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MODEL–BASED DAMAGE DETECTION, LOCALIZATION AND ASSESSMENT OF STRUCTURES

THE EXTENDED SYSTEM IDENTIFICATION METHODOLOGY

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System analysis and system identification are the basis of damage detection, localization and assessment. The (global) dynamic mathematical model with generally many fewer degrees of freedom than the static model is used for stability and dynamic qualification. It is also the basis for identification, which refers to the correction and validation of the mathematical model by using measurements of the existing structure. When this is done at several instances in the lifetime of the structure, these validated models can be taken as a knowledge base for damage detection, localization, and for making decisions for further actions. No better knowledge base exists than that of a validated mathematical model using the recent state of the structure itself. Assessment of the structure in every state can be done with the dynamic model based on fault investigations. The possible application of neural networks and fuzzy logic in this context is mentioned here. Active and smart structures are also open for further study with the use of the measuring devices and control circuits.

1. Introduction

Man–made systems from civil, mechanical, naval, aeronautical etc. engineering in operation are dynamically loaded during their lifetimes in several ways: internal and external forces (including those from the environment) stress the system, which leads to modifying dynamic behaviour during the lifetime due to modified model parameters dependent on lifetime. If the modifications attain corresponding thresholds with respect to stress or strain limits, to production quality limits, or to comfort conditions etc., the system is said to be damaged, has faults or the process is faulty. These brief introductory remarks provide motivation for a holistic consideration of the problem of the damage detection, location and assessment of structural systems. It has to be noted that a complete theory of holistic dynamics does not yet exist, but only an engineering approximation which distinguishes between fast and slow time coordinates is available (see, for example (Cempel and Natke, 1993b)). The cross–impact of the holistic dynamic investigations on the

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design and development phase can be that the lifetime of the system is enlarged, or the inspection costs, the maintenance or servicing costs and repair costs can be reduced drastically.

In addition to theoretical investigations, lab tests and tests with the existing system (sometimes during construction work) have to be done. The goals for testing are the validation of the theoretical models and of their related assumptions, and the tests serve for system qualification. During the operating phase the monitoring of symptoms or periodical inspection and such on request produce data for the detection of system modifications or of modified system behaviour. If such a modification indicates a fault, then further investigations concerning localization and causes lead to an assessment. Based on these results a decision has to be made which leads to corresponding actions.

As can be imagined from the problems mentioned above, diagnostics is an interdisciplinary field and can be classified as a part of applied system engineering. The particular fields are:

- holistic dynamics (long–term, short–term dynamics): modelling, prediction, simulation, optimization,
- system identification: dynamics, measuring technique (data acquisition), excitation technique, process computation (hardware and software), signal processing,
- AI (expert systems),
- mathematics: algorithms, inverse problems (posedness), stochastics, estimators, computational engineering.

Design and construction techniques including material science influence the system properties which, however, belong to the particular engineering task. The interrelations are obvious, and system identification itself is an interdisciplinary field.

The state of the art of diagnostics is documented in the proceedings of, for example, the international conference SAFEPROCESS'91 (Isermann, 1991), and in the books (Natke and Yao, 1988; Natke *et al.*, 1993). In (Ben–Haim *et al.*, 1993) various methods are enumerated, discussed and a first attempt is made at evaluation. For highly complex systems one may conclude that diagnosis with physics–based mathematical models can be recommended: the best available knowledge basis is a validated mathematical model. This model must

- be adjusted with respect to the lifetime,
- include the symptom description within the fast time, coordinates[1],
- fulfil the three requirements concerning validation (Craemer, 1985; Natke, 1992c):
  - verification (model reliability),
  - validation (homomorphy between model and system),

[1] That means the model must be detailed enough in order to be able to describe model parameter modifications.

- usability (negligible systematic errors, sufficiently small standard deviations of the estimates),

- enable the applicant to assess system modifications by simulations,
- with some additional effort lead to trend predictions.

Consequently, without neglecting the difficulties that arise, model–based diagnostics is favoured. This method is discussed and reviewed in the paper in hand.

2. The Methodology

Damage occurs due to system modification(s). Severe system modification results in a modification of the system behaviour. It is assumed that these modifications can be described by parameter modifications of a structured mathematical model, and by model structure modifications including parameter modifications when compared with the parameters of the previous model.

