the unconstrained optimal control signal actually applied is given by

$$\Delta u_t^o = \Delta u_{t-1}^o + q_1^T (r - f), \tag{32}$$

where q_1^T denotes the first row of the matrix $(G^T G + q_u I)^{-1} G^T$. This means that in the self-tuning control only q_1 and predictions f have to be recalculated at each discrete time instant t along with the standard derivation of the GPC algorithm. Obviously, the implementation of a constrained self-tuning controller increases the computational load. In this case the performance and stability of the closed-loop system depend strongly on the initial parameter uncertainty and constraints.

At present, there are no rigorous theoretical results regarding the stability of finite horizon STGPC when amplitude and/or rate constraints are imposed on the input. If the system (1) is unstable, the global stability cannot be achieved due to the constraints even in the noise-free case, i.e. $e_t = 0$. However, some closed-loop stability boundaries could be evaluated in terms of initial uncertainties (initial parameter estimates) as well as initial conditions and constraints if a noise-free or a noise-bounded system is considered.

6. Simulations

The following examples are taken for simulation:

1. The second-order stable ARIMAX/ARMAX system with numerical values of parameters $a_1 = -1.8,\ a_2 = 0.9,\ b_0 = 1.0,\ b_1 = 0.5$,

2. The second-order unstable ARIMAX/ARMAX system with numerical values of parameters $a_1 = 1.8,\ a_2 = -0.9,\ b_0 = 1.0,\ b_1 = 0.5$,

3. The second-order non-minimum phase ARIMAX/ARMAX system with numerical values of parameters $a_1 = -1.5,\ a_2 = 0.7,\ b_0 = -1.0,\ b_1 = 2.0$.

The polynomial C, is taken as $C = 1$, and the noise variance σ_e^2 was set to 0.01. System parameters θ are identified using the standard RELS method with initial estimates $\hat{\theta}_0$ close to zero. Prior to the control process, an off-line pre-identification was performed, which yields initial estimates for constrained STGPC to be started up.

Simulation runs were performed for a square wave as a reference signal given by

$$r_{20N+t+5} = 5(-1)^N, \quad t = 0, 1, \ldots, 39, \quad N = 0, 1, \ldots .$$

The design parameters were set at $N_u = N_y = 3$, $q_u = 0.1$ unless stated otherwise. It should be noted that for $N_y > N_u$ the tracking performance is generally worse than for $N_y = N_u$, because in the former case the matrix G is truncated. Programmes were written in MATLAB for both the presented methods, and selected simulation runs are presented below. Obviously, both the methods yield the same optimal constrained control inputs when the non-adaptive, noise-free case is considered for a given set of design parameters.

6.1. Rate Constraint

Figures 1 and 2 show the control behaviour (LM) for the ARIMAX and ARMAX models of Example 1, respectively. Exemplary plots of hard-constrained control signals with $du_c = 2$ (ARIMAX) and $du_c = 2.4$ (ARMAX) are included. Self-tuning disables good tracking in the initial phase of the control process. An example of one estimation run with initial estimates $\hat{a}_1 = -1.25$, $\hat{a}_2 = 0.40$, $\hat{b}_0 = 0.33$, $\hat{b}_1 = 0.20$ is shown in Fig. 3 for a stable ARIMAX case. Simulation (LCP) of the ARIMAX of Example 2 is shown in Fig. 4 with control signal constrained at $du_c = 6.7$. The tracking deterioration takes place for $du_c \leq 7$. The ARIMAX of Example 3 is simulated (LM) in Fig. 5 for $N_u = N_y = 5$ and $q_u = 0.2$, where the control signal constrained by $du_c = 7$ is also shown. Poor tracking occurs for $du_c \leq 6$. Here and in other simulations of non-minimum phase systems, a longer prediction horizon and a larger value of the weight q_u were needed in order to stabilize the control system.

6.2. Amplitude Constraint

The results for the ARIMAX (LM) of Example 1 are shown in Fig. 6, while in Fig. 7 they correspond to the ARMAX (LCP). In both the cases, hard-constrained control signals with $u_c = 0.8$ and $u_c = 0.5$ are given. Simulation for the ARMAX (LCP) of Example 2 is shown in Fig. 8 including the control signal with $u_c = 7$. The tracking deteriorates very much when the constraint is imposed on the control signal. This is also the case for an ARIMAX model. The ARIMAX (LM) of Example 3 is simulated in Fig. 9, where hard constraints $u_c = 2, 3$ are considered. In this case, the tracking is not satisfactory; however, for larger values of u_c (not included here) a good tracking was observed.

6.3. Energy Constraint

The control behaviour for the ARIMAX of Example 1 (LM) is shown in Fig. 10 for $e_c = 4$, where an on-line adjustment of λ is also reflected. The multiplier λ plays here the role of the weight q_u which is tuned along with the control process.

