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FEEDBACK STABILIZATION OF CONTINUOUS SYSTEMS BY ADDING AN INTEGRATOR

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This note is devoted to the problem of global stabilization of continuous systems by adding an integrator. The goal is to prove that if a continuous non-linear system $\dot{x} = f(x, u)$ is globally asymptotically stable at the origin for $u \equiv 0$, then the augmented system obtained by adding an integrator is stabilizable by means of a continuous feedback.

Keywords: feedback stabilization, nonlinear systems, continuous systems, Lyapunov function

1. Introduction and the Stability Result

This note is a contribution to the global stabilization problem for the non-linear control system

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = u, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ and y, $u \in \mathbb{R}^m$. The objective is to prove a result of stabilization when f is only a continuous map.

It is well-known that, as long as f is smooth (i.e. of class C^∞), if the system

$$\dot{x} = f(x, 0) \tag{2}$$

is globally asymptotically stable at the origin (we write it GAS for short), then (1) is globally asymptotically stabilizable (see e.g. Byrnes and Isidori, 1989; Koditschek, 1987; Tsinias, 1989).

When f is linear (i.e. $f(x, y) = Ax + By$), the proof of the above result is very simple. Indeed, it suffices to take $u(x, y) = Dy$, where D is any Hurwitz matrix with suitable dimensions. If f is not linear, the proof is not difficult, but is more ingenious. It is based on the fact that if $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is such that $g(x, 0) \equiv 0$, then g has the following decomposition:

$$g(x, y) = G(x, y)y \quad \text{for all} \quad (x, y) \in \mathbb{R}^n \times \mathbb{R}^m \tag{3}$$

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where G is the smooth function defined by

$$G(x,y) = \int_0^1 \frac{\partial g}{\partial y}(x,ty)\,\mathrm{d}t$$

Now, if f is continuous but not of class C^1, this proof is not valid. The main reason is that for a continuous (not C^1) function g, the decomposition given by (3) is not always possible. Indeed, a simple counterexample is $g(x,y) \equiv y^{1/3}$.

The main goal of this paper is to prove the following result:

Theorem 1. *If f is continuous and (2) is GAS, then (1) is stabilizable by means of a continuous feedback law.*

The proof of the above result, given in the next section, is based on a lemma, proved also in that section, which states that a continuous function $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ which fulfils $g(x,0) \equiv 0$ can be decomposed as $g(x,y) = G(x,y)h(y)$, where h is a suitable function from which one can construct a Lyapunov function. Note that since we assume that f is continuous, and we do not impose that this function is Lipschitz, the obtained closed-loop system is ensured to be only continuous. This means that the existence of solutions is guaranteed but not necessarily their uniqueness.

The problem of global feedback stabilization of system (1) received much attention (see e.g. Byrnes and Isidori, 1989; Coron and Praly, 1991; Iggidr and Sallet, 1994; Koditschek, 1987; Kokotovic and Sussmann, 1989; Outbib, 1991; Outbib and Jghima, 1996; Outbib and Sallet, 1998; Rosier, 1993; Tsinias, 1989). Generally speaking, in those papers it is assumed that f is at least of class C^1 and the reduced system $\dot{x} = f(x,v)$ is stabilizable by means of a smooth feedback law. Note that the smoothness of the stabilizing feedback is very important. For instance, even for smooth systems, if the stabilizing feedback for the reduced system is only continuous, the question of the stabilizability of (1) remains open (cf. some interesting answers in Coron and Praly, 1991).

This paper is dedicated to the feedback stabilization of (1) when f is only continuous and the goal is slightly different from those of the papers mentioned above. The objective is to show that the assumption of smoothness of f can be weakened. This work is motivated, in part, by the fact that the models of some practical systems are only continuous (Richard *et al.*, 1997). Another aim of this work is to show that some classical results concerning feedback stabilization of smooth systems even if their proofs are, in part, depending on the smoothness of the system, can possess versions for systems that are only continuous.

The paper is organized as follows. Section 2 is divided into two subsections: in the first we state and prove a key lemma and in the other, we give the proof of Theorem 1. In order to illustrate our result, an example is given in Section 3. The last section contains conclusions.

