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MODEL REFERENCE ADAPTIVE CONTROL OF UNKNOWN PLANTS USING DYNAMIC NEURAL NETWORKS

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A direct non-linear adaptive controller solving the tracking problem for unknown dynamic systems which are modelled by dynamic neural networks is discussed. A complete model matching case is examined and convergence of the control error to zero plus boundedness of all signals in the closed loop are ensured.

1. Introduction

The application of artificial neural networks to control has already gained considerable attention within the control systems community, mainly due to their massive parallelism, very fast adaptability and inherent approximation capabilities. In the past four–five years the field has experienced a great amount of research activity, which has led to numerous applications and, furthermore, to the development of certain control architectures, based on neural network models. A beautiful survey of the above-mentioned techniques can be found in a paper by Hunt *et al.* (1992), where links between the field of control science and neural networks were explored and key areas for future research were proposed.

The main problem in the application of neural networks to control is the fact that very interesting simulation results that are provided lack theoretical verification. Crucial properties like stability, convergence and robustness of the overall system must be developed and/or verified. However, recently, interesting theoretical results have begun to emerge, aiming at filling the gap between theory and applications.

The problem of controlling an unknown non-linear dynamic system has been approached from various angles using both direct and indirect adaptive control structures and employing different neural network models. However, all works share the same key idea, that is, since neural networks can approximate arbitrarily well static and dynamic highly non-linear systems, the unknown system can be substituted by a neural network model, which is of known structure but contains a number of unknown parameters (synaptic weights) plus a modelling error term. The unknown parameters may appear both linear or non-linear with respect to the network non-linearities, thus transforming the original problem into a non-linear robust adaptive control problem.

Several answers to the problem of non-linear, but not necessarily robust, adaptive control exist in the literature with typical examples (Campion and Bastin, 1990;

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Kanellakopoulos *et al.*, 1991a; 1991b; Marino and Tomei, 1993a; Pomet and Praly, 1992; Sastry and Isidori, 1989; Taylor *et al.*, 1989). A common assumption made in the above works is that of linear parameterization. Although sometimes it is quite realistic, it constraints considerably the application field. An attempt to relax this assumption and provide a global adaptive output feedback control for a class of non-linear systems, determined by specific geometric conditions, is given by Marino and Tomei (1993b).

Dynamic neural networks for identification and control, as a concept, were first introduced by Narendra and Parthasarathy (1990). They proposed dynamic backpropagation schemes, which are static backpropagation neural networks, connected either in series or in parallel with linear dynamic systems. However, their method requires a great deal of computational time and furthermore, lacks theoretical verification that simulations provide. Sanner and Slotine (1992) incorporate Gaussian radial-basis-function neural networks with sliding mode control and linear feedback to formulate a direct adaptive tracking control architecture for a class of continuous time non-linear dynamic systems. However, the use of sliding mode, which is a discontinuous control law, generally creates various problems, such as the existence and uniqueness of solutions (Polycarpou and Ioannou, 1993), introduction of chattering phenomena (Utkin, 1978) and possibly excitation of high frequency unmodelled dynamics (Young *et al.*, 1977). Polycarpou and Ioannou (1991) employed Lyapunov stability theory to develop stable adaptive laws for identification and control of dynamic systems with unknown non-linearities, using various neural network architectures. Their control results were restricted to SISO feedback linearizable systems. Recently, Rovithakis (1994) and Rovithakis *et al.* (1993; 1994), developed an indirect adaptive control scheme for unknown non-linear dynamical systems, with certain restrictions on the form of the unknown non-linearities. However, although not all the plant states were assumed to be available for measurement, the restrictions imposed on the system need to be relaxed, in order to be more widely applied.

In this paper, dynamic neural networks, of the form that will be described in Section 2, are used as models of the unknown plant, practically transforming the originally unknown system to a dynamic neural network model, which is of known structure but contains a number of unknown constant parameters, known as synaptic weights. Thus we relax the need of the two-stage, identification based control in (Rovithakis, 1994; Rovithakis *et al.*, 1993; 1994), while we broaden the application field. Furthermore, the direct adaptive setting we propose does not require extensive computations and has an easier implementation.

When the dynamic neural network model matches the unknown plant, we provide a comprehensive and rigorous analysis of the stability properties of the closed loop system. Convergence of the control error to zero plus boundedness of all signals in the closed loop are guaranteed, without the need of parameter (weight) convergence, which is assured only if a sufficiency of excitation condition is satisfied.

The paper is organized as follows: in Section 2, we state precisely the problem and the form of the dynamic neural network model, while in Section 3, the complete matching case is examined and basic control and update laws are developed.