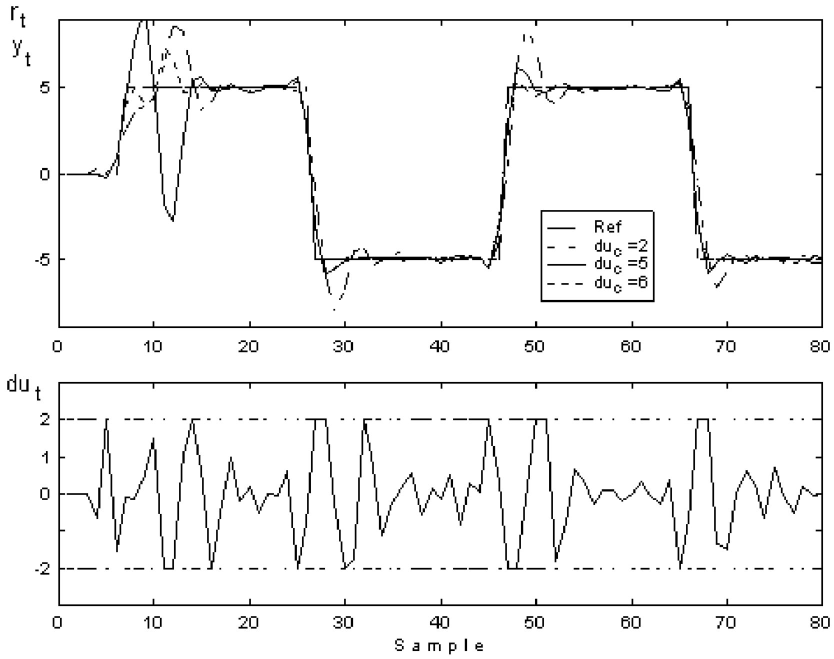

Fig. 1. STGPC under rate constraints (Example 1, ARIMAX).

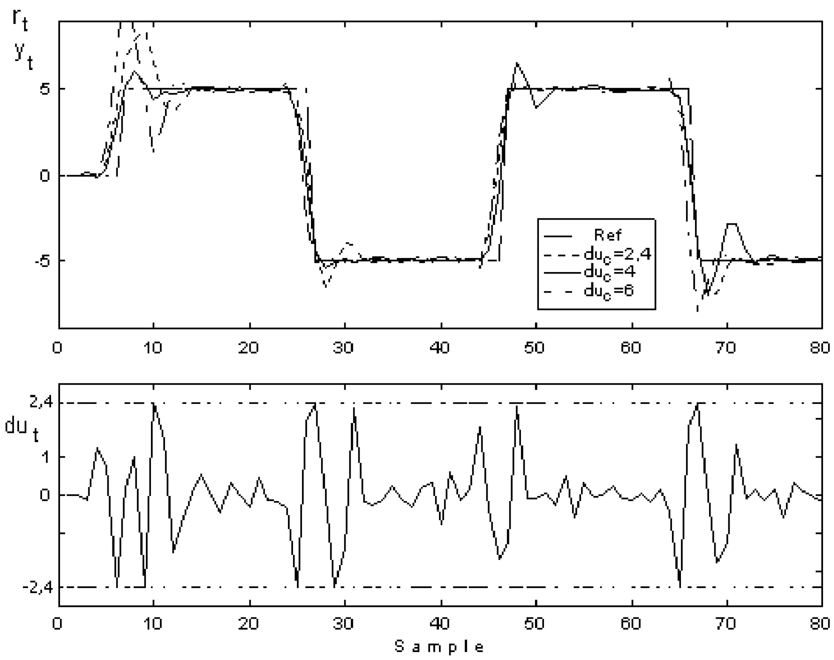

Fig. 2. STGPC under rate constraints (Example 1, ARMAX).

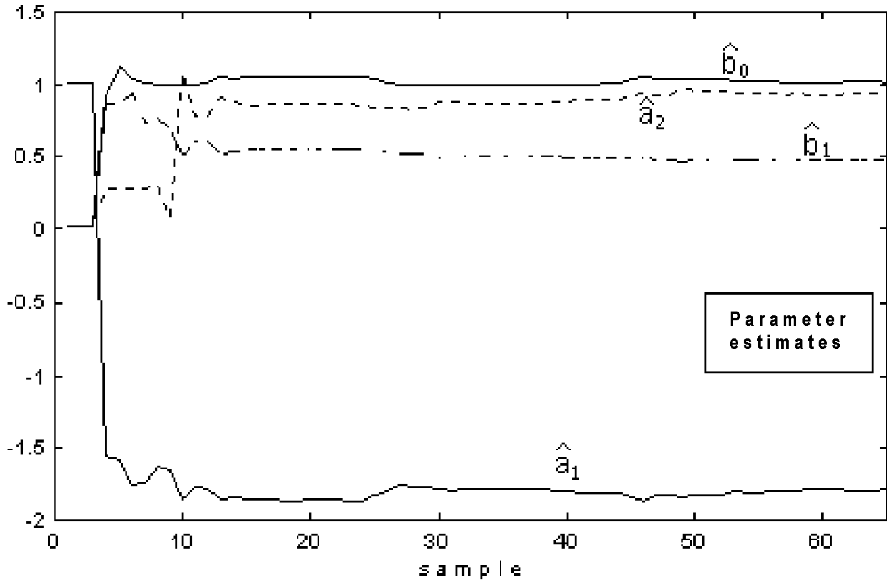

Fig. 3. Run of parameter estimates.

