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FLC DESIGN FOR MULTI-OBJECTIVE SYSTEMS

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For a system with multiple objectives, designing the rule base for a fuzzy-logic controller (FLC) is not an easy task. In particular, when the fuzzy if-then rules are obtained from human operators, they can be inconsistent due to possibly conflicting multiple objectives. In this paper, we propose an FLC design scheme that is suitable for this type of systems. The proposed controller consists of a supervisory controller and many subcontrollers. Each subcontroller is a typical FLC responsible for the corresponding control objective and the supervisory controller coordinates the subcontrollers by adjusting the weights. Some learning mechanism is also introduced for the supervisory controller to enhance its performance. Then, the stability issue of the proposed FLC is discussed when it is used for a regulation system. By means of a simple example, the proposed algorithm is shown to be effective for multi-objective systems.

1. Introduction

The fuzzy logic controller is often designed by directly modelling actions of the human operator in a human-in-loop system (Seaman *et al.*, 1994; Wang and Mendel, 1992). However, when there are several control objectives to satisfy in this system, it is not an easy task to design the rule base for a fuzzy-logic controller (FLC). Usually, when the fuzzy if-then rules are obtained by interviewing the operators, we may end up with several groups of rules satisfying partial control objectives instead of rules satisfying all the control objectives simultaneously. This can be due to complexities and/or uncertainties of the plant, i.e. when the plant is complex and uncertain, the human tends to decompose the system into several subsystems in reference to the multiple control objectives and tries to render control rules for each simplified subsystem. Once some form of control rules is obtained for each subsystem, then the remaining task is to design a coordinator by which the multiple groups of rules are fused appropriately.

The design problem for a complex system with multiple objectives has been dealt with by a number of researchers. For example, a design scheme is presented in (Katai *et al.*, 1994; Yu and Bien, 1994) in which all the obtained groups of rules were simply accumulated without discrimination to form a new rule base for inference. But this technique may result in a biased control output toward some special control objectives (Katai *et al.*, 1994). The methods in (Kim and Kim, 1994; Tanaka *et al.*, 1993; Wang and Mendel, 1992) are based on assigning certainty factors or weights to the rules.

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But such an assignment is often a tedious and difficult task in case there are many rules and/or in case *a priori* knowledge is weak.

In this paper, it is proposed that an FLC be designed to handle each control objective and then the outputs of all sub-FLCs are combined by means of variable weights. The weights are varied according to the results of evaluating the outputs of the system. This strategy is adopted from the observation that human operators usually change the priorities of the control objectives based on the outputs of the plant. That is to say, an operator tends to consider each group of rules satisfying each corresponding control objective as an independent module and then tries to accomplish the multiple control objectives simultaneously by assigning some weight to each module, not to each rule. The proposed FLC design scheme is a realization of such a control strategy of human operators when there are multiple control objectives.

The paper is organized as follows. In Section 2, a design scheme of the proposed FLC is first specifically described. Then, the structure of the proposed FLC and the decision making mechanism for the weights are given. In addition, the scheme is applied to an example system to show the effectiveness for multi-objective systems. The stability of the proposed FLC for regulator problems is studied in Section 3. Concluding remarks are given in Section 4.

2. Multi-Objective FLC Design

2.1. Problem Description

Consider the plant described by the state equation

$$\dot{x}(t) = f(x(t),\ u(t)) \tag{1}$$

$$y(t) = g(x(t),\ u(t)) \tag{2}$$

where $x(t)$ is an n-dimensional state vector, $y(t)$ is a p-dimensional output vector, $u(t)$ is a scalar input, and $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^p$ are non-linear mappings, respectively. Suppose there are M control objectives to satisfy:

$$Q_i = q_i(x(\cdot), u(\cdot)), \qquad i = 1, 2, \ldots, M \tag{3}$$

where $q_i(\cdot)$ denotes some functional of the function arguments $x(\cdot)$ and $u(\cdot)$. Then, the control problem may be formulated as a multi-objective optimization problem to find a control output which maximizes the M control objective functions Q_i, $i = 1, 2, \ldots, M$ in (3) with constraints (1), (2). Some authors investigated the problem under the assumption that the multiple objective functions were combined in a linear quadratic form (Guez and Ahmad, 1991; Kyr, 1988; Li, 1990; 1993). But, it is pointed out (Sakawa *et al.*, 1994; Seaman *et al.*, 1994);) that it is often inadequate to represent the objective functions only as a linear quadratic form.

To handle effectively such a case of multiple objective optimization problem, one may adopt the concept of a satisfaction degree (Zimmermann, 1991) of each control objective. Here, the satisfaction degree of a control objective is defined as the value of a criterion selected to represent the degree of how much the control objective under consideration is satisfied (Sakawa *et al.*, 1994). Let the satisfaction degree of the i-th control objective be denoted by P_i ($i = 1, 2, \ldots, M$), where $P_i \in [0, 1]$. Then the problem in this context is to design a controller such that the overall satisfaction degree J is maximized, where

$$J = \min_{1 \leq i \leq M} P_i \tag{4}$$

Mathematically speaking, the above formulation becomes a max-min problem which renders very few tractable solutions (Li, 1990; Sakawa *et al.*, 1994; Seaman *et al.*, 1994). This is due to difficulties in applying the existing multi-objective programming methods to the problem.