1.1. Preliminaries

The following notations and definitions will extensively be used throughout the paper. I denotes the identity matrix. $|\cdot|$ is the usual Euclidean norm of a vector. In case where y is a scalar, $|y|$ denotes its absolute value. If A is a matrix, then $\|A\|$ denotes the Frobenius matrix norm (Golub and Van Loan, 1989), defined as

$$\|A\|^2 = \sum_{ij} |a_{ij}|^2 = \text{trace}\{A^T A\}$$

where trace$\{\cdot\}$ denotes the trace of a matrix. Now let $d(t)$ be a vector function of time. Then,

$$\|d\|_2 \doteq \left(\int_0^\infty |d(\tau)|^2 \, \mathrm{d}\tau \right)^{1/2}$$

and

$$\|d\|_\infty \doteq \sup_{t \geq 0} |d(t)|$$

We will say that $d \in L_2$ when $\|d\|_2$ is finite. Similarly, we will say that $d \in L_\infty$ when $\|d\|_\infty$ is finite.

2. Problem Formulation and the Dynamic Neural Network

We consider affine in the control, non-linear dynamic systems of the form

$$\dot{x} = f(x) + G(x)u \tag{1}$$

where the state x, living in an n-dimensional smooth manifold $\mathcal{M}$, is assumed to be completely measured, the control u is in $\mathbb{R}^n$, f is an unknown smooth vector field called the drift term and G is a matrix with columns being the unknown smooth controlled vector fields g_i, $i = 1, 2, \ldots, n$; $G = [g_1 \; g_2 \; \cdots \; g_n]$.

The tracking problem consists in forcing the state of the system to follow the output of a given stable linear dynamic system of the form

$$\dot{x}_m = -A_m x_m + B_m u_m \tag{2}$$

for every possible input $u_m \in \mathbb{R}^n$. The state of the reference model belongs to $\mathcal{M}$; $-A_m$ is a stable matrix which for simplicity is assumed to be diagonal.

However, the problem, as it is stated above for system (1), is very difficult or even impossible to be solved since the vector fields f, g_i, $i = 1, 2, \ldots, n$ are assumed to be completely unknown. Therefore, it is obvious that in order to provide solution to our problem, it is necessary to have a more accurate model for the unknown plant. For that purpose we apply dynamic neural networks.

Dynamic neural networks are recurrent, fully interconnected nets, containing dynamic elements in their neurons. Therefore, they are described by the following set of differential equations

$$\dot{\hat{x}} = -A\hat{x} + WS(x) + W_{n+1}S'(x)u$$

where $\hat{x} \in \mathcal{M}$; the inputs $u \in \mathbb{R}^n$; W – an $n \times n$ matrix of adjustable synaptic weights; W_{n+1} – an $n \times n$ diagonal matrix of adjustable synaptic weights of the form $W_{n+1} = \text{diag}[w_{1,n+1} w_{2,n+1} \cdots w_{n,n+1}]$ and A is an $n \times n$ matrix with positive eigenvalues which for simplicity can be diagonal. Finally, $S(x)$ is an n-dimensional vector and $S'(x)$ is an $n \times$ diagonal matrix, with elements $s(x_i)$ and $s'(x_i)$, respectively, both smooth (at least twice differentiable) monotone increasing functions which are usually represented by sigmoidals of the form

$$s(x_i) = \frac{k}{1 + e^{-lx_i}}, \qquad s'(x_i) = \frac{k}{1 + e^{-lx_i}} + \lambda$$

for all $i = 1, 2, \ldots, n$, where k, l are parameters representing the bound (k), and slope (l), of sigmoid's curvature and $\lambda > 0$ is a strictly positive constant that shifts the sigmoid, such that $s'(x_i) > 0$ for all $i = 1, 2, \ldots, n$.

Due to the approximation capabilities of the dynamic neural networks, we can assume, with no loss of generality, that the unknown system (1) can be completely described by a dynamic neural network plus a modelling error term $\omega(x, u)$. In other words, there exist weight values $W^\star$ and $W^\star_{n+1}$ such that system (1) can be written as

$$\dot{x} = -Ax + W^\star S(x) + W^\star_{n+1} S'(x) u + \omega(x, u) \tag{3}$$

Therefore, the tracking problem is analyzed for system (3) instead of (1). Since $W^\star$ and $W^\star_{n+1}$ are unknown, our solution consists in designing a control law $u(W, W_{n+1}, x)$ and appropriate update laws for W and W_{n+1} to guarantee convergence of the state to x_m.

In this paper, however, we will treat only the case of complete matching. The presence of a modelling error term and its effect on the stability and robusteness properties of our control scheme are left as open problems.