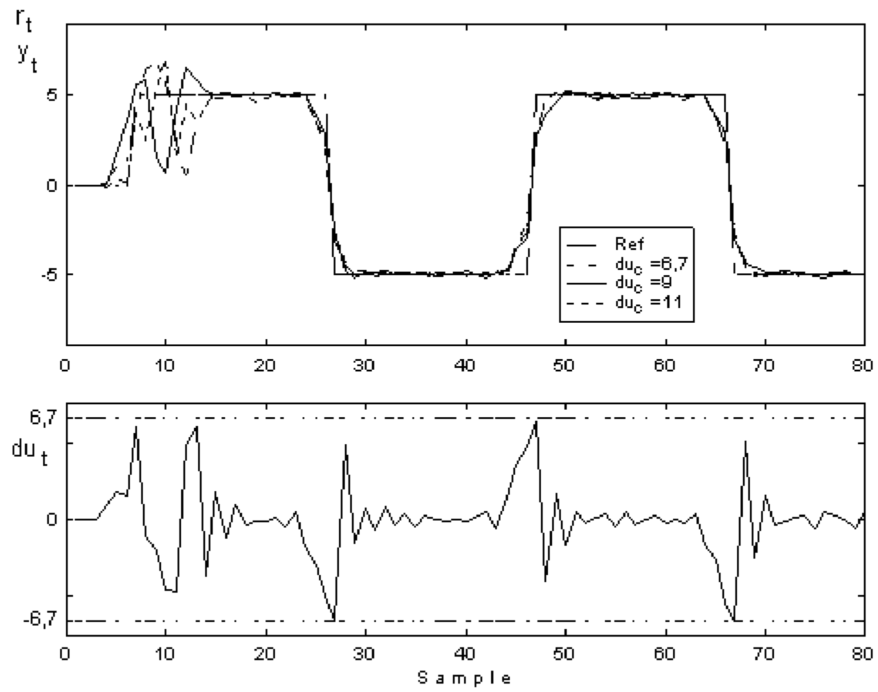

Fig. 4. STGPC under rate constraints (Example 2, ARIMAX).

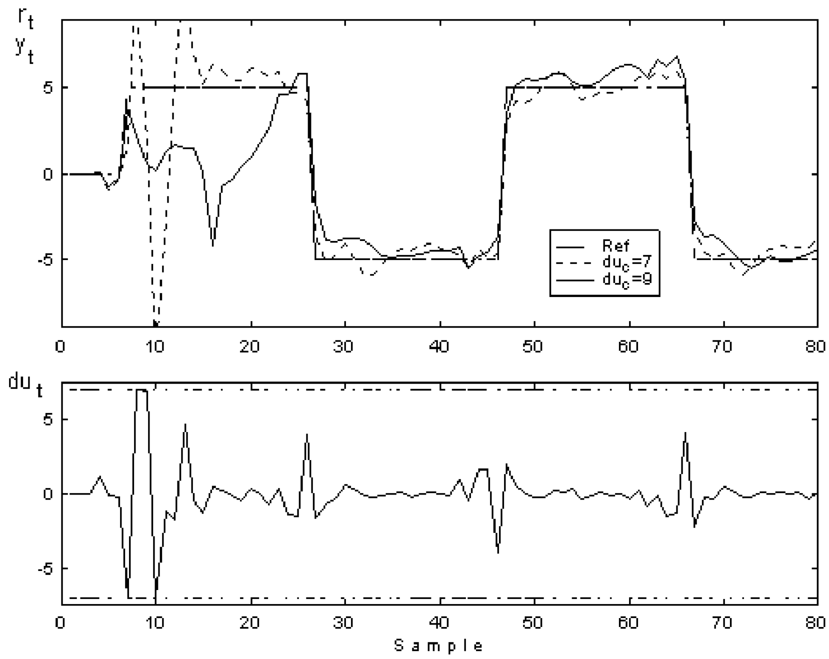

Fig. 5. STGPC under rate constraints (Example 3, ARIMAX).