As a practical way, a heuristic approach based on a reasoning paradigm and knowledge base of the human operator in a human-in-loop system is considered in this paper. The advantage of this approach is to lessen the computational complexities efficiently and to deal effectively with the imprecise data caused by uncertainties and/or non-linearities in the dynamic system.

For this purpose, in our approach it is necessary to obtain the if-then rules from the human operator. However, it is not an easy task to obtain a control rule base maximizing the overall satisfaction degree (4) at once since there can be inconsistencies among the fuzzy if-then rules, and accordingly, it is not clear how some P_i is given more weight. For example, when interviews were conducted with the operators of the overhead cranes installed in Pohang Iron and Steel Company in Korea (Yu and Bien, 1994; Yu *et al.*, 1993), it was found that the operators could explain their behaviour of positioning the trolley only without referring to reducing the swing at one time while explaining their behaviour of reducing the swing only without considering the positioning of the trolley (Yu *et al.*, 1993). As a result, the fuzzy if-then rules for overhead crane control were obtained as two groups of rules, i.e. the group of rules for positioning the trolley and the group of rules for reducing the swing, respectively. When the two groups of rules were simply integrated to form a rule base, we found that the FLC based on such a rule base showed a biased control output toward one control objective (Yu and Bien, 1994).

In (Kim and Kim, 1994) a design scheme is proposed in which constant but different weights are assigned to all the rules. Since the performance of a control system is determined by its collective rule base, it can be difficult and tedious to assign a weight to each of the rules when there are many rules in the rule base and/or there are complexities and uncertainties in the relationship between the rules and the control objectives.

As a means to alleviate the difficulties mentioned above, it is proposed in this paper that the obtained groups of rules be treated as modules and then some appropriate weight is assigned to each module. This approach is based on our experience (Yu *et al.*, 1993) of observing how the operators of the overhead crane achieve a satisfactory performance by changing the weights between the objective of positioning the trolley and that of reducing the swing according to the outputs of the plant. For example, the operators tend to concentrate on the position control when the trolley is far from the target position and change their priority to the swing control as the trolley gets near to the target position. Summing up, we may suggest the following steps in designing an FLC for plants with M multiple objectives:

Step 1. Design independently sub-FLCs such that each of them maximizes the corresponding satisfaction degree for an objective.

Step 2. Determine the weights for M sub-FLCs. If w_i denotes the weight for the i-th sub-FLC $(i = 1, 2, \ldots, M)$, the control output $u(t)$, which is also the input of the plant, is a weighted sum of the control outputs $u_i(t)$, $i = 1, 2, \ldots, M$, of the sub-FLCs:

$$u(t) = \sum_{i=1}^{M} w_i \, u_i(t) \tag{5}$$

The problem is now reduced to the determination of the weights w_i, $i = 1, 2, \ldots, M$, for the sub-FLCs. In this paper, it is suggested to determine w_i as a function of the output of the plant, i.e.

$$w_i \equiv w_i(y) = w_i(y_1, y_2, \ldots, y_p) \tag{6}$$

The design scheme implied in (6) will be described in what follows.

2.2. Design Scheme

As shown in Fig. 1, the proposed multi-objective fuzzy-logic controller (MOFLC) consists of a supervisory controller and subcontrollers. Each subcontroller is a typical FLC while the supervisory controller is a coordinator to determine the weights for the subcontrollers.

The weight decision part in the supervisory controller utilizes the output of the plant as the input, and generates weights suitable for the current control situation as the final output of the supervisory controller. The weight decision part is made up of the rule base and the inference engine. The rule base is expressed in the following linguistic description:

If y_1 is $L_{y_1}^{(l)}$ and ... and y_k is $L_{y_k}^{(l)}$ and ... and y_p is $L_{y_p}^{(l)}$, then w_1 is $L_{w_1}^{(l)}$ and ... and w_i is $L_{w_i}^{(l)}$ and ... and w_M is $L_{w_M}^{(l)}$

Here, (l), $l = 1, 2, \ldots, L$, is the rule number; y_k is the k-th output of the plant and $L_{y_k}^{(l)}$, $L_{w_i}^{(l)}$ are linguistic labels of y_k and w_i in the (l)-th rule, respectively. Mamdani's method (Mamdani, 1974) is utilized for the inference engine. Therefore, the weight decision making part produces w_i^*'s as follows:

$$w_i^* = \frac{\int w_i c(w_i)\, \mathrm{d}w_i}{\int c(w_i)\, \mathrm{d}w_i}, \qquad i = 1, 2, \ldots, M \tag{7}$$