3. Tracking in the Complete Matching Case

In this section we investigate the adaptive model reference control problem when the modelling error term $\omega(x, u)$ is zero or, in other words when we have complete model matching. Under this assumption the unknown system can be written as

$$\dot{x} = -Ax + W^\star S(x) + W^\star_{n+1} S'(x) u \tag{4}$$

Further, assume that we want the unknown system states to follow the states of the model

$$\dot{x}_m = -A_m x_m + B_m u_m \tag{5}$$

as stated in the previous section. From (4) and (5) we obtain the error equation

$$\dot{e} = -Ax + W^\star S(x) + W^\star_{n+1} S'(x) u + A_m x_m - B_m u_m \tag{6}$$

where we have defined

$$e \doteq x - x_m$$

If we add and subtract the term Ax_m, expression (6) becomes

$$\begin{aligned}\dot{e} &= -Ax + W^\star S(x) + W^\star_{n+1}S'(x)u + A_m x_m - B_m u_m + Ax_m \\ &- Ax_m - Ae + W^\star S(x) + W^\star_{n+1}S'(x)u + \tilde{A}x_m - B_m u_m \qquad (7)\end{aligned}$$

where

$$\tilde{A} = A_m - A$$

Let us take a function $H(e)$ of class C^2, from $\mathcal{M}$ to $\mathbb{R}^+$, whose derivative with respect to time is

$$\dot{H}(t) = \frac{\partial H}{\partial e}[-Ae + W^\star S(x) + W^\star_{n+1}S'(x)u + \tilde{A}x_m - B_m u_m]$$

The above equation is linear with respect to $W^\star$ and $W^\star_{n+1}$ and can be written as

$$\dot{H} + \frac{\partial H}{\partial e}Ae - \frac{\partial H}{\partial e}\tilde{A}x_m + \frac{\partial H}{\partial e}B_m u_m = \frac{\partial H}{\partial e}W^\star S(x) + \frac{\partial H}{\partial e}W^\star_{n+1}S'(x)u \qquad (8)$$

Define

$$\nu \doteq \frac{\partial H}{\partial x}WS(x) + \frac{\partial H}{\partial x}W_{n+1}S'(x)u - \dot{H} - \frac{\partial H}{\partial x}Ax + \frac{\partial H}{\partial e}\tilde{A}x_m - \frac{\partial H}{\partial e}B_m u_m$$

where W and W_{n+1} are the estimates of $W^\star$ and $W^\star_{n+1}$, respectively, obtained by update laws which are to be designed later. However, the above signal cannot be measured since $\dot{H}$ is unknown. To round this problem, we use the following filtered version of ν:

$$\begin{aligned}\dot{\xi} + r\xi &= \nu \\ &= \frac{\partial H}{\partial e}WS(x) + \frac{\partial H}{\partial e}W_{n+1}S'(x)u - \dot{H} - \frac{\partial H}{\partial e}Ae + \frac{\partial H}{\partial e}\tilde{A}x_m - \frac{\partial H}{\partial e}B_m u_m \\ &= -\dot{H} + \frac{\partial H}{\partial e}[-Ae + WS(x) + W_{n+1}S'(x)u + \tilde{A}x_m - B_m u_m] \qquad (9)\end{aligned}$$

where r – a strictly positive constant. To implement (9), we take

$$\xi \doteq \eta - H \qquad (10)$$

Employing (10), equation (9) can be written as

$$\dot{\eta} + r\eta = rH + \frac{\partial H}{\partial e}[-Ae + WS(x) + W_{n+1}S'(x)u + \tilde{A}x_m - B_m u_m] \qquad (11)$$

with the state $\eta \in \mathbb{R}$. This method is referred to as error filtering. Furthermore, we choose $H(e)$ to be

$$H(e) = \frac{1}{2}|e|^2$$

Hence, (11) becomes

$$\dot{\eta} + r\eta = rH - e^T Ae + e^T WS(x) + e^T W_{n+1}S'(x)u + e^T \tilde{A}x_m - e^T B_m u_m \quad (12)$$

To continue, consider the Lyapunov-like function

$$\mathcal{L} = \frac{1}{2}\xi^2 + \frac{1}{2}\text{trace}\{\tilde{W}^T\tilde{W}\} + \frac{1}{2}\text{trace}\{\tilde{W}_{n+1}^T\tilde{W}_{n+1}\} \quad (13)$$

where

$$\tilde{W} = W - W^\star, \qquad \tilde{W}_{n+1} = W_{n+1} - W_{n+1}^\star$$

If we take the derivative of (13) with respect to time, we obtain

$$\dot{\mathcal{L}} = \xi\dot{\xi} + \text{trace}\{\dot{W}^T\tilde{W}\} + \text{trace}\{\dot{W}_{n+1}^T\tilde{W}_{n+1}\} \quad (14)$$