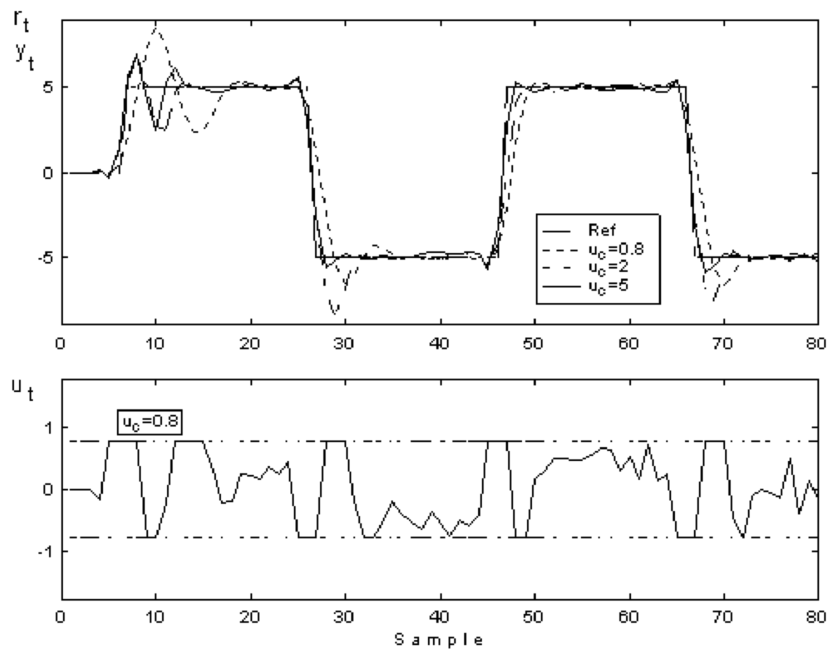

Fig. 6. STGPC under amplitude constraints (Example 1, ARIMAX).

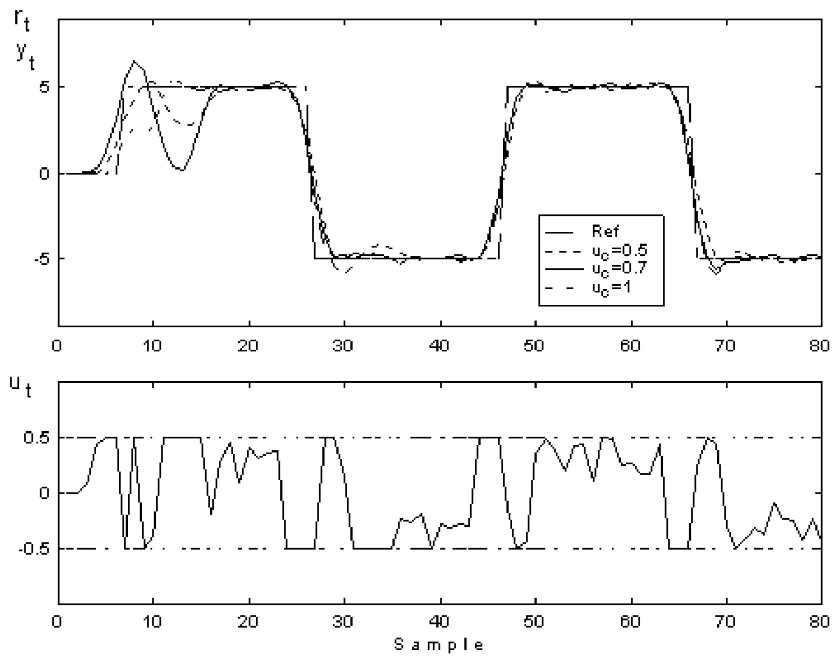

Fig. 7. STGPC under amplitude constraints (Example 1, ARMAX).

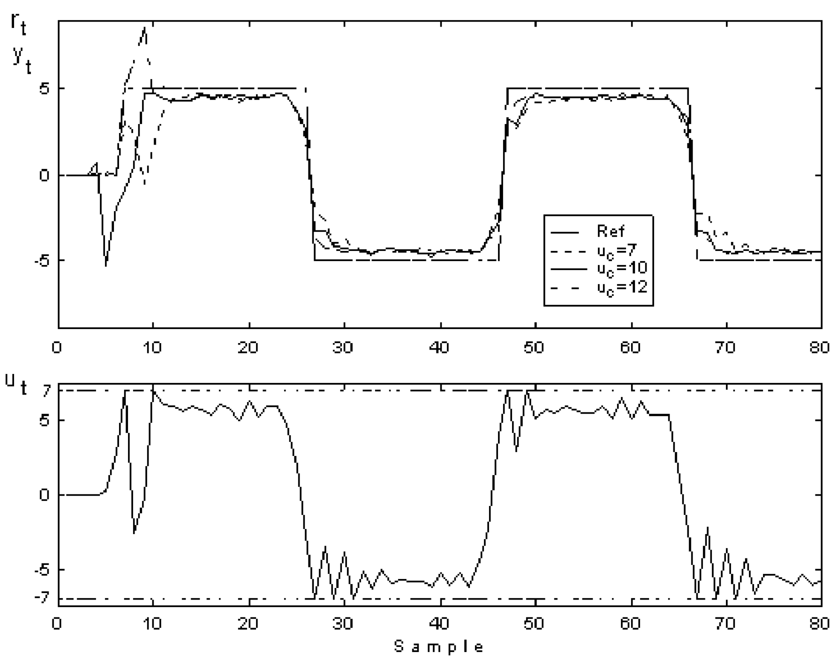

Fig. 8. STGPC under amplitude constraints (Example 2, ARMAX).