Throughout the paper, as usual, $||\cdot||$ denotes the Euclidean norm, $|\cdot|$ is the absolute value of a real number, and $\langle\cdot,\cdot\rangle$ denotes the scalar product. For a vector v,

v^T is its transpose. From now on we write C for the class of continuous functions $h:\mathbb{R}\to\mathbb{R}$ such that

- $h(y)y \geq 0$,
- $h(y)=0 \iff y=0$,
- $\lim\limits_{|y|\to+\infty} |h(y)| = +\infty$.

We will also use the symbol $\mathcal{M}_{n\times m}$ to denote the set of matrices with n rows and m columns, where the components are scalar functions.

2. Proof of the Result

2.1. Key Lemma

In order to prove our main result of this subsection, we need the following lemma:

Lemma 1. *Let $\varphi:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}$ be a continuous function with $\varphi(x,0)\equiv 0$ $(x\in\mathbb{R}^n)$. Assume that φ is bounded for any bounded y. Then there exists $h\in C$ such that*

$$|h(y)| \geq |\varphi(x,y)| \quad \textit{for all} \quad (x,y)\in\mathbb{R}^n\times\mathbb{R}. \tag{4}$$

Proof. Let $(a_k)_{k\geq 1}$ and $(b_k)_{k\geq 1}$ be two sequences of real numbers defined respectively by

$$\begin{cases} a_1 = \sup\left(1, \sup\limits_{(x,y)\in\mathbb{R}^n\times[-2,2]} |\varphi(x,y)|\right) \\ a_k = \sup\left(\dfrac{a_{k-1}}{2}, \sup\limits_{(x,y)\in\mathbb{R}^n\times[-1/k,1/k]} |\varphi(x,y)|\right) \quad \text{for} \quad k\geq 2, \end{cases}$$

and

$$b_k = \sup\left(k, \sup_{(x,y)\in\mathbb{R}^n\times[-k-1,k+1]} |\varphi(x,y)|\right) \quad \text{for} \quad k\geq 1.$$

Introduce h as follows:

- $h(0)=0$;
- $h(y) = -h(-y)$ for $y<0$;
- for $y\in\,]0,5/12]$

$$h(y) = \begin{cases} a_k & \text{for} \quad y\in\left[\dfrac{1}{k+1}, \dfrac{1}{k} - \dfrac{1}{2k(k+1)}\right] \\ \alpha_k y + \beta_k & \text{for} \quad y\in\left[\dfrac{1}{k+1} - \dfrac{1}{2(k+1)(k+2)}, \dfrac{1}{(k+1)}\right], \end{cases}$$

where

$$\begin{cases} \alpha_k = 2(k+1)(k+2)(a_k - a_{k+1}) \\ \beta_k = (3-2k)a_k + 2(k+2)a_{k+1}, \quad k \geq 2; \end{cases}$$

- for $y \in]5/12, 1/2]$

$$h(y) = 12(a_1 - a_2)y + 6a_2 - 5a_1;$$

- for $y \in]1/2, 1]$

$$h(y) = a_1;$$

- for $y \in]1, +\infty[$

$$h(y) = \begin{cases} b_k & \text{for} \quad y \in [k, k+1/2] \\ \gamma_k y + \delta_k & \text{for} \quad y \in [k+1/2, k+1[, \end{cases}$$

where

$$\begin{cases} \gamma_k = 2(b_{k+1} - b_k) \\ \delta_k = 2(k+1)b_k - (2k+1)b_{k+1}, \quad k \geq 1. \end{cases}$$