2.1. General Description

The extended system identification methodology is published in the Preface of (Natke and Yao; 1988; Fig. 1). Starting with the operation of the existing system, it is assumed that a validated mathematical model exists which describes the dynamic behaviour sufficiently accurately with known errors: $\mathcal{M}_0$. This mathematical model of the design, development and prototype phase of the system serves for studies of possible faults and the resulting parameter modifications including the modifications of the dynamic behaviour. These lead to particular features and patterns due to specific faults, from which symptoms (sensitive quantities to be measured, sensitive with respect to expected parameter modifications) can be derived for monitoring. This prior knowledge enables us to design the monitoring of the system under operation and to predict the stresses etc.

Theoretical system analysis gives the necessary data for thresholds, bounds for symptoms etc. during monitoring, or the first inspection intervals etc. This is an estimation with respect to the lifetime τ. The measuring of dynamic responses (e.g. by the symptoms) at the lifetime τ_i, $i = 1(1)N$, $N \in \mathcal{N}$, and adjustment of the prior mathematical model (see Fig. 2) gives a holistic dynamic description of the system in discrete steps of the lifetime: $\mathcal{M}_i$. Comparison of the resulting model structure and the parameters with those of the model $\mathcal{M}_{i-1}$ in time step τ_{i-1} permits a decision concerning significant model deviations. If the deviations are significant, then dependent on the pattern and the existing catalogue of faults a first diagnosis can be made. If this working step is unsuccessful, the mathematical model $\mathcal{M}_i$ serves for further investigations: localization and assessment of the detected deviations. Reliability analysis, trend predictions with the help of the lifetime models $\mathcal{M}_i$ for the previous lifetime steps, and decision analysis in combination with the inference machine gives the experienced engineer responsible the information needed for possible actions. These can consist of further operation for a fixed time before repair etc. The iterative procedure is shown in Figure 3 as a flow chart.

FOR COMPLEX AND HIGHLY NONLINEAR STRUCTURES

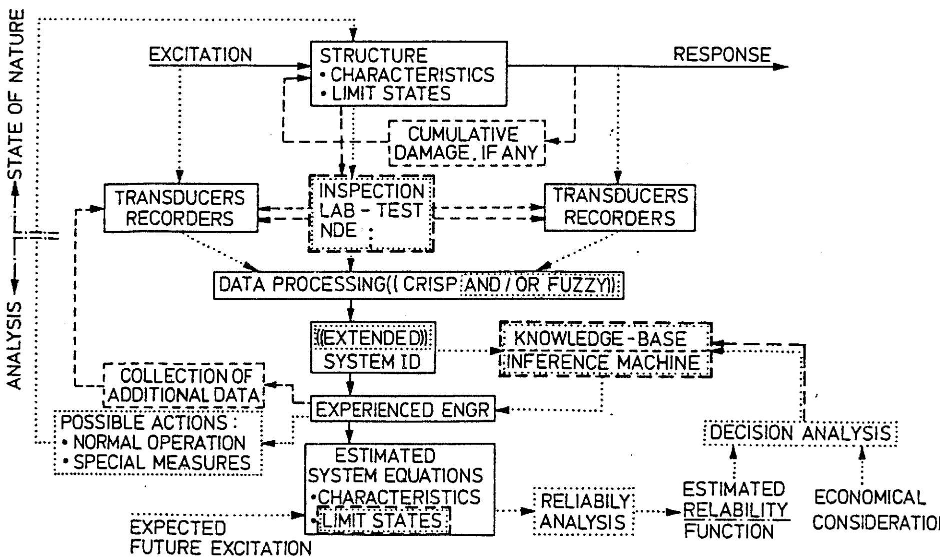

Fig. 1. The extended system identification methodology.

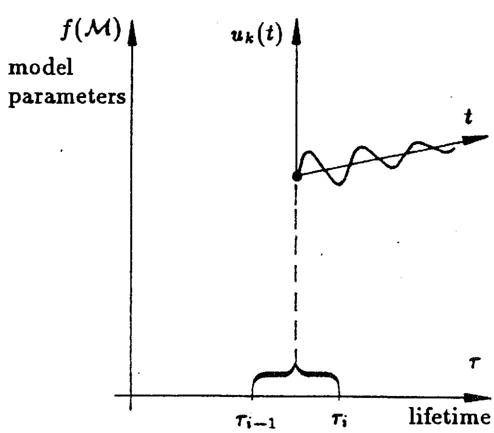

Fig. 2. Lifetime concept and zooming out the dynamics.