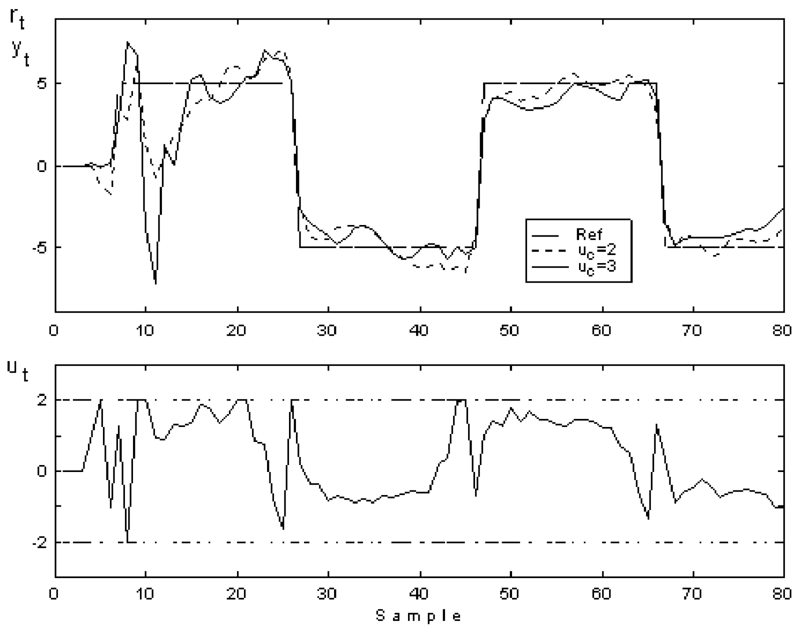

Fig. 9. STGPC under amplitude constraints (Example 3, ARMAX).

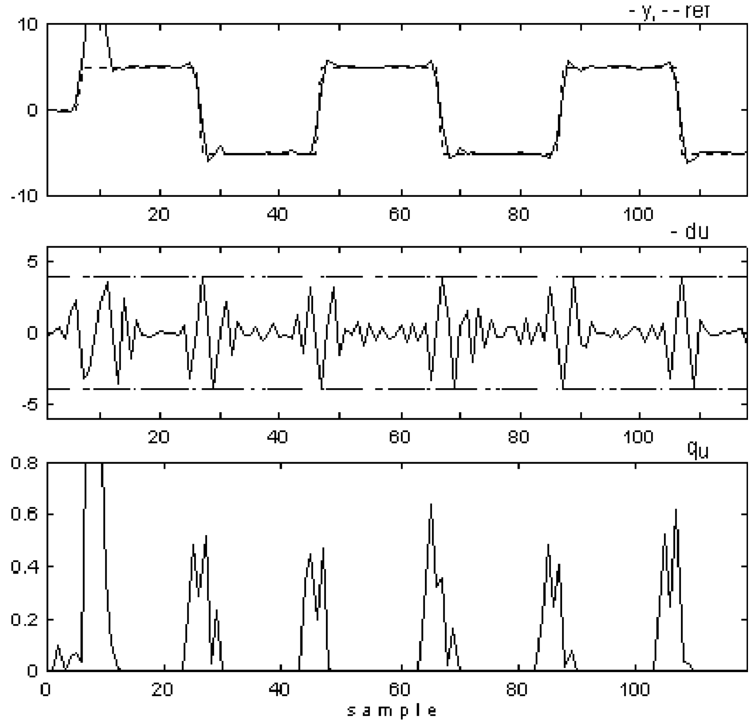

Fig. 10. STGPC under energy constraints (Example 1, ARIMAX).

6.4. Output Constraint

In some cases, imposing a constraint on the system output may be required, e.g. for safety reasons. This kind of constraint can be especially useful in the control of non-minimum systems. The output constraint has the form

$$y_{\min} \le y_t \le y_{\max},$$

where $y_{\min} = r_t - y_c$ and $y_{\max} = r_t + y_c$, cf. (23). It is worth noticing that the output constraint included into Lemke's algorithm makes sense when the system output is noise-free. In this case, imposing the output constraint can keep the system output within the limits $(y_{\min}, y_{\max})$ even when the control system destabilizes.

Figure 11 shows the simulation for the noise-free ARIMAX (LCP) system of Example 2, where $du_c = 8,\ u_c = 10,\ y_c = 3$. One can see that under constraints du_c and u_c comparable to those of Figs. 4 and 8, the output y_t attains $y_{\min} = 2$ only at one point. Moreover, this takes place in the initial phase of identification, when a substantial self-tuning occurs.

6.5. Simultaneous Rate and Amplitude Constraints

In Figs. 12 and 13, both the constraints ($du_c = 4.5,\ u_c = 3.7$) are considered for the ARIMAX of Example 1 simulated by LM and LCP methods, respectively. The tracking is satisfactory; however, both constraints are not much comparable with those in Figs. 1 and 6.