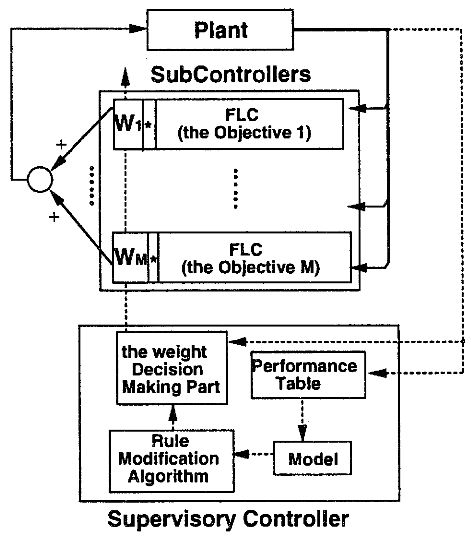


Fig. 1. Structure of MOFLC.

$$c(w_i) = \vee_{l=1,2,\ldots,L}\left\{\mu^{(l)}_{L_{y_1}}(y_1^*) \wedge \cdots \wedge \mu^{(l)}_{L_{y_p}}(y_p^*) \wedge \mu^{(l)}_{L_{w_i}}(w_i)\right\} \tag{8}$$

where $\vee$ is the max operator and $\wedge$ is the min operator; $\mu^{(l)}_{L_{y_k}}(y_k^*)$ is the membership value of y_k^* in $L^{(l)}_{y_k}$ and $\mu^{(l)}_{L_{w_i(w_i)}}$ is the grade membership of w_i in $L^{(l)}_{w_i}$. The quantity $c(w_i)$ is the union of the inferenced fuzzy sets when $y_1^*, y_2^*, \ldots, y_p^*$ (the crisp output values of the plant) are given. Furthermore, w_i^*'s are the outputs of the weight decision making part, which are the crisp values. If the sub-FLCs yield u_1^*, $u_2^*, \ldots, u_M^*$ as crisp outputs, the final output of MOFLC reduces to

$$u = \sum_{i=1}^{M} w_i^* u_i^* \tag{9}$$

It is worth noticing that it is often difficult to determine the membership functions of the weight decision rule base, and accordingly the self-organizing mechanism (Pedrycz, 1993) is adopted for the learning part in the supervisory controller to modify the membership functions. As shown in Fig. 1, the performance table part, the model part and the rule modification algorithm part in the supervisory controller constitute a self-organizing controller (SOC). The learning procedure in the supervisory controller is as follows:

First, the performance table part evaluates the control result according to the satisfaction degree of each control objective.

Secondly, the following condition is checked:

$$\min_{i=1,..,M} P_i \geq \alpha \tag{10}$$

Here, $\alpha \in [0,1]$ is the least satisfaction limit which the designer chooses. If condition (10) is satisfied, i.e. if the current MOFLC achieves the overall satisfaction degree, the procedure is stopped. If not, proceed to the next step.

Thirdly, the model part determines the membership functions to be modified. They are the membership functions of the consequents in the rule base, i.e. the membership functions of weights. Let the shapes of the membership functions be those of isosceles triangles, and λ_{ij} be the central value of the j-th membership function of w_i $(j = 1, 2, \ldots, J)$. To determine the membership functions to be modified, let $P_{avg} = \sum_{i=1}^{M} P_i/M$. Then P_{avg} offers a reference value to determine which of the control objectives is accounted for modification. If $P_i \leq P_{avg}$, then λ_{ij} moves to a larger value. In the same manner, if $P_i \geq P_{avg}$, then λ_{ij} moves to a smaller value. The amount of change in λ_{ij} is determined by

$$\delta_i = \eta \frac{P_{avg} - P_i}{\sum_{l=1}^{M} |P_{avg} - P_l|} \tag{11}$$

where η is a learning gain.

Finally, the rule modification algorithm part utilizes δ_i as the input and updates the central values of the membership functions of w_i as follows:

$$\lambda_{ij,new} = \lambda_{ij} + \delta_i \tag{12}$$

where $j = 1, 2, \ldots, J$. In this manner, the satisfaction degree gets improved by modifying the central values of the corresponding membership functions. The SOC part modifies the membership functions of the weight decision rule base repetitively if condition (10) is not satisfied.

2.3. Simulation Example

The proposed MOFLC is applied for the overhead crane control problem (Nakatsuyama *et al.*, 1994; Yamada and Fujikawa, 1989; Yu *et al.*, 1993). The dynamics of the overhead crane is as follows:

$$\ddot{x} = \frac{f}{M} \tag{13}$$

$$\ddot{\theta} = \frac{-g\sin\theta + \ddot{x}\cos\theta}{l} \tag{14}$$

Here, f is the input force (N), x is the trolley position [m], θ is the angle of the load [rad], l is the length of the rope [m], M is the mass of the trolley [kg] and, g is the gravity constant [m/s^2].

In this model, it is assumed that $M = 1\,\text{kg}$, $l = 1\,\text{m}$ and $g = 9.8\,\text{m/s}^2$. The first control objective in this control problem is to reduce the swing angle from 0.7 rad to zero with tolerance 0.05 rad. The second control objective is to decrease the trolley position from 1.0 m to zero with tolerance 5 cm.