As can be concluded:

- uncertainties of models play a great role; they must be overcome by additional modelling,
- the relationship between model errors and damage effects influences the damage detection,
- system identification (model structure identification and parameter estimation) is the key of the model–based methodology,

- the resulting validated mathematical model provides various theoretical investigations with respect to the detection, localization, and assessment of faults (⤳ knowledge base),
- lifetime dependent mathematical models (holistic dynamics) serve for trend statements,
- reliability analysis and decision–making is based on predictions and simulation.

These topics are discussed in some detail in the following.

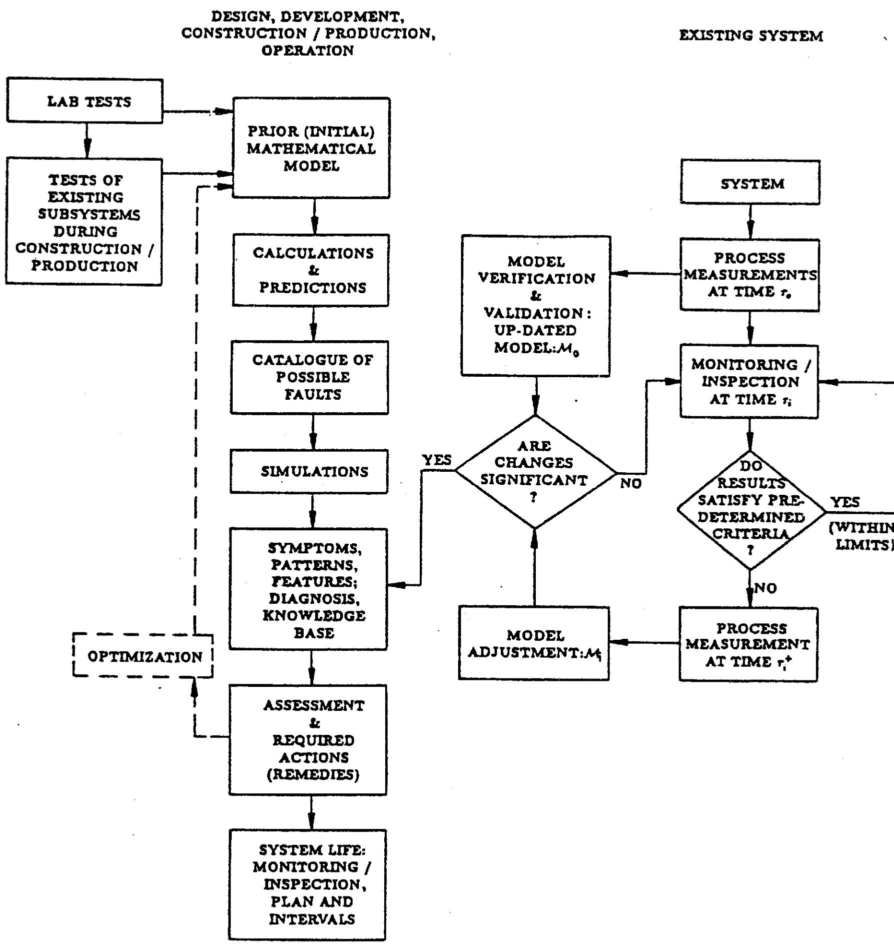

Fig. 3. Flow chart of the iterative procedure of diagnosis.

2.2. The Relationship between Model Error and Damage Detection

The mathematical model of a system in general is uncertain. The model uncertainty includes the model structure and the parameters. In addition, the environmental uncertainties have to be considered. The model uncertainties can be reduced by applying system identification for well-defined environmental conditions. Such a verified mathematical model can be used under additional conditions (validation and usability, see Sect. 1) for the design of diagnostic procedures in order to predict damage symptoms, features and damage severity, dependent on type and location.

Uncertainty models can also be more or less erroneous. Sometimes these errors can be removed by measurements. Uncertainty models describing damage processes are necessary and important for the design and optimization of diagnostic algorithms. They should be able, first, to detect faults which are significant; this means they should lead to parameter modifications in $\mathcal{M}_i$ compared with the parameters of $\mathcal{M}_{i-1}$ larger than the corresponding parameter errors (standard deviations as well as biases). Additionally, they should allow one to distinguish between various classes of faults. The combination of dynamic models and uncertainty models perform the ability to predict the behaviour of the system. Inclusion of the lifetime, an indirect or direct modelling of the fault evolution (Cempel and Natke; 1993a) with their particular uncertainties, results in monitoring/inspection by demand rather then by a rigid schedule.