Table 1. Destabilizing constraint values.

Example $1,2,3$	Rate Constr. du_{dest}	Ampl.Constr. u_{dest}	Energy Constr. e_{dest}
1 ARIMAX	2.0	0.8	0.6
1 ARMAX	2.0	0.3	0.5
2 ARIMAX	6.7	7.0	140
2 ARMAX	9.5	7.0	55
3 ARIMAX	6.0	1.5	0.7
3 ARMAX	6.0	1.8	0.9

6.6. Stability Analysis

Figures 14, 15 and 16 show the plots of the pole $\max_i |z_i|$ versus q_u obtained from (31) for Examples 1, 2 and 3 (all ARIMAX), respectively, for different horizons $N_u = N_y$. In all cases, it is easier to assure stability for long horizons. This is particularly evident for non-minimum phase systems. It can be seen from Figs. 14 and 15 that for the design parameters $N_u = N_y = 3,\ q_u = 0.1$ considered in the above simulations, the closed-loop stability of an unconstrained non-adaptive control system is assured for Systems 1 and 2. From Fig. 16 it can be observed that for a non-minimum phase system (Example 3), a longer prediction horizon $N_y = 5$ and a larger weighting constant $q_u = 0.2$ are needed to ensure the stability. As already mentioned in Section 4, the analytical discussion of stability is not feasible when constraints are present and active.

For the presented simulation examples of constrained control, the stability limits are evaluated through simulations for all the open-loop systems considered. The rate, amplitude and energy constraint values for which the stgpc control system destabilizes are denoted by du_{dest} and u_{dest} and e_{dest}, respectively. The approximate values of destabilizing constraints imposed separately on the input are given in Table 1 for all simulated examples. Obviously, one can observe much larger values of $du_{\text{dest}},\ u_{\text{dest}}$ for unstable systems (Example 2) when compared with stable and non-minimum phase systems (Examples 1 and 3). Surprisingly large values of $e_{\text{dest}} = 140, 55$ occur for unstable ARIMAX and ARMAX systems. However, in the non-adaptive case the corresponding values were found to be $e_{\text{dest}} = 5, 6$, respectively.

6.7. Computation Analysis

In Table 2, a comparison of the computation amount measured by the number of floating-point operations required by the LM method (separate constraint handling) and by Lemke's algorithm (simultaneous constraint handling) is given for a non-adaptive control of a stable noise-free ARIMAX system where $N_u = N_y = 3,\ q_u = 0,$ and for the total number of control steps set at 80. The number of flops which is automatically registered by MATLAB is presented in terms of constraints given as a percentage of the unconstrained (full range) control signal. One can see the computational advantage of the LM method over Lemke's algorithm, which is even more evident when hard constraints are imposed on the input.

Table 2. Number of flops.

Constraints –	Range of input signal %	LM *Flops*	LCP *Flops*
unconstrained	100	8168	13619
$du_c = 11.5$	50	8381	27132
$du_c = 3.3$	14	10246	48756
$u_c = 6.6$	50	8405	20270
$u_c = 2.66$	20	8781	30586
$u_c = 0.93$	7	9140	69848
$du_c = 15.2,\quad u_c = 4$	66, 30	8595	27325
$du_c = 5.75,\quad u_c = 4$	25, 30	9008	36178
$du_c = 4.6,\quad u_c = 2.66$	20, 20	9089	48475

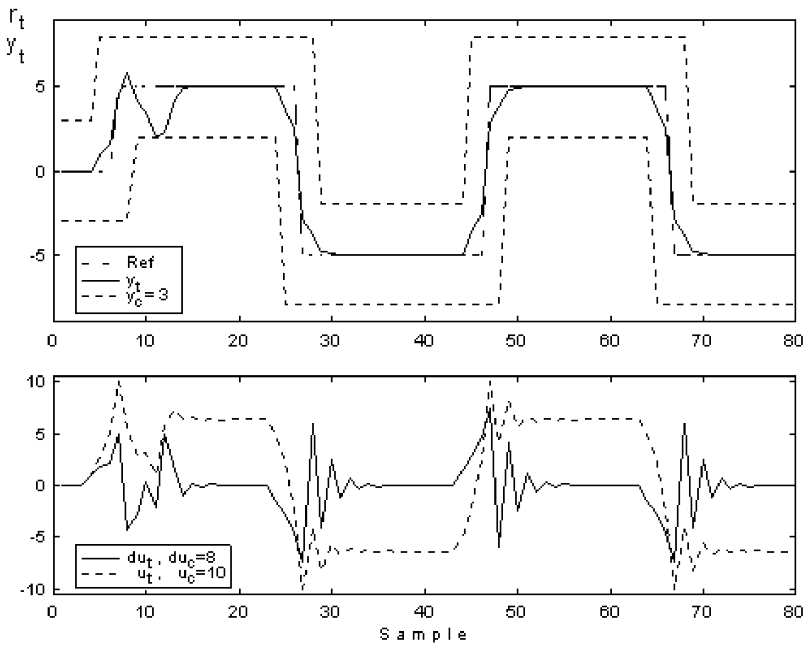

Fig. 11. STGPC under input (rate, amplitude) and output constraints.