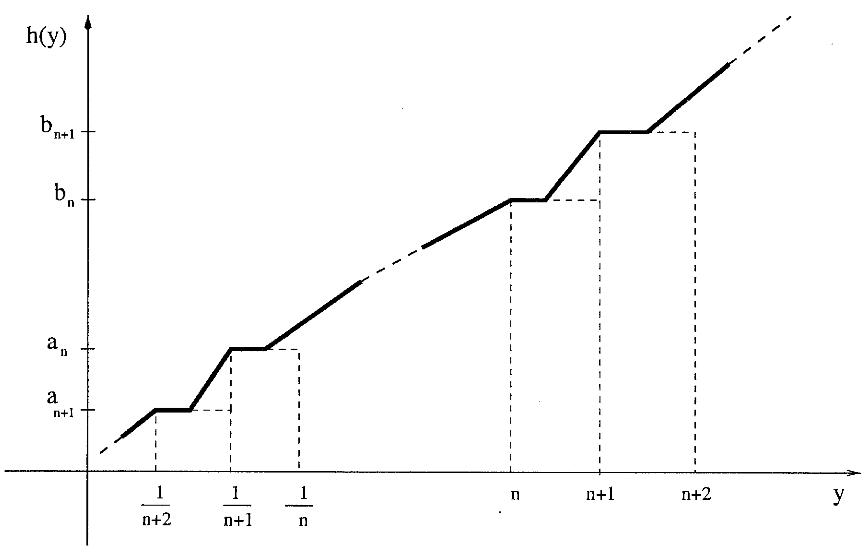

Fig. 1. Behaviour of the function h.

By construction, h is continuous on $\mathbb{R} \setminus \{0\}$, verifies (4) and is such that

$$h(y)y > 0 \quad \text{for} \quad y \neq 0 \qquad \text{and} \qquad \lim_{y \to +\infty} |h(y)| = +\infty\,,$$

Note that h is also continuous at zero. Indeed, since

$$\lim_{k \to +\infty} a_k = 0$$

we see that

$$\lim_{|y| \to 0} h(y) = 0\,.$$

This finishes the proof of the lemma. ∎

Remark 1. As is shown in the proof of Lemma 1, the construction of h is very technical. Note that the proposed function h is not unique and there is no particular reason behind such a specific choice. Indeed, to get another function, it suffices e.g. to multiply h by any smooth scalar function h_1 which verifies $h_1(y) \geq 1$ for all $y \in \mathbb{R}$.

Now we state and prove the main result of this subsection.

Lemma 2. *Let $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ be a continuous function such that $f(x,0) \equiv 0$. Then there exist continuous functions $H = (H_1, \ldots, H_m)^T : \mathbb{R}^m \to \mathbb{R}^m$ and $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathcal{M}_{n \times m}$ such that*

(L1) *$H_i(y) = h_i(y_i)$, where $h_i \in \mathcal{C}$, $i = 1, \ldots, m$, and*

(L2) *$f(x,y) = G(x,y)H(y)$, $\forall\, (x,y) \in \mathbb{R}^n \times \mathbb{R}^m$.*

Proof. Without loss of generality, we assume that f is bounded. Indeed, f can be rewritten as

$$f = f^1\left(1 + \|f\|^2\right)$$

where $f^1 = f/(1 + \|f\|^2)$. If $f^1 = G^1 H$, where G^1 and H are suitable functions which verify the conditions of the lemma, then $f = GH$ with $G = G^1\left(1 + \|f\|^2\right)$.

First we give the proof for $m = 1$. From Lemma 1 there exist $h_i \in \mathcal{C}$ $(i = 1, \ldots, n)$ such that

$$|h_i(y)| \geq \left| f_i^{\frac{1}{3}}(x,y) \right|, \quad \forall\, (x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1, \ldots, n\,. \tag{5}$$

We have

$$f_i(x,y) = f_i^{\frac{2}{3}}(x,y)\, h_i(y)\, h_i^{-1}(y)\, f_i^{\frac{1}{3}}(x,y), \quad \forall\, (x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1, \ldots, n\,.$$

Furthermore, we define $\hat{f}$ by

$$\hat{f}_i(x,y) = h_i^{-1}(x,y) f_i^{\frac{1}{3}}(x,y), \quad \forall\, (x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1, \ldots, n\,.$$

Clearly, $\hat{f}$ is continuous for $y \neq 0$ and, in view of (5), it is bounded.