Employing (9), equation (14) becomes

$$\begin{aligned}\dot{\mathcal{L}} &= -re^2 + \xi[-\dot{H} - e^T Ae + x^T WS(x) + e^T W_{n+1}S'(x)u \\ &+ e^T\tilde{A}x_m - e^T B_m u_m] + \text{trace}\{\dot{W}^T\tilde{W}\} + \text{trace}\{\dot{W}_{n+1}^T\tilde{W}_{n+1}\} \quad (15)\end{aligned}$$

which together with (8) gives

$$\begin{aligned}\dot{\mathcal{L}} &= -re^2 + \xi[-e^T W^\star S(x) - e^T W_{n+1}^\star S'(x)u + e^T WS(x) \\ &+ e^T W_{n+1}S'(x)u] + \text{trace}\{\dot{W}^T\tilde{W}\} + \text{trace}\{\dot{W}_{n+1}^T\tilde{W}_{n+1}\}\end{aligned}$$

or equivalently

$$\begin{aligned}\dot{\mathcal{L}} = -r\xi^2 + \xi e^T\tilde{W}S(x) + \xi e^T\tilde{W}_{n+1}S'(x)u + \text{trace}\{\dot{W}^T\tilde{W}\} \\ +\text{trace}\{\dot{W}_{n+1}^T\tilde{W}_{n+1}\} \quad (16)\end{aligned}$$

Hence, if we choose

$$\text{trace}\{\dot{W}^T\tilde{W}\} = -\xi e^T\tilde{W}S(x) \quad (17)$$

$$\text{trace}\{\dot{W}_{n+1}^T\tilde{W}_{n+1}\} = -\xi e^T\tilde{W}_{n+1}S'(x)u \quad (18)$$

then $\dot{\mathcal{L}}$ becomes

$$\dot{\mathcal{L}} = -r\xi^2 \leq 0 \quad (19)$$

It can be easily verified that (17), (18) can be written element-wise as

$$\dot{w}_{ij} = -\xi e_i s(x_j), \qquad \dot{w}_{in+1} = -\xi e_i s'(x_i)u_i \quad (20)$$

for all $i, j = 1, 2, \ldots, n$, and in matrix form as

$$\dot{W} = -\xi e S^T(x), \qquad \dot{W}_{n+1} = -\xi e' S'(x)U \quad (21)$$

where

$$e' = \text{diag}[e_1, e_2, \ldots, e_n], \qquad U = \text{diag}[u_1, u_2, \ldots, u_n]$$

Now we can prove the following Lemma

Lemma 1. *Consider the system*

$$\dot{x} = -Ax + W^\star S(x) + W^\star_{n+1} S'(x) u$$

$$\dot{x}_m = -A_m x_m + B_m u_m$$

$$\dot{\eta} = -r\eta + rH - e^T A e + e^T W S(x) + e^T W_{n+1} S'(x) u + e^T \tilde{A} x_m - e^T B_m u_m$$

$$\xi = \eta - H$$

$$H = \frac{1}{2}|e|^2$$

$$e = x - x_m$$

The update laws

$$\dot{W} = -\xi e S^T(x)$$

$$\dot{W}_{n+1} = -\xi e' S'(x) U$$

guarantee the following properties

- $\xi, |e|, W, W_{n+1}, \eta \in L_\infty$
- $|\xi| \in L_2$
- $\lim_{t\to\infty} \xi(t) = 0, \quad \lim_{t\to\infty} \dot{W}(t) = 0, \quad \lim_{t\to\infty} \dot{W}_{n+1}(t) = 0$

provided that $u \in L_\infty$.

Proof. We have that $\mathcal{L} \in L_\infty$ which implies $\xi, W, W_{n+1} \in L_\infty$. Since $u \in L_\infty$, we get $e \in L_\infty$, hence $H \in L_\infty$. Furthermore, $\xi = \eta - H$, hence $\eta \in L_\infty$. Since $\mathcal{L}$ is a monotone decreasing function of time and is bounded from below, $\lim_{t\to\infty} \mathcal{L}(t) = \mathcal{L}_\infty$ exists. Therefore, by integrating $\dot{\mathcal{L}}$ from 0 to ∞, we have

$$\int_0^\infty |\xi|^2 \, dt = \frac{1}{r}[\mathcal{L}(0) - \mathcal{L}_\infty] < \infty$$

which implies that $|\xi| \in L_2$. We also have that

$$\dot{\xi} = -r\xi + e^T \tilde{W} S(x) + e^T \tilde{W}_{n+1} S'(x) u$$

Hence, $\dot{\xi} \in L_\infty$ provided that $u \in L_\infty$. Having in mind that $\xi \in L_2 \bigcap L_\infty$ and $\dot{\xi} \in L_\infty$, by applying Barbalat's Lemma (Rouche *et al.*, 1977) we conclude that $\lim_{t\to\infty} \xi(t) = 0$. Now using the boundedness of u, $S(x)$, $S'(x)$, x and the convergence of $\xi(t)$ to zero, we have that $\dot{W}$, $\dot{W}_{n+1}$ also converge to zero. ∎

Remark 1. From Lemma 1 we cannot conclude anything about the convergence of the weights W and W_{n+1} to their optimal values $W^{\star}$ and $W^{\star}_{n+1}$, respectively. In order to guarantee convergence, $S(x)$, $S'(x)u$ need to satisfy a persistency of excitation condition. A signal $z(t) \in \mathbb{R}^n$ is persistently exciting in $\mathbb{R}^n$ if there exist positive constants β_0, β_1, T such that

$$\beta_0 I \leq \int_t^{t+T} z(\tau) z^T(\tau)\,\mathrm{d}\tau \leq \beta_1 I, \quad \forall t \geq 0$$

However, such a condition cannot be verified *a priori* since $S(x)$ and $S'(x)u$ are non-linear functions of the state x.