2.3. Model Adjustment at Various Lifetimes

As Figure 3 indicates, at several lifetimes τ_i the mathematical model has to be adjusted by the use of measurements at this time. The adjusted model is considered to be spatially discretized with n degrees of freedom:

$$\mathcal{M}_i : \ \hat{M}_i\ddot{u}(t) + \hat{C}_i\dot{u}(t) + \hat{K}_i u(t) = p(t), \quad i = 0, 1, 2, ..., N, \tag{1}$$

with the quadratic parameter matrices (of the prior, not adjusted model) M_i, C_i, K_i of order n describing inertias, dampings and stiffnesses, respectively. $u(t)$ is the vector of displacements, dots indicate differentiation with respect to time t , and $p(t)$ is the vector of external forces. All vectors consist of n components.

Note: of course, description of the system in the state space also is possible.

Model adjustment methods are discussed in (Natke, 1992a), the reader can find further details in (Natke *et al.*, 1993). These methods assume a sufficiently accurate model structure, so that the identification is reduced to parameter estimation. One characteristic of the adjustment methods is error modelling within the frame of submodelling. The parameter matrices are partitioned into sums of matrices describing submodels. In order to avoid confusion, in the following the subscript i is suppressed with respect to τ_i.

$$\left.\begin{aligned} M(a_M) &= \textstyle\sum_{\sigma=1}^{S} a_{M\sigma} M_\sigma, \\ C(a_C) &= \textstyle\sum_{\rho=1}^{R} a_{C\rho} C_\rho, \\ K(a_K) &= \textstyle\sum_{\iota=1}^{I} a_{K\iota} K_\iota. \end{aligned}\right\} \tag{2}$$

where a_M, a_C, a_K are vectors of a total of $S + R + I = J$ components. The prior mathematical model has the parameters identical to one, assembled in the vector e; the mathematical model to be updated with the parameter matrices (2) contains the vector a to be estimated. The (extended) weighted least squares estimator is widely applied (Isenberg, 1979; Natke, 1992a; Natke, 1992b; Natke *et al.*, 1993). The estimates $\hat{a}$ inserted into (2) result in the estimated parameter matrices $\hat{M}$, $\hat{C}$, $\hat{K}$ for every lifetime step. In addition, an error estimation has to be performed in order to make certain that the resulting adjusted model is usable (see Sect. 1).

Model adjustment in the manner described here has some difficulties that have to be overcome. One difficulty is the decomposition of the parameter matrices, and the other difficulties result from the fact that here the identification is an inverse problem which is generally ill–posed. The problem of submodelling is unsolved for the general system. In the particular case sufficient prior knowledge is generally available in order to assemble matrix elements of the same error magnitudes, and in order to decompose by real subsystems with respect to special faults (dependent on their sensitivities to be detected). Investigations within theoretical analysis will answer questions concerning the decomposition due to the faults expected.

Application of the weighted least squares within matrix equations of inverse problems together with erroneous measurements, as already mentioned, will lead to ill–conditioned equations. Therefore, regularization methods have to be applied. A powerful regularization is to include additional information, that means extending the relative information content. One method already introduced above is the reduction of the number of parameters to be estimated by submodelling. The relative (with respect to the number of parameters) information content for estimation is enlarged. Another method is to apply the *extended* least squares,

$$J(a) = v^{*T}(a)G_v v(a) + (a - e)^T G_e(a - e), \tag{3}$$

with

$$\left.\begin{array}{ll} v(a) & \text{the vector of residuals dependent on } a \\ v^{*T} & \text{the conjugate and transposed residual vector } v \\ a & \text{the parameter vector to be estimated} \\ e & \text{the prior information of } a \\ G_v & \text{the weighting matrix with respect to } v\text{, equal to the inverse} \\ & \text{covariance matrix } C_{vv}^{-1} \\ G_e & \text{the weighting matrix with respect to } e\text{, equal to the inverse} \\ & \text{covariance matrix } C_{ee}^{-1}. \end{array}\right\} \tag{4}$$

This estimator enables us to include a priori information, for example from the prior mathematical model, as the penalty term. The problem introduced additionally is to determine the regularization parameter or a corresponding matrix of the required weighting. Here cross–validation is recommended if a sufficient number of measurements are available. Otherwise a prior determination of the regularization parameter has to be performed. Regularization in system identification is discussed in (Natke, 1992b; Natke *et al.*, 1993).