Firstly, we set the satisfaction degrees as follows:

$$P_i(t_{elapsed}) = \begin{cases} 1 & \text{for } t_{elapsed} \leq t_{\min} \\ 1 - \dfrac{t_{elapsed} - t_{\min}}{t_{\max} - t_{\min}} & \text{for } t_{\min} \leq t_{elapsed} \leq t_{\max} \\ 0 & \text{for } t_{\max} \leq t_{elapsed} \end{cases} \tag{15}$$

Here, i=1, 2; P_1 is the satisfaction degree of the position control objective and P_2 is that of the swing control objective. Moreover, $t_{elapsed}$ is the time elapsed after which each control objective remains within the tolerance limit, $t_{\min}$ is 10 s and $t_{\max}$ is 20 s.

We are given two independent groups of rules based on the operator's knowledge. One group of rules is designed for positioning the trolley and the other group of rules is designed for reducing the swing. Both are used in the rule base for the corresponding subcontrollers. Figures 2(a) and (b) show the corresponding groups of rules — the group of the position control rules and that of the swing control rules.

$\dot{x}$ \ x	NB	NM	NS	ZE	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	ZE
NM	PB	PB	PB	PM	PS	ZE	NS
NS	PB	PB	PM	PS	ZE	NS	NM
ZE	PB	PM	PS	ZE	NS	NM	NB
PS	PM	PS	ZE	NS	NM	NB	NB
PM	PS	ZE	NS	NM	NB	NB	NB
PB	ZE	NS	NM	NB	NB	NB	NB

(a) Position control rules

$\dot{\theta}$ \ θ	NB	NM	NS	ZE	PS	PM	PB
NB	ZE	ZE	ZE	NB	ZE	ZE	ZE
NM	ZE	ZE	ZE	NM	ZE	ZE	ZE
NS	ZE	ZE	ZE	NS	ZE	ZE	ZE
ZE	ZE	ZE	ZE	ZE	ZE	ZE	ZE
PS	ZE	ZE	ZE	PS	ZE	ZE	ZE
PM	ZE	ZE	ZE	PM	ZE	ZE	ZE
PB	ZE	ZE	ZE	PB	ZE	ZE	ZE

(b) Swing control rules

Fig. 2. Obtained groups of rules.

After the subcontrollers are designed based on these groups of rules, we design the weight decision making part in the supervisory controller. There are two weights, w_1 and w_2, to be determined; w_1 is for the position controller and w_2 is for the swing angle controller. We construct the weight decision rules whose antecedents are the trolley position and the swing angle, whereas the consequents are w_1 and w_2. The weight decision rule tables and the membership functions of the variables are shown in Figs. 3(a), (b) and (c), respectively. Note that the fuzzy labels of the weights are {*SMall, MiDdle, BiG*} and the initial central values of the membership functions are {0.0, 0.5, 1.0}.

In addition, we set the parameters in the the block concerning the learning capability, i.e. the learning gain (η) is set to 0.02 and the least satisfaction limit (α) is set to 0.7. Thus the design process is completed, and the MOFLC is put to control the overhead crane under the above initial condition till condition (10) is satisfied.

Swing Angle

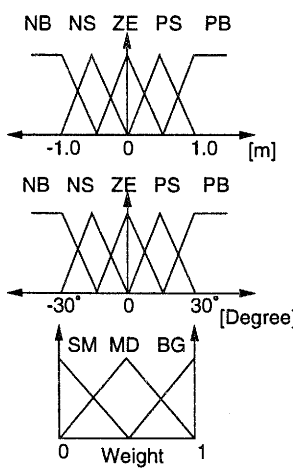
Position	NB	NS	ZE	PS	PB
NB	BG	BG	BG	SM	MD
NS	BG	BG	SM	MD	SM
ZE	BG	SM	MD	SM	BG
PS	SM	MD	SM	BG	BG
PB	MD	SM	BG	BG	BG