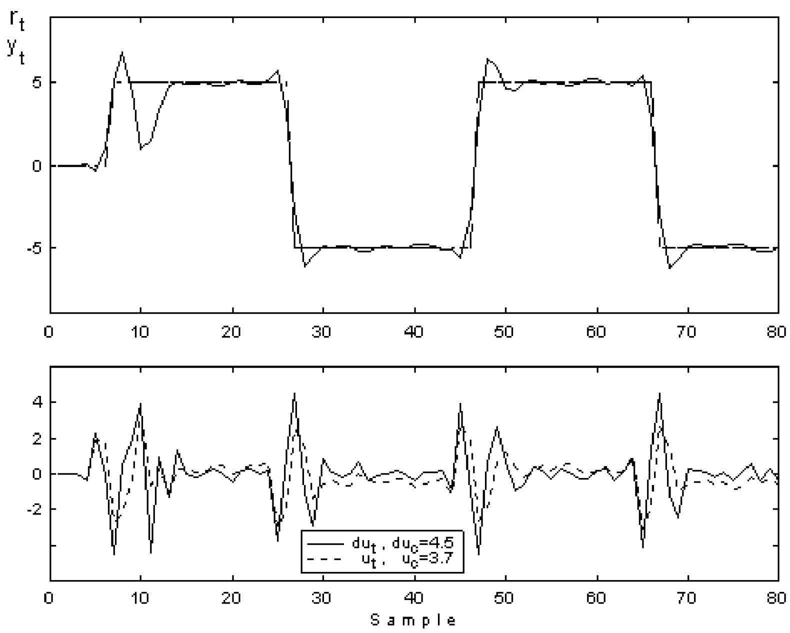

Fig. 12. STGPC under input constraints (Example 1, ARIMAX).

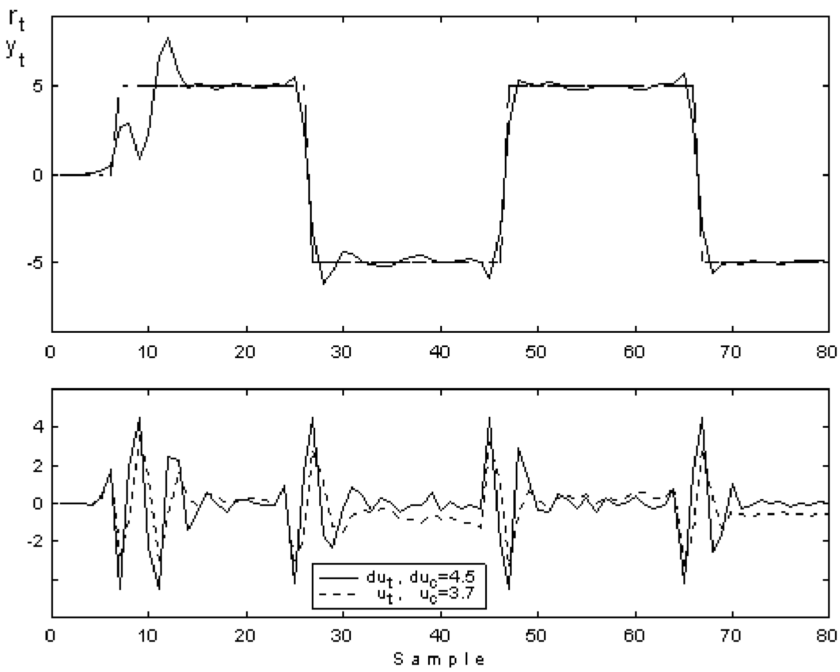

Fig. 13. STGPC under input constraints (Example 1, ARIMAX).

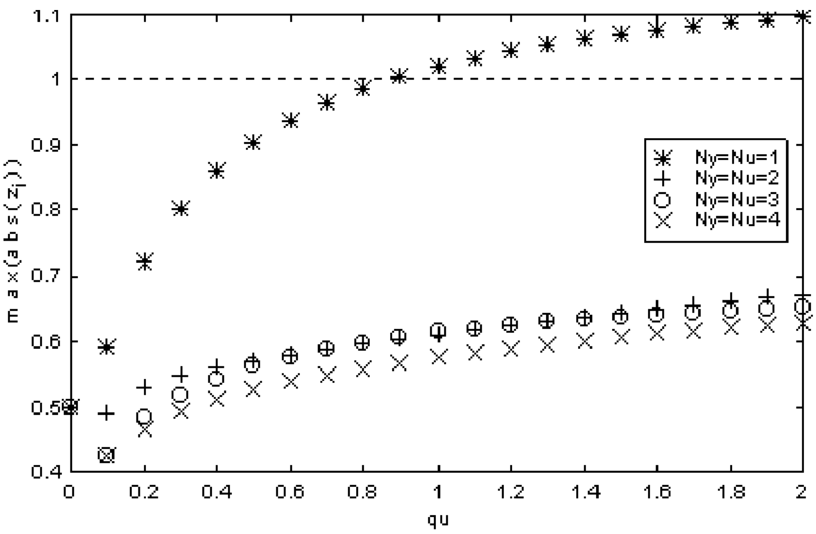

Fig. 14. Poles of the closed-loop system (Example 1, ARIMAX).