If model structure identification is necessary during the lifetime of the system, then the problem difficulties increase immensely. Model structure modifications have first to be detected. This can possibly be done by finding physical non–interpretable parameter estimates, or by estimating additional eigenquantities (mainly eigenfrequencies). As in the observer theory shown (Gertler, 1991; Mook, 1992), and as stated in Subsection 2.2, the structure uncertainty has to be modelled. Nonlinearities can be modelled by an additional parametric term to be estimated. If the system behaves linearly, then additional degrees of freedom will result in additional equations of motions. There is no detailed scientific investigation available yet with respect to structural systems. Structure identification with respect to systems behaving nonlinearly and related problems are discussed in (Natke *et al.*, 1993). A summary states that due to the diversity of nonlinear phenomena there is no unique approach for safety evaluation. Detection and localization of nonlinearities in such systems, for example due to faults, needs more detailed investigation for each application.

The extended system identification methodology shown in Figure 1 indicates the use of limit states as estimates. These quantities can be handled as local properties (information) combined with the global estimation procedure (see, e.g. (Orkisz, 1992)). This is a part of regularization, too, included in the extended weighted least squares, for instance.

3. Resulting Possibilities from the Adjusted Mathematical Model

The adjusted mathematical dynamical models $\mathcal{M}_i$ are models with known confidence and sufficiently small errors of the type (1). Dynamic models are restricted to a finite frequency band, and therefore they describe only a small number of degrees of freedom, in general less than 100.

3.1. Detection

$\mathcal{M}_i$ may be adjusted by the measurement set $\mathcal{T}_i$. The new measurements at the lifetime $\tau_{i+1}, \mathcal{T}_{i+1}$, serve for adjusting $\mathcal{M}_i$. Parts of $\mathcal{T}_{i+1}$, taken directly or the manipulated (information condensation by, for example, correlation or spectral characteristics, symptoms) measurements, and also first estimates (for example, eigenfrequencies) can serve for the detection of (significant) system modifications by their comparison with the corresponding quantities of the model $\mathcal{M}_i$. Additionally, the resulting residuals of certain quantities from $\mathcal{M}_{i+1}$ and $\mathcal{M}_i$ can be taken for the detection of system modifications. Global detection uses scalars, such as the norms of the residuals. Local detection is performed by the components of the residuals. The deviations between estimated $\mathcal{T}_i$ and the measurements $\mathcal{T}_{i+1}$ and the model residuals in Section 2.3 can be understood as errors of the model

$\mathcal{M}_i$ (Natke, 1991). Therefore the problem is transferred to one to detect model errors as discussed in (Natke, 1991).

3.2. Localization

The resulting local residuals, which are the deviations between the parameter matrix elements of the adjusted models $\mathcal{M}_{i+1}$ and $\mathcal{M}_i$, indicate the fault locations (in the model). If the parameter matrices stem from finite element modelling then, for example, the stiffness matrix is the superposition of several element submatrices, and the system locations may not be easily determined. Spectral decompositions of the parameter matrices can also be investigated; the products of the eigenvectors will define locations of modifications by means of modified eigenmodes. However, with the prior knowledge of modelling and dynamic system behaviour, the localization of modifications in the system due to those in the model can be assumed as being solved in particular cases of application.

3.3. Assessment

The dynamic responses (for example, velocities, stresses) predicted by the last adjusted mathematical model $\mathcal{M}_{i+1}$ is generally taken for assessment, that means by comparison with the foregoing state at time τ_i. If the state is changed in such a way that detailed local analysis has to be performed, then it can happen that the dynamic model is not detailed enough for diagnosis purposes. In this case an interface to a static model with a sufficient number of (static) degrees of freedom must be used in order to find out maximum stresses for assessment. It may also be necessary to use this model in order to study causes, and consequently to work out countermeasures.

At the design and development phase some of this analysis was done for possible faults and is already stored in the knowledge base. Only unexpected faults with their modifications therefore need additional analysis.

Trend predictions on the base of $\mathcal{M}_i$, $i = 0(1)N' < N$, N' the current lifetime, use extrapolated data of a model $\mathcal{M}_\tau$ with extrapolated (in τ) model parameters. If a probabilistic model is available, for example coming from fault evolution (Cempel and Natke, 1993a), then these trends can be given with some probabilistic limits. This result leads into the reliability analysis.