(a) w_1 decision rule table

Swing Angle

Position	NB	NS	ZE	PS	PB
NB	SM	SM	SM	BG	MD
NS	SM	SM	BG	MD	BG
ZE	SM	BG	MD	BG	SM
PS	BG	MD	BG	SM	SM
PB	MD	BG	SM	SM	SM

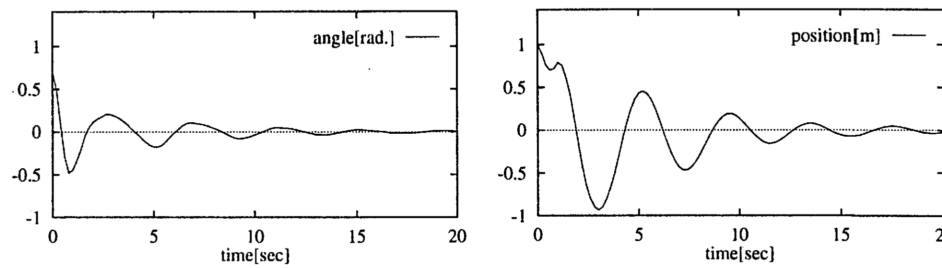
(b) w_2 decision rule table



(c) Membership function

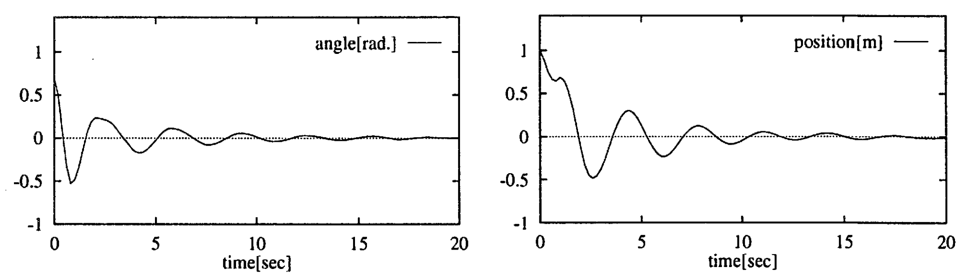
Fig. 3. Weight decision making part.

Figure 4 shows the control result at the first iteration. We obtain $P_1 = 0.3$, $P_2 = 0.87$, i.e. the overall satisfaction degree is below the least satisfaction limit 0.7. In addition, P_1 is small in comparison with P_2, while P_2 is large. As a result of the learning mechanism, the central values of the memberships of w_1 are shifted to a larger value by 0.1. Also, those of the memberships of w_2 are shifted to a smaller value by -0.1. After five iterations, the control result is shown in Fig. 5. Figure 5 shows that P_1 is 0.8 which is larger than the initial value, while P_2 is as good as the initial value. Moreover, the overall satisfaction degree becomes larger than the least satisfaction limit. In comparison with the result obtained by applying the technique in (Yu and Bien, 1994) in which the two groups of rules are simply added to form a new rule base, the result of the proposed MOFLC is less biased and more satisfactory with larger satisfaction degrees as shown in Fig. 6. In Fig. 7 the transitions of the weight w_1 before and after the learning process are shown.



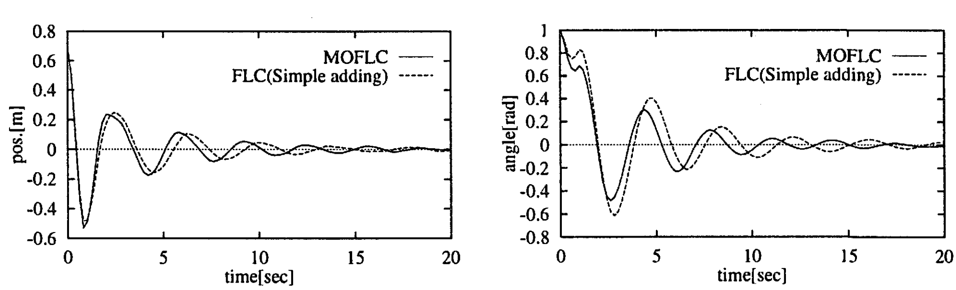
(a) Position (b) Swing angle

Fig. 4. Initial control result.



(a) Position (b) Swing angle

Fig. 5. Control result after learning.



(a) Position (b) Swing angle

Fig. 6. Comparison of MOFLC with the previous FLC.

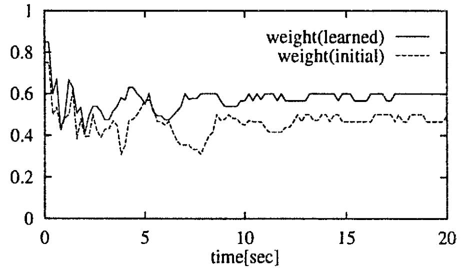


Fig. 7. Transition of w_1.

As can be seen from this example, for multi-objective control the control parameters in the proposed scheme can be determined without great difficulties by referring to the expert's knowledge and/or operator's experience. The control system designed in this way exhibits the multiple objectives satisfied impartially. For designing a real control plant which often involves multiple objectives and for which some forms of expert knowledge are available to handle uncertainty, the design method of MOFLC proposed here can be a useful tool.

3. Stability of MOFLC and Regulator Problem

In this section, the stability of the MOFLC system is studied for a regulator problem. Although there are several results (Aracil and Ollero, 1989; Bouslama and Ichikawa, 1992; Farinwata, 1994; Kitamura and Kurozumi, 1991; Tanaka and Sugeno, 1992; Vidyasagar, 1993) concerning the FLC stability analysis, it is found difficult to apply directly those results to the MOFLC system. In this paper, a sufficient condition for the MOFLC stability is presented in the context of Lyapunov stability. This result not only provides a tool for guaranteed stability, but also gives a hint to determine the range of the weights to stabilize the MOFLC system.