To proceed further, we observe that $\dot{H}$ can be written in the form

$$\dot{H} = e^T[-Ae + WS(x) + W_{n+1}S'(x)u + \tilde{A}x_m - B_m u_m] \\ -e^T\tilde{W}S(x) - e^T\tilde{W}_{n+1}S'(x)u$$

Hence, if we choose the control input u to be

$$u = -[W_{n+1}S'(x)]^{-1}[WS(x) + \tilde{A}x_m - B_m u_m] \tag{22}$$

then $\dot{H}$ becomes

$$\dot{H} = -e^T Ae - e^T\tilde{W}S(x) - e^T\tilde{W}_{n+1}S'(x)u \tag{23}$$

Moreover, (23) can be written in the form

$$\dot{H} \leq -\frac{c}{2}|\xi|^2 - \dot{\xi} - r\xi \tag{24}$$

where

$$c = 2n\lambda_{min}(A)$$

and $\lambda_{min}(A)$ denotes the minimum eigenvalue of the matrix A. Observe that (24) is equivalent to the condition

$$\dot{H} \leq -cH - \dot{\xi} - r\xi \tag{25}$$

Furthermore,

$$H = \eta - \xi$$

hence (25) becomes

$$\dot{\eta} \leq -c\eta + c\xi - r\xi \leq -c\eta + (c+r)|\xi| \tag{26}$$

However, in order to apply the control law (22), we have to assure the existence of $[W_{n+1}S'(x)]^{-1}$. Since W_{n+1} and $S'(x)$ are diagonal matrices and $s'(x_i) > 0$, $\forall i = 1, 2, \ldots, n$, all we need to establish is $w_{in+1}(t) \neq 0$, $\forall t \geq 0$, $\forall i = 1, 2, \ldots, n$. Hence, $w_{in+1}(t)$, $i = 1, 2, \ldots, n$, are confined through the use of a projection algorithm (Goodwin and Mayne, 1987; Ioannou and Datta, 1991; Narendra and Annaswamy,

1989) to the set $\mathcal{W}' = \{w_{in+1} : 0 < \varepsilon \leq w_{in+1} \leq w^m\}$ where ε, w^m are appropriately chosen positive constants[1]. In particular, the standard update law defined by (20) is modified to

$$\dot{w}_{in+1} = \begin{cases} -\xi e_i s'(x_i) u_i & \text{if } w_{in+1} \in \mathcal{W}' \text{ or } w_{in+1}\mathrm{sgn}(w^*_{in+1}) = \varepsilon \\ & \text{and } \xi e_i s'(x_i) u_i \mathrm{sgn}(w^*_{in+1}) \leq 0 \\ 0 & \text{if } w_{in+1}\mathrm{sgn}(w^*_{in+1}) = \varepsilon \text{ and} \\ & \xi e_i s'(x_i) u_i \mathrm{sgn}(w^*_{in+1}) > 0 \\ -\xi e_i s'(x_i) u_i & \text{if } w_{in+1} \in \mathcal{W}' \text{ or } w_{in+1}\mathrm{sgn}(w^*_{in+1}) = w^m \\ & \text{and } \xi e_i s'(x_i) u_i \mathrm{sgn}(w^*_{in+1}) \geq 0 \\ 0 & \text{if } w_{in+1}\mathrm{sgn}(w^*_{in+1}) = w^m \text{ and} \\ & \xi e_i s'(x_i) u_i \mathrm{sgn}(w^*_{in+1}) < 0 \end{cases} \tag{27}$$

for all $i, j = 1, 2, \ldots, n$, where the update law is written element-wise for easier understanding. The following Lemma presents in detail the properties of the projection algorithm.

Lemma 2. *The update law (20) with the projection modification (27) can only make* $\dot{\mathcal{L}}$ *more negative and, in addition, guarantee that* $w_{in+1} \in \mathcal{W}'$ *for all* $i = 1, 2, \ldots, n$, *provided that* $w_{in+1}(0) \in \mathcal{W}'$ *and* $w^*_{in+1} \in \mathcal{W}'$.

Proof. The proof of Lemma 2 is given in Appendix A. ■

In principle, the projection modification does not alter $\dot{w}_{in+1}$ given by (20) if w_{in+1} is in the interior $\mathcal{W}'_{in}$ of $\mathcal{W}'$ or if w_{in+1} is at the boundary $\partial(\mathcal{W}')$ of $\mathcal{W}'$ and has the tendency to move inward. Otherwise, it subtracts a vector normal to the boundary so that we get a smooth transformation from the original vector field, to an inward or tangent to the boundary, vector field.