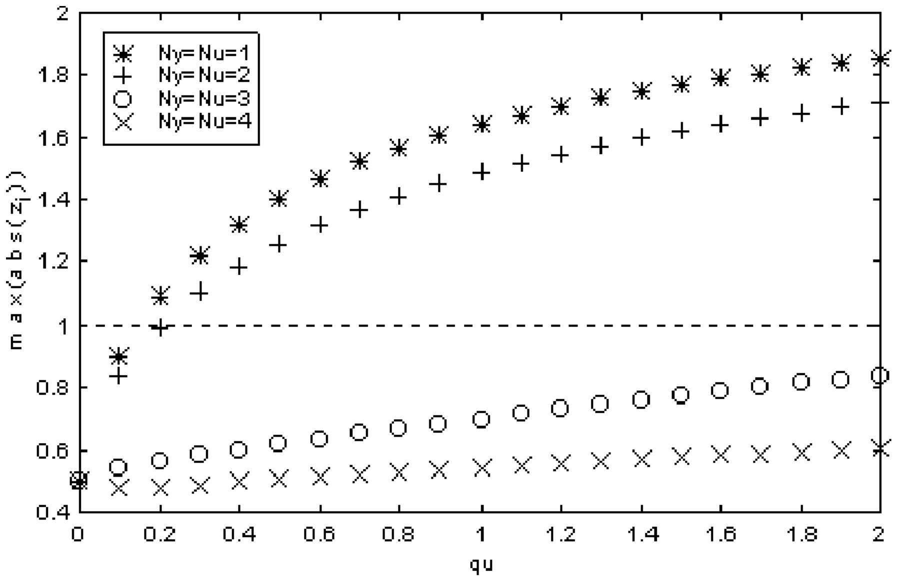

Fig. 15. Poles of the closed-loop system (Example 2, ARIMAX).

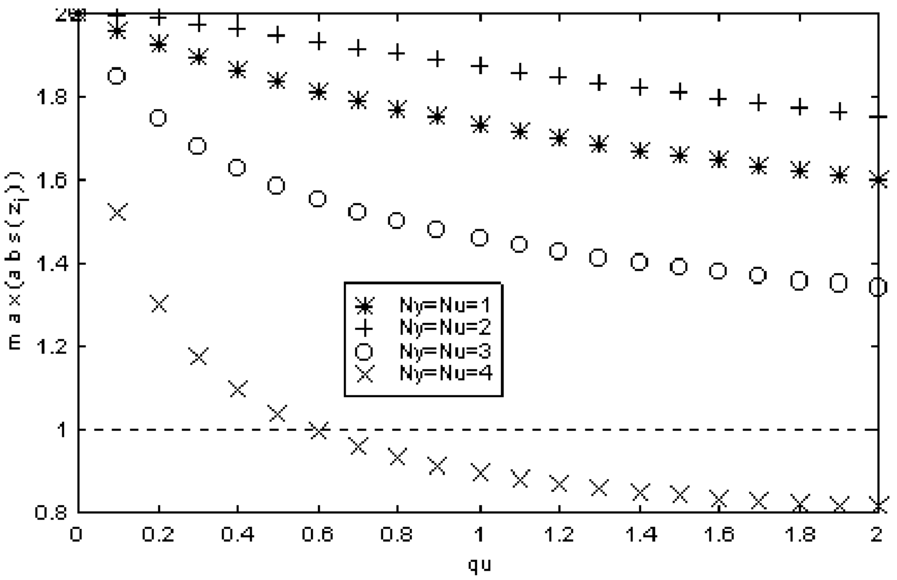

Fig. 16. Poles of the closed-loop system (Example 3, ARIMAX).

7. Conclusions

An STGPC problem is presented for rate, amplitude and energy-constrained inputs. The self-tuning of constrained GPC is implemented in an indirect way. Second-order stable, unstable and non-minimum phase examples are analyzed and simulated for different configurations of design parameters. Simulation results reveal an essential influence of design parameters and constraints on the stability and control performance. This is especially crucial in the analysis of the closed-loop stability of unstable systems. For assumed initial conditions the destabilizing values of constraints can be established through simulations. Finally, from a comparison of computation loads for both the methods considered, a conclusion can be drawn that the separate handling of constraints is advantageous. However, this usually takes place at the cost of some optimality deterioration.

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