

## CORRECTIONS TO “CONVERGENCE ANALYSIS FOR PRINCIPAL COMPONENT FLOWS”†

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Equation (8) in Theorem 1 is incorrect and has to be replaced by the correct condition

$$X_\infty^T A X_\infty D (X_\infty^T X_\infty - I_k) = 0. \quad (1)$$

Our mistake was to use the incorrect claim  $X^T A X D + D X^T A X \geq 0$ . A correct proof for (1) works with the Lyapunov function  $V_D(X) := \frac{1}{2} \text{tr}[D(X^T X - I_k)]^2$  instead of  $V(X) := \frac{1}{2} \text{tr}(X^T X - I_k)^2$ . The Lie derivative of  $V_D$  is easily computed as ( $Q := D^{1/2}(X^T X - I)D^{1/2}$ )

$$L_F V_D(X) = -\text{tr}[Q D^{1/2} X^T A X D^{1/2} Q] \leq 0$$

from which we conclude the desired condition

$$L_F V_D(X) = 0 \iff X^T A X D (X^T X - I) = 0.$$

In order to prove Theorem 2, we used the incorrect equation  $X_\infty^T A X_\infty (X_\infty^T X_\infty - I_k) = 0$ . However, we can get the same result that  $\Sigma = I_r$  by using the correct equation (1) and the result of Theorem 2 is still valid.

We also take this opportunity to correct some typographical errors:

P.224, 1.-14: Read “ $\dot{X} = A X D - X D X^T A X$ ” for “ $\dot{X} = (I - X D X^T) A X D$ ”.

P.226, 1.3: Read “ $L = U \Sigma V^T$ ,  $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_l, 0, \dots, 0)$ ” for “ $L = U \Sigma U^T$ ,  $\Sigma = \text{diag}(\sigma_1, \dots, \sigma_l)$ ”.

P.228, in Theorem 2: Replace “ $\Phi_0$ ” by “ $\Psi_0$ ”.

P.229, 1.11: Read “ $X = \Psi \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \Phi$ ” for

$$“X = \Psi \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = \Phi”.$$

P.229, 1.14: Read “ $\tilde{D} := \Phi D \Phi^T$ ” for “ $\tilde{D} := \Phi A \Phi^T$ ”.

P.230, 1.3: Read “ $\Psi$  for “ $\Phi$ ”.

P.230, 1.16: Read “ $P \Gamma P^T = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \in \mathbf{R}^{k \times k}$  for

$$“P \Gamma P^T = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \quad k \in \mathbf{R}^{k \times k}.”$$

P.234, in Proposition 2: Replace “ $D$ ” by “ $D^{-1}$ ”.

P.234, in the proof: Read “ $A$ ” for “ $N$ ”.

P.234, 1.1: Read “ $D$ ” for “ $H$ ”.

P.234, 1.-5: Omit  $\frac{1}{2}$  in front of the first summation.

We would like to thank P. Absil for pointing out the error in the proof of Theorem 1.

### References

- [1] S.Yoshizawa, U. Helmke, and K. Starkov, *Convergence analysis for principal component flows*, 2001, Vol. 11, No.1, pp.223–236.

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