3.1. Stability Checking Mechanism

Consider the following MOFLC system:

$$\dot{x}(t) = f(x(t),\, u(t)) \tag{16}$$

$$u(t) = \sum_{i=1}^{M} w_i\, u_i(t) \tag{17}$$

where it is assumed that $f(\cdot,\cdot)$ is a continuously differentiable non-linear function and that $f(\mathbf{0},0) = \mathbf{0}$, i.e. $\mathbf{0}$ is the equilibrium point of the MOFLC system. Here w_i is the weight for the i-th subcontroller in the neighbourhood of $\mathbf{0}$, and u_i is the control output of the i-th subcontroller. Suppose the control problem is to regulate all the outputs of the plant. Here, we have M control objectives, namely the i-th control objective is to regulate the i-th output as fast as possible ($i = 1, 2, \ldots, M$).

Let us suppose that the control input of the i-th subcontroller $u_i(t)$ utilizes only the state relating the i-th output (x^i) and its time derivative ($\dot{x}^i$) for feedback. Then, it is typical that the mapping relation of the i-th subcontroller, denoted by $\phi_i(x^i,\ \dot{x}^i)$, has the following properties (Lee, 1990; Mamdani, 1974; Yi, 1994):

(i) The diagonal term of $\phi_i(x^i,\ \dot{x}^i)$ is zero while the mapping is depicted in Fig. 8. That is, there exist g_1^i and g_2^i such that $g_1^i x^i + g_2^i \dot{x}^i = 0$. In particular, $\phi_i(0,\ 0) = 0$.

(ii) The upper-left triangular term of $\phi_i(x^i,\ \dot{x}^i)$ is negative and the lower-right triangular term is positive as shown in Fig. 8.

(iii) The farther the distance from the diagonal, the larger $|\phi_i(x^i,\ \dot{x}^i)|$ is.

		x^i				
		NB	NS	ZE	PS	PB
$\dot{x}^i$	NB	NB	NB	NS	NS	ZE
	NS	NB	NS	NS	ZE	PS
	ZE	NB	NS	ZE	PS	PB
	PS	NS	ZE	PS	PS	PB
	PB	ZE	PS	PS	PB	PB

Fig. 8. General control rule table.

If the rule base of the i-th subcontroller satisfies the above conditions, there exist k_1^i and k_2^i such that $\phi_i(g_1^i x^i,\ g_2^i \dot{x}^i)$ belongs to the extended sector $(k_1^i \sigma_i,\ k_2^i \sigma_i)$ (Kitamura and Kurozumi, 1991). Here, the extended sector is an extended version of a sector used to specify a non-linear function in the two-dimensional space (Slotine and Li, 1991) and, in this paper, it refers to a limited space between two planes in the tree-dimensional space to which the non-linear function $\phi_i(x^i,\ \dot{x}^i)$ with two input variables always belongs (Kitamura and Kurozumi, 1991).

For our problem, we have

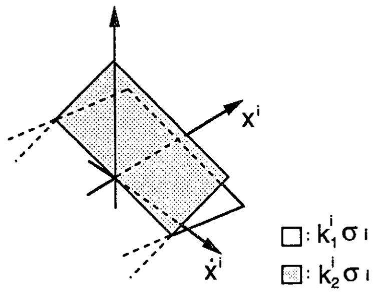
$$k_1^i {\sigma_i}^2 \leq \phi_i(g_1^i x^i,\ g_2^i \dot{x}^i)\sigma_i \leq k_2^i {\sigma_i}^2 \tag{18}$$

where $\sigma_i = g_1^i x^i + g_2^i \dot{x}^i$ and

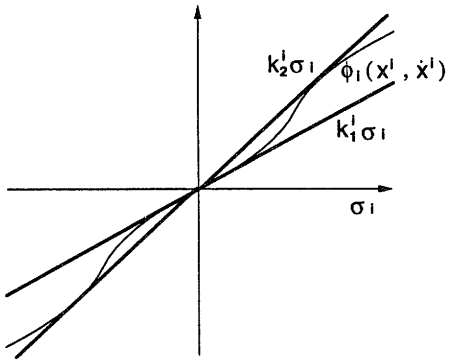
$$k_1^i = \inf_{x^i, \dot{x}^i} \frac{\phi_i(g_1^i x^i,\ g_2^i \dot{x}^i)}{g_1^i x^i + g_2^i \dot{x}^i} \tag{19}$$

$$k_2^i = \sup_{x^i, \dot{x}^i} \frac{\phi_i(g_1^i x^i,\ g_2^i \dot{x}^i)}{g_1^i x^i + g_2^i \dot{x}^i} \tag{20}$$

Here, $k_1^i \sigma_i$ corresponds to the lower limit and $k_2^i \sigma_i$ to the upper limit.



(a) Extended sector



(b) Profile

Fig. 9. Extended sector.

Figure 9 shows that the non-linear function is contained within the extended sector. Since the MOFLC combines all the subcontrollers whose mappings belong to some extended sectors, we can define the bounds corresponding to the mapping of the MOFLC.