Remark 2. Let us note that in order to apply the projection algorithm (27) we need to know the sign of the unknown parameter w^*_{in+1}.

To continue, we need to recall the following well-known Lemma (Desoer and Vidyasagar, 1975).

Lemma 3. *Let* ζ *be a* C^1*-function defined on* $[0, T)$, *where* $0 \leq T \leq \infty$, *satisfying*

$$\dot{\zeta} \leq -c\zeta + \alpha(t)\zeta + \beta(t)$$

where c *is a strictly positive constant and* $\alpha(t)$ *and* $\beta(t)$ *are two positive functions of time belonging to* $L_2(0, T)$, *that is*

$$\int_0^T \alpha^2(t)\,\mathrm{d}t \leq M_1 < \infty$$

[1] Observe that w_{in+1} can also be confined to be negative. However, the above choice does not harm the generality.

and

$$\int_0^T \beta^2(t)\,\mathrm{d}t \le M_2 < \infty$$

Under this assumption, $\zeta(t)$ *is bounded from above on* $(0,T)$ *and precisely*

$$\zeta(t) \le e^{\frac{M_1}{c}}\left[\zeta(0) + \sqrt{\frac{2}{c}}\sqrt{M_2}\right], \qquad \forall t \in [0,T)$$

Moreover, if T *is infinite, then*

$$\limsup_{t\to\infty} \zeta(t) \le 0$$

■

Let us note that inequality (26) with $r=1$ becomes

$$\dot\eta \le -c\eta + (1+c)|\xi| \tag{28}$$

However, from Lemma 1 we have that $|\xi| \in L_2$. Therefore, we obtain $(1+c)|\xi| \in L_2$. Furthermore, observe that T can be obviously extended to be infinite. Hence, Lemma 3 can be applied to (28) with $M_1 = 0$ to obtain

$$\limsup_{t\to\infty} \eta(t) \le 0 \tag{29}$$

Moreover, since $H = \eta - \xi$ and $H \ge 0$, we have

$$\eta(t) \ge \xi(t)$$

or

$$-\eta(t) \le -\xi(t) \tag{30}$$

However, from Lemma 1 we have

$$\lim_{t\to\infty} \xi(t) = 0 \tag{31}$$

Hence, (30) together with (29) and (31) give

$$\lim_{t\to\infty} \eta(t) = 0 \tag{32}$$

Furthermore, since $H = \eta - \xi$, (31) and (32) yield

$$\lim_{t\to\infty} H(e(t)) = 0$$

which by the definition of $H(e)$ finally implies that

$$\lim_{t\to\infty} |e(t)| = 0$$

Therefore, we have proven the following theorem

Theorem 1. *The closed loop system*

$$\dot{x} = -Ax + W^{\star}S(x) + W^{\star}_{n+1}S'(x)u$$

$$\dot{x}_m = -A_m x_m + B_m u_m$$

$$\dot{\eta} = -r\eta + rH - e^T Ae + e^T WS(x) + e^T W_{n+1}S'(x)u + e^T \tilde{A}x_m - e^T B_m u_m$$

$$u = -[W_{n+1}S'(x)]^{-1}[WS(x)\tilde{A}x_m - B_m u_m]$$

$$\xi = \eta - H$$

$$H = \frac{1}{2}|e|^2$$

$$r = 1$$

together with the update laws

$$\dot{w}_{ij} = -\xi e_i s(x_j)$$

$$\dot{w}_{in+1} = \begin{cases} -\xi e_i s'(x_i)u_i & \text{if } w_{in+1} \in \mathcal{W}' \text{ or } w_{in+1}\mathrm{sgn}(w^{\star}_{in+1}) = \varepsilon \\ & \text{and } \xi e_i s'(x_i)u_i \mathrm{sgn}(w^{\star}_{in+1}) \leq 0 \\ 0 & \text{if } w_{in+1}\mathrm{sgn}(w^{\star}_{in+1}) = \varepsilon \text{ and} \\ & \xi e_i s'(x_i)u_i \mathrm{sgn}(w^{\star}_{in+1}) > 0 \\ -\xi e_i s'(x_i)u_i & \text{if } w_{n+1} \in \mathcal{W}' \text{ or } w_{in+1}\mathrm{sgn}(w^{\star}_{in+1}) = w^m \\ & \text{and } \xi e_i s'(x_i)u_i \mathrm{sgn}(w^{\star}_{in+1}) \geq 0 \\ 0 & \text{if } w_{in+1}\mathrm{sgn}(w^{\star}_{in+1}) = w^m \text{ and} \\ & \xi e_i s'(x_i)u_i \mathrm{sgn}(w^{\star}_{in+1}) < 0 \end{cases}$$

for all $i, j = 1, 2, \ldots, n$ *guarantee that*

$$\lim_{t\to\infty} |e(t)| = 0$$

■

4. Conclusions

A direct adaptive model reference controller, for unknown non-linear dynamic systems, that are modelled by dynamic neural networks is discussed. It is shown that if the dynamic neural network perfectly models the unknown system, then convergence of the control error to zero plus boundedness of all signals in the closed loop are assured, while parameter convergence is not required. The effect of a modelling error term on the stability and robusteness of our control scheme is left as an open problem.

