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LINEAR ADAPTIVE STRUCTURE FOR CONTROL OF A NONLINEAR MIMO DYNAMIC PLANT

STANISŁAW BAŃKA, PAWEŁ DWORAK, KRZYSZTOF JAROSZEWSKI

Faculty of Electrical Engineering
West Pomeranian University of Technology, Szczecin, 26 Kwietnia 10, 71-126 Szczecin, Poland
e-mail: {stanislaw.banka,pawel.dworak,krzysztof.jaroszewski}@zut.edu.pl

In the paper an adaptive linear control system structure with modal controllers for a MIMO nonlinear dynamic process is presented and various methods for synthesis of those controllers are analyzed. The problems under study are exemplified by the synthesis of a position and yaw angle control system for a drillship described by a 3DOF nonlinear mathematical model of low-frequency motions made by the drillship over the drilling point. In the proposed control system, use is made of a set of (stable) linear modal controllers that create a linear adaptive controller with variable parameters tuned appropriately to operation conditions chosen on the basis of two measured auxiliary signals. These are the ship's current forward speed measured in reference to the water and the systematically calculated difference between the course angle and the sea current (yaw angle). The system synthesis is carried out by means of four different methods for system pole placement after having linearized the model of low-frequency motions made by the vessel at its nominal "operating points" in steady states that are dependent on the specified yaw angle and the sea current velocity. The final part of the paper includes simulation results of system operation with an adaptive controller of (stepwise) varying parameters along with conclusions and final remarks.

Keywords: MIMO multivariable control systems, nonlinear systems, modal control.