Definition 1. Let the mapping of the MOFLC be represented by $\Psi(w,x)$, where

$$\Psi(w,x) = \sum_{i=1}^{M} w_i u_i = \sum_{i=1}^{M} w_i \phi_i(x^i, \dot{x}^i) = w\Phi(x) \tag{21}$$

Here, $w = [w_1\ w_2 \cdots w_M]$ and $\Phi^T(x) = [\phi_1(x^1,\dot{x}^1)\ \phi_2(x^2,\dot{x}^2)\cdots\phi_M(x^M,\dot{x}^M)]$. If each $\phi_i(x^i,\dot{x}^i)$ has the corresponding sector $(k_1^i\sigma_i,\ k_2^i\sigma_i)$, then the *bounds of MOFLC* are defined by the set $\{wk_i x \mid i = 1,2,\ldots,2^M\}$. Here, k_i is determined according to the combination of the two limits of each extended sector.

In general, if the bounds of the MOFLC are represented by $\{wk_i x \mid i = 1,2,\ldots,2^M\}$, the following sufficient condition for the MOFLC stability is obtained.

Theorem 1. *(Local Stability): Consider the MOFLC system $\dot{x}(t) = f(x(t),u(t))$ with $u(t) = \Psi(w,x)$, where x is an n-dimensional state vector and u is a scalar. It is assumed that f is continuously differentiable and $f(\mathbf{0},0) = \mathbf{0}$. Let*

$$A = \left(\frac{\partial f}{\partial x}\right)_{\substack{x=\mathbf{0}\\u=0}}, \qquad b = \left(\frac{\partial f}{\partial u}\right)_{\substack{x=\mathbf{0}\\u=0}}$$

Suppose that there exist bounds $\{wk_i x \mid i = 1,2,\ldots,2^M\}$ in the sense of Def. 1. Let $A_i = A + bwk_i$, $i = 1,2,\ldots,2^M$. If there exists a positive-definite symmetric matrix P such that $x^T({A_i}^T P + PA_i)x \le 0$ for all A_i, then $\mathbf{0}$ is an asymptotically stable equilibrium point of the resulting system $\dot{x}(t) = f(x(t),\Psi(w,x))$.

Proof. The system in a neighbourhood of $\mathbf{0}$ is approximately expressed by the linear equation, $\dot{x}(t) = Ax(t) + bu(t)$. Let $V(x) = x^T Px$. Then

$$\begin{aligned} \dot{V}(x) &= \dot{x}^T Px + x^T P\dot{x} \\ &= \Big(Ax + b\Psi(w,x)\Big)^T Px + x^T P\Big(Ax + b\Psi(w,x)\Big) \\ &= x^T(A^T P + PA)x + 2x^T Pb\Psi(w,x) \\ &\leq x^T(A^T P + PA)x + \max_i \Big\{2x^T Pbwk_i x\Big\} \\ &= \max_i \Big\{x^T(A + bwk_i)^T Px + x^T P(A + bwk_i)x\Big\} \\ &= \max_i \Big\{x^T(A_i^T P + PA_i)x\Big\} \\ &\leq 0 \end{aligned}$$

Since $\Psi(w,x)$ is limited by the bounds of MOFLC, the above inequality is valid. Thus $\mathbf{0}$ is globally asymptotically stable. ■

In utilizing the above result, note that the existence of $P \geq \mathbf{0}$ is essential. We may summarize the above result as a procedure if A_1, and A_2 are selected as follows:

(i) Linearize the plant in a neighbourhood of $\mathbf{0}$. Matrices A and b can be obtained in this step.

(ii) Find the bounds for the MOFLC, $\{wk_i x \mid i = 1, 2, \ldots, 2^M\}$.

(iii) Check whether A_i's are Hurwitz matrices ($A_i = A + bwk_i$).

(iv) Check whether $(A_i + A_j)$'s are Hurwitz matrices.

(v) Select P such that $A_i^T P + PA_i < \mathbf{0}$ for $i = 1$.

(vi) Check whether $A_i^T P + PA_i < \mathbf{0}$ holds for $i = i + 1$. Repeat it until $i = 2^M$. If it holds, we say that the designed MOFLC system is stable. Otherwise, go to Step (v).

The following lemma may be used in sorting out infeasible candidates.

Lemma 1. *Let us suppose that A_i, $i = 1, 2, \ldots, M$ are Hurwitz matrices. If there exists a positive-definite symmetric matrix P such that $A_i{}^T P + PA_i < \mathbf{0}$ $\forall i$, then $\sum_{i=1}^M A_i$ is a Hurwitz matrix.*

The proof of the above lemma is straightforward and therefore is omitted.

For example, consider the existence of a common P matrix in the case where

$$A_1 = \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix}$$

Since A_1 and A_2 are both Hurwitz matrices, by the antithesis of Lemma 1 it follows that there does not exist such P because $A_1 + A_2$ is not Hurwitz.