Appendix A

Proof of Lemma 2. In order to prove that the projection modification given by (27) can only make $\dot{\mathcal{L}}$ more negative, we observe that the update law (27) has the same form as the one without the projection except the additional term

$$\mathcal{R} = \begin{cases} \xi e_i s'(x_i)u_i & \text{if } w_{in+1}\text{sgn}(w^\star_{in+1}) = \varepsilon \text{ and } \xi e_i s'(x_i)u_i\text{sgn}(w^\star_{in+1}) > 0 \\ & \text{or if } w_{in+1}\text{sgn}(w^\star_{in+1}) = w^m \text{ and } \xi e_i s'(x_i)u_i\text{sgn}(w^\star_{in+1}) < 0 \\ 0 & \text{otherwise} \end{cases}$$

in the expression for $\dot{w}_{in+1}$ for all $i = 1, 2, \ldots, n$.

Thus $\dot{\mathcal{L}}$ is augmented by the following quantity

$$\mathcal{R}_a = \sum_{i=1}^{n} \xi e_i s'(x_i)u_i\tilde{w}_{in+1}$$

Furthermore, assume without loss of generality that $w^\star_{in+1} > 0$. Hence, $\text{sgn}(w^\star_{in+1}) = 1$. Now, having in mind that $\tilde{w}_{in+1} = w_{in+1} - w^\star_{in+1}$, we obtain

$$\mathcal{R}_a = \sum_{i=1}^{n} \xi e_i s'(x_i)u_i(w_{in+1} - w^\star_{in+1})$$

- **Case 1:** $w_{in+1} = \varepsilon$

  Now $\mathcal{R}_a$ becomes

$$\mathcal{R}_a = \sum_{i=1}^{n} \xi e_i s'(x_i)u_i(\varepsilon - w^\star_{in+1})$$

  However, $\varepsilon - w^\star_{in+1} < 0$ and $\xi e_i s'(x_i)u_i > 0$ hold by definition. Hence, $\mathcal{R}_a < 0$.

- **Case 2:** $w_{in+1} = w^m$

  In this case we have

$$\mathcal{R}_a = \sum_{i=1}^{n} \xi e_i s'(x_i)u_i(w^m - w^\star_{in+1})$$

  However, we have $w^m - w^\star_{in+1} > 0$ and $\xi e_i s'(x_i)u_i < 0$. Therefore again $\mathcal{R}_a < 0$.

Thus $\dot{\mathcal{L}}$ is augmented in either case by a negative term. Furthermore, to verify that $w_{in+1} \in \mathcal{W}'$ $\forall t \geq 0$, we examine the sign of $\dot{w}_{in+1}$ when w_{in+1} reaches the boundary of $\mathcal{W}'$. With no loss of generality we assume again that $w^\star_{in+1} > 0$.

- **Case 1:** $w_{in+1} = \varepsilon$

  In this case we have

$$\dot{w}_{in+1} = \begin{cases} -\xi e_i s'(x_i) u_i & \text{if } w_{in+1} \in \mathcal{W}' \text{ or } w_{in+1} = \varepsilon \text{ and } \xi e_i s'(x_i) u_i \leq 0 \\ 0 & \text{if } w_{in+1} = \varepsilon \text{ and } \xi e_i s'(x_i) u_i > 0 \end{cases}$$

  Hence, when $w_{in+1} = \varepsilon$ we have $\dot{w}_{in+1} \geq 0$, which implies that w_{in+1} is directed towards the interior of $\mathcal{W}'$, provided that $w^*_{in+1} \in \mathcal{W}'$ and $w_{in+1}(0) \in \mathcal{W}'$.

- **Case 2:** $w_{in+1} = w^m$

  In this case

$$\dot{w}_{in+1} = \begin{cases} -\xi e_i s'(x_i) u_i & \text{if } w_{n+1} \in \mathcal{W}' \text{ or } w_{in+1} = w^m \text{ and } \xi e_i s'(x_i) u_i \geq 0 \\ 0 & \text{if } w_{in+1} = w^m \text{ and } \xi e_i s'(x_i) u_i < 0 \end{cases}$$

  Hence, when $w_{n+1} \in \mathcal{W}'$, we have $\dot{w}_{in+1} \leq 0$, which again implies that w_{in+1} is directed towards the interior of $\mathcal{W}'$, provided that $w^*_{in+1} \in \mathcal{W}'$ and $w_{in+1}(0) \in \mathcal{W}'$. ■

References

Campion G. and Bastin G. (1990): *Indirect adaptive state feedback control of linearly parametrized non-linear systems.* — Int. J. Adap. Contr. Signal Processing, v.4, pp.345–358.