4. Decision–Making

4.1. Deterministic and Probabilistic–Based Decisions

Decisions are needed in many manners. Firstly, with regard to damage detection one has to decide whether the symptoms/features satisfy predetermined criteria. This can be probabilistic–based, or deterministic error bounds will be used. Secondly, decisions on the severity of detected faults and their effects are necessary. Dependent on the accuracy of the models used, these decisions are often deterministic-based. The uncertainty model (see Subsection 2.2.) may be probabilistic, but within the uncertainty bounds the decision is made deterministically. The deterministic logic for decision-making does not need any explanation. The probabilistic decision-making needs samples of data and assumptions of the corresponding (joint) probability density functions, and it seems to be hard to verify these assumptions.

4.2. Fuzzy Logic-Based Decisions

Fuzzy sets (Zadeh, 1965) and fuzzy logic (Zadeh, 1973) deal with linguistic variables, the possibility of events happening, and approximate reasoning. Several attempts were made to apply fuzzy sets in assessing structural damage in the early eighties (e.g., Ishizuka *et al.*, 1982; Fu *et al.*, 1982; Yao, 1985). Results of these studies indicated the potential usefulness of such applications. However, it was not continued because of the extreme complexity of the practical problem involved. With the maturity of the fuzzy set theory (e.g., Zimmermann, 1991) and importance of damage and fault detection using system identification approaches today (Natke and Yao, 1988; Natke *et al.*, 1993), it is timely to apply such an approach again to solve practical problems.

There are various ways of describing the damaged condition of a structure. For example, an expert may decide whether an existing structure is "totally collapsed", "partially collapsed", "severely damaged", "moderately damaged", "lightly damaged", or "not damaged". All these linguistic descriptions are meaningful conclusions of an expert following careful and extensive studies.

With the exception of total or partial collapse, which is obvious to every observer, all other damage states are not clearly defined and are thus fuzzy events. On the basis of process measurements and the monitoring/inspection of the existing structure, we may compare the resulting mathematical model $\mathcal{M}_i$ at time τ_i with the model $\mathcal{M}_{i+1}$ at time τ_{i+1}. The changes in parameters of theses two models may be classified as "negative large", "negative small", "almost zero", "positive small", or "positive large". These data may be used in the subsequent process of diagnosis in deciding the damaged state of the structure. Recent attempts along these lines are given by Natke and Yao (1991), Yao and Natke (1992).

5. Summary and Outlook

Damage detection, localization and assessment are important engineering problems which remain to be solved. Solution of these problems would influence safety aspects as well as ones of economics. The recommended procedure is the extended system identification methodology which is a model-based diagnostic procedure. System monitoring/inspection will preserve data to detect significant system/process modifications, and required process[2] measurements will serve for identification purposes, which means mainly for model correction in the initial phase and for model adjustment during the lifetime of the system with its modifications. These verified

[2] input/output description

and validated mathematical models including their extrapolations (for trend predictions), perform the knowledge base, which is assumed to be the best available.

An expert system for supporting the monitoring and measurements of structures (Doll *et al.*, 1993) is under development. In the first stage it will be applicable to bridges and dwellings. Problems have to be solved here, such as detecting sensor failures (see the related papers using the state space formulation and the measuring equation within the observer theory). An interface to finite element modelling is foreseen in order to continue with system identifications, this is in the recursive formulation a self–learning part.

Problems within system identification are those of model decomposition, which seems to be worth recommending due to the incomplete measurements as a part of regularization, and also with regard to system modifications which introduce additional degrees of freedom into the freequency band considered or additional types of forces (model structure modification). The latter concerns their detection less than the introduction into the model. However, for the particular system under investigation this problem can be solvable with a detailed study of possible defects and faults.

The mathematical model here is a model describing the dynamics behaviour: a dynamic model with a few degrees of freedom compared with a static model. For detailed diagnostic purposes and for safety assessment (local stress evaluation) an interface to the static model should be provided for.

The inclusion of intelligent, smart structures, which are equipped a priori with sensors and actuators, will complicate the equations of motion and the failure detection of the related devices. However, this equipment can possibly be used to compensate additional forces due to faults. The optimum decision–making process can be a combination of deterministic, probabilistic and fuzzy approaches with the possible use of neural nets.

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