3.2. Simple Example

Consider the control problem of a cart-pole system given in the form

$$\ddot{z} = \frac{u}{M} \tag{22}$$

$$\ddot{\theta} = \frac{Mg\sin\theta + \cos\theta(u - m_l l\dot{\theta}^2 \sin\theta)}{\frac{4}{3}Ml - m_l l\cos^2\theta} \tag{23}$$

where z [m] is the cart position, θ [rad] is the angle of the pole, u [N] is the input force, l [cm] is the length of the pole, M [kg] is the mass of the cart and the pole, m_l [g] is the mass of the pole and g [m/s^2] is the gravity constant. It is assumed that l is 15 cm, M is 1.57 kg, m_l is 70 g, g is 9.8 m/s^2.

Let us suppose that there are two control objectives. The first one is to set the cart position as close as possible to the zero position and the other one is to set the pole as vertical as possible. Two controllers are designed independently to satisfy the corresponding control objectives. Using the stability checking procedure mentioned above, we can check the designed MOFLC stability.

Firstly, the system in the neighbourhood of **0** is linearized as the following state equation:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 50.68 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.64 \\ 0 \\ 3.29 \end{bmatrix} u(t) \tag{24}$$

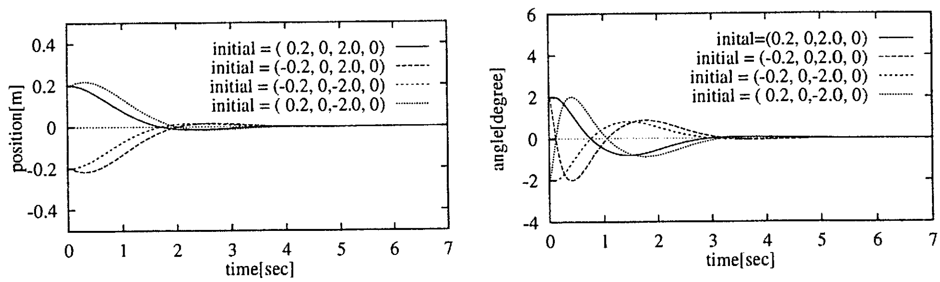
where $x = [x^1\ \dot{x}^1\ x^2\ \dot{x}^2]^T$, $x^1 = z$, and $x^2 = \theta$.

Secondly, the bounds for MOFLC are obtained. The extended sector of the position controller is $(17.0x^1 + 17.0\dot{x}^1, 19.0x^1 + 19.0\dot{x}^1)$ and that of the angle controller is $(120.0x^2 + 20.0\dot{x}^2, 130.0x^2 + 21.6\dot{x}^2)$. Suppose the weight for the position controller is -0.5 and the weight for the angle controller is 0.5. Then we can find the bounds of the designed MOFLC as follows:

$$\Big\{ -9.5x^1 - 9.5\dot{x}^1 + 60.0x^2 + 10.0\dot{x}^2,\ -9.5x^1 - 9.5\dot{x}^1 + 65.0x^2 + 10.8\dot{x}^2,$$
$$-8.5x^1 - 8.5\dot{x}^1 + 60.0x^2 + 10.0\dot{x}^2,\ -8.5x^1 - 8.5\dot{x}^1 + 65.0x^2 + 10.8\dot{x}^2 \Big\}$$

Thirdly, we check that A_i's are Hurwitz matrices. Since the eigenvalues of A_i's lie in the NHP (Negative Half-Plane), A_i's are Hurwitz matrices. In the fourth step, we check that $(A_i + A_j)$'s are Hurwitz matrices. This can be checked in the same manner as in step (iii). In the fifth step, we can find an appropriate P:

$$P = \begin{bmatrix} 3.33 & 2.33 & -7.06 & -0.47 \\ 2.33 & 2.85 & -9.28 & -0.65 \\ -7.06 & -9.28 & 51.01 & 2.49 \\ -0.47 & -0.65 & 2.49 & 0.34 \end{bmatrix}$$



(a) Swing angle (b) Position

Fig. 10. Control result in case $w_1 = -0.5$, $w_2 = 0.5$.

Since there exists a common P, the $\mathbf{0}$ of the MOFLC system is stable. Figure 10 shows the response in the neighbourhood of $\mathbf{0}$ of the designed MOFLC system from four different initial states. Therefore we can verify whether the designed MOFLC system is stable by using the proposed stability checking mechanism.

4. Concluding Remarks

In this paper, we proposed an FLC design scheme for a case when multiple groups of rules satisfying partial control objectives are given. In the proposed MOFLC, each subcontroller is designed to satisfy the corresponding control objective only and then the outputs of the subcontrollers are combined by varying the weights for the subcontrollers. Moreover, owing to a self-organizing algorithm, MOFLC can learn the weight decision rules effectively. In addition, we analysed the stability of the MOFLC for regulation by using Lyapunov's linearized method and the bounds of MOFLC. The results in the paper, however, should be elaborated further. The heuristics employed in designing the weights in the controller should further be investigated and the stability analysis should be extended be applying to a general control problem.

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