As $f(x,0) \equiv 0$, we deduce that the function $\tilde{f}$, defined by

$$\tilde{f}_i(x,y) = f_i^{\frac{2}{3}}(x,y)\hat{f}_i(x,y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1,\dots,n\,,$$

is continuous. Therefore,

$$f_i(x,y) = h_i(y)g_i(x,y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1,\dots,n\,,$$

where $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is the continuous function defined by

$$g_i(x,y) = f_i(x,y)h_i^{-1}(y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1,\dots,n.$$

Let h be the continuous function given by

$$h(y) = \begin{cases} \sqrt{\sum_{i=1}^{n} h_i^2(y)} & \text{if } \; y \geq 0 \\ h(y) = -h(-y) & \text{if } \; y < 0 \end{cases}.$$

Clearly, $h \in C$ and

$$|h(y)| \geq |h_i(y)|\,, \quad \forall\, y \in \mathbb{R}, \quad i = 1,\dots,n\,.$$

We rewrite f as

$$f(x,y) = h(y)\hat{g}(x,y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R},$$

where

$$\hat{g}_i(x,y) = g_i(x,y)h_i(y)h^{-1}(y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}, \quad i = 1,\dots,n.$$

The function $\tilde{h}_i$ $(i = 1,\dots,n)$ defined by

$$\tilde{h}_i(y) = h_i(y)h^{-1}(y), \quad \forall\, y \in \mathbb{R}, \quad i = 1,\dots,n,$$

is continuous for $y \neq 0$ and bounded. Now, $g(x,0) \equiv 0$, so $\hat{g}$ is a continuous function. This completes the proof for the case $m = 1$.

Assume that $m > 1$. We prove that there exist continuous functions $G_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ $(i = 1,\dots,n)$, such that

$$f_i(x,y) = \langle H(y), G_i(x,y)\rangle\,, \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m, \quad i = 1,\dots,n, \tag{6}$$

where $H : \mathbb{R}^m \to \mathbb{R}^m$ is a suitable function which verifies (L1).

In order to prove (6) we show that, for a given fixed integer k, if (6) is satisfied for $m = k$, then (6) is also satisfied for $m = k+1$. Clearly, since (6) holds for $m = 1$, this implies that (6) is verified for all $m \geq 1$.

Let $y = (y_1, \bar{y})$, $y_1 \in \mathbb{R}$ and $\bar{y} \in \mathbb{R}^k$. Abusing the notation, we write

$$f(x,y) = f(x,y_1,\bar{y})\,.$$

We see that

$$f(x,y_1,\bar{y}) = f(x,y_1,0) + \bar{f}(x,y_1,\bar{y}), \quad \forall\,(x,y_1,\bar{y}) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^k,$$

where $\bar{f}$ is defined by

$$\bar{f}(x,y_1,\bar{y}) = f(x,y_1,\bar{y}) - f(x,y_1,0), \quad \forall\,(x,y_1,\bar{y}) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^k\,.$$

From the case of $m = 1$ and since $f(x,0) \equiv 0$, we get

$$f(x,y_1,0) = H^1(y_1)G^1(x,y_1), \quad \forall\,(x,y_1) \in \mathbb{R}^n \times \mathbb{R}, \tag{7}$$

where $H^1 \in \mathcal{C}$ and $G^1 : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuous function.

Since (6) is satisfied for k and $\bar{f}(x,y_1,0) \equiv 0$, there exist a continuous function $\bar{H} : \mathbb{R}^k \to \mathbb{R}^k$ which verifies (L1) and continuous functions $\bar{G}_i : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^k$ $(i = 1,\ldots,n)$ such that

$$\bar{f}_i(x,y_1,\bar{y}) = \big\langle \bar{H}(\bar{y}), \bar{G}_i(x,y)\big\rangle \quad \forall\,(x,y_1,\bar{y}) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^k, \quad i = 1,\ldots,n. \tag{8}$$

Using (7) and (8), we obtain

$$\begin{aligned} f_i(x,y) &= f_i(x,y_1,0) + \bar{f}_i(x,y_1,\bar{y}) \\ &= H^1(y_1)G_i^1(x,y) + \big\langle \bar{H}(\bar{y}), \bar{G}_i(x,y)\big\rangle = \langle H(y), G_i(x,y)\rangle \end{aligned}$$

for $i = 1,\ldots,n$, where $H = (H^1, \bar{H})$ and $G_i = (G^1, \bar{G}_i)$. Therefore,

$$f(x,y) = G(x,y)H(y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m$$

with $G = [G_1\, G_2 \ldots G_n]^T$. This completes the proof of the lemma. ■

Remark 2. We would like to mention that the proposed decomposition of the function g is not a generalization of that used in the particular case of a smooth function f. Indeed, for the case of a smooth function we do not necessarily have $h(y) = y$.

2.2. Proof of Theorem 1

We have

$$f(x,y) = f(x,0) + \bar{f}(x,y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m$$

where

$$\bar{f}(x,y) = f(x,y) - f(x,0), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m\,.$$

is a continuous function such that $\bar{f}(x,0) \equiv 0$. Then, from Lemma 2, there exist continuous functions $H = (H_1, H_2, \dots, H_m)^T : \mathbb{R}^m \to \mathbb{R}^m$ and $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathcal{M}_{n\times m}$ such that

$$\bar{f}(x,y) = G(x,y)H(y), \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m$$

with $H_i(y) = h_i(y)$, where $h_i \in \mathcal{C}$, $i = 1, \dots, m$.

Let

$$W(x,y) = V(x) + \sum_{i=1}^{m} \int_0^{y_i} h_i(s)\,\mathrm{d}s, \quad \forall\,(x,y) \in \mathbb{R}^n \times \mathbb{R}^m$$

where V is a positive-definite and proper function such that

$$\dot{V}_{(2)}(x) = \langle f(x,0), \nabla V(x)\rangle < 0 \quad \text{for} \quad x \neq 0\,.$$

$\dot{V}_{(2)}$ being the time derivative of V along the trajectories of system (2). Note that since (2) is GAS, such a function always exists.

It is clear that W is positive definite and proper. The time derivative of W along the trajectories of the closed- loop system defined from (1) with

$$u(x,y) = -\nabla V(x)G(x,y) - y \tag{9}$$

is given by

$$\dot{W}(x,y) = \dot{V}_{(2)}(x) - \langle y, H(y)\rangle\,.$$

Clearly, we have $\dot{W}(x,y) < 0$ for $(x,y) \neq (0,0)$. Therefore, the system (1), (9) is GAS.

3. Example

Let us consider the special case $n = m = 1$ and assume that

$$f(x,y) = f_0(x) + \theta(x)g(y), \quad \forall\,(x,y) \in \mathbb{R}^2 \tag{10}$$

where f_0, $\theta \in C^0(\mathbb{R})$ such that $\dot{x} = f_0(x)$ is GAS and $g \in C^0(\mathbb{R}) \cap C^1(\mathbb{R}\backslash\{0\})$, with $g(0) = 0$ and $\{\mathrm{d}g/\mathrm{d}y\}$ is bounded in a neighborhood of $y = 0$.

System (10) is not necessarily of class C^1, so the classical approach (Byrnes and Isidori, 1989) cannot be used to investigate its stabilizability. The goal here is to show how our methodology can be used to get stabilizing feedback for (10).

First, we prove that there exists $\bar{g} \in C^0(\mathbb{R})$ such that

$$g(y) = y^{\frac{5}{7}}\,\bar{g}(y), \quad \forall\, y \in \mathbb{R}. \tag{11}$$

Indeed, since $g \in C^1(\mathbb{R}\backslash\{0\})$, we have $g^{\frac{7}{5}} \in C^1(\mathbb{R}\backslash\{0\})$. On the other hand,

$$\frac{\mathrm{d}g^{\frac{7}{5}}}{\mathrm{d}y}(y) = \frac{7}{5}g^{\frac{2}{5}}(y)\frac{\mathrm{d}g}{\mathrm{d}y}(y), \quad \forall y \in \mathbb{R}.$$

Thus

$$\lim_{y\to 0} \frac{\mathrm{d}g^{\frac{7}{5}}}{\mathrm{d}y}(y) = 0.$$

and hence $g^{\frac{7}{5}} \in C^1(\mathbb{R})$. Accordingly, there is $g_1 \in C^0(\mathbb{R})$ such that

$$g^{\frac{7}{5}}(y) = y g_1(y), \quad \forall y \in \mathbb{R},$$

which clearly implies (11).

Therefore, using the methodology developed in this paper, the feedback defined by

$$u(x,y) = -x\,\theta(x)\,\bar{g}(y) - y \tag{12}$$

stabilizes the system (1), (10) and a strict Lyapunov function for the closed-loop system (1), (10) with (12) is given by

$$W(x,y) = \frac{1}{2}x^2 + \frac{7}{12}y^{\frac{12}{7}}.$$

In this paper we have proved a result on the stabilization of continuous systems by adding an integrator. The proposed stabilizing feedback is guaranteed to be continuous. However, in some cases the stabilizing feedback can be selected to be smooth even if the system is only continuous.

To illustrate our remark, we prove that (1), (10) is stabilizable by means of a smooth feedback. Let k be any smooth function such that

$$k^2(x,y) \geq \left(\bar{g}^2(y)+1\right)\left(\theta^2(x)+1\right)\delta(x) \tag{13}$$

where δ is a smooth function which fulfils

$$\delta(x) \geq \begin{cases} 1 & \text{if } \; x \in [-1,1] \\ \dfrac{|f_0(1)|+|f_0(-1)|}{f_0(x)} & \text{if } \; |x| \geq 1 \end{cases} . \tag{14}$$

The closed-loop system (1), (10) with

$$u(x,y) = -(y+y^3)k^2(x,y) - y \tag{15}$$

is GAS. Indeed, let

$$W_1(x,y) = \int_0^x l(s)\,\mathrm{d}s + \frac{3}{4}y^{\frac{4}{3}}$$

where l is the continuous function defined by

$$l(x) = \begin{cases} -\frac{1}{2} f_0(1) & \text{if} \quad x > 1, \\ -\frac{1}{2} f_0(x) & \text{if} \quad |x| \le 1, \\ -\frac{1}{2} f_0(-1) & \text{otherwise.} \end{cases} \tag{16}$$

The derivative of W_1 along the trajectories of the closed-loop system (1), (10) with (15) is given by

$$\dot{W}_1(x,y) = l(x) f_0(x) + y^{\frac{5}{7}} l(x)\theta(x)\bar{g}(y) - \left(y^{\frac{4}{3}} + y^{\frac{10}{3}}\right) k^2(x,y) - y^{\frac{4}{3}}.$$

Since

$$y^{\frac{4}{3}} + y^{\frac{10}{3}} \ge y^{\frac{10}{7}} \qquad \text{for all} \quad y \in \mathbb{R},$$

we get

$$\begin{aligned} \dot{W}_1(x,y) &\le l(x) f_0(x) + y^{\frac{5}{7}} l(x)\theta(x)\bar{g}(y) - y^{\frac{10}{7}} k^2(x,y) - y^{\frac{4}{3}} \\ &= -\left(y^{\frac{5}{7}} k(x,y) - \frac{1}{2} l(x)\theta(x)\bar{g}(y)\right)^2 \\ &\quad + \left(\frac{l(x)\theta(x)\bar{g}(y)}{k(x,y)}\right)^2 + l(x) f_0(x) - y^{\frac{4}{3}}. \end{aligned}$$

Now, using (13), we obtain

$$\dot{W}_1(x,y) \le l(x) \left(\frac{l(x)}{\delta(x)} + f_0(x)\right) - y^{\frac{4}{3}}.$$

A simple reasoning using (14) and (16) shows that

$$x\left(\frac{l(x)}{\delta(x)} + f_0(x)\right) < 0 \qquad \text{for all} \quad x \ne 0.$$

This clearly implies that

$$\dot{W}_1(x,y) < 0 \qquad \text{for} \quad (x,y) \ne (0,0).$$

4. Conclusion

We have presented a result on feedback stabilization of continuous systems by adding an integrator. Its proof is based on a technical lemma related to continuous functions. Works are in progress to show how other classical results on feedback stabilization of smooth systems can be extended to systems that are only continuous.

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