Desoer C.A. and Vidyasagar M. (1975): *Feedback Systems: Input-Output Properties.* — New-York: Academic Press.

Golub G.H. and Van Loan C.F. (1989): *Matrix Computations.* — The John Hopkins Univ. Press, (2nd Edition).

Goodwin G.C. and Mayne D.Q. (1987): *A parameter estimation perspective of continuous time model reference adaptive control.* — Automatica, v.23, No.1, pp.57–70.

Hunt K.J., Sbarbaro D., Zbikowski R. and Gawthrop P.J. (1992): *Neural networks for control systems – a survey.* — Automatica, v.28, No.6, pp.1083–1112.

Ioannou P.A. and Datta A. (1991): *Robust adaptive control: design, analysis and robustness bounds.* — In: P.V. Kokotovic (Ed.), Foundations of Adaptive Control, pp.71–152, Berlin: Springer-Verlag.

Kanellakopoulos I., Kokotovic P.V. and Marino R. (1991a): *An extended direct scheme for robust adaptive non-linear control.* — Automatica, v.27, No.2, pp.247–255.

Kanellakopoulos I., Kokotovic P.V. and Morse A.S. (1991b): *Systematic design of adaptive controllers for feedback linearizable systems.* — IEEE Trans. Automat. Contr., v.36, No.11, pp.1241–1253.

Marino R. and Tomei P. (1993a): *Global adaptive output feedback control of non-linear systems, part I: linear parameterization.* - – IEEE Trans. Automat. Contr., v.38, No.1, pp.17–32.

Marino R. and Tomei P. (1993b): *Global adaptive output feedback control of non-linear systems, part II: non-linear parameterization.* — IEEE Trans. Automat. Contr., v.38, No.1, pp.33– 48.

Narendra K.S. and Annaswamy A.M. (1989): *Stable Adaptive Systems.* — Englewood Cliffs: Prentice Hall.

Narendra K.S. and Parthasarathy K. (1990): *Identification and control of dynamical systems using neural networks.* — IEEE Trans. Neural Networks, v.1, No.1, pp.4–27.

Polycarpou M.M. and Ioannou P.A. (1991): *Identification and control of non-linear systems using neural network models: design and stability analysis.* — Tech. Rep. 91-09-01, Dept. Elec. Eng.-Systems, Univ. of Southern Cal., Los Angeles.

Polycarpou M.M. and Ioannou P.A. (1993): *On the existence and uniqueness of solutions in adaptive control systems.* — IEEE Trans. Automat. Contr., v.38, No.3, pp.474–479.

Pomet J.-B. and Praly L. (1992): *Adaptive non-linear regulation: estimation from the Lyapunov equation.* — IEEE Trans. Automat. Contr., v.37, No.6, pp.729–740.

Rouche N., Habets P. and Laloy M. (1977): *Stability Theory by Liapunov's Direct Method.* — New York: Springer-Verlag.

Rovithakis G.A. (1994): *Recurrent Neural Network Architectures for Adaptive Control of Unknown Plants.* — M.Sc. Thesis, Technical University of Crete, Dept. of Electronic & Computer Engineering, Greece.

Rovithakis G.A. and Christodoulou M.A. (1994): *Adaptive control of unknown plants using dynamical neural networks.* — IEEE Trans. Syst., Man, Cybern., v.24, No.3, pp.400–412.

Rovithakis G.A., Kosmatopoulos E.B. and Christodoulou M. A. (1993): *Robust adaptive control of unknown plants using recurrent high order neural networks — application to mechanical systems.* — IEEE Systems, Man and Cybernetics Conf., Le Touquet, (France) October 17–20, pp.57–62.

Sanner R.M. and Slotine J.-J. (1992): *Gaussian networks for direct adaptive control.* — IEEE Trans. Neural Networks, v.3, No.6, pp.837–863.

Sastry S. and Isidori A. (1989): *Adaptive control of linearizable systems .* — IEEE Trans. Automat. Contr., v.34, No.11, pp.1123–1131.

Taylor D.G., Kokotovic P.V., Marino R. and Kanellakopoulos I. (1989): *Adaptive regulation of non-linear systems with unmodeled dynamics.* — IEEE Trans. Automat. Contr., v.34, No.4, pp.405–412.

Utkin V.I. (1978): *Sliding Modes and their Applications to Variable Structure Systems.* — Moscow: MIR Publishers.

Young K.D., Kokotovic P.V. and Utkin V.I. (1977): *A singular perturbation analysis of high gain feedback systems.* — IEEE Trans. Automat. Contr., v.AC–22, No.6, pp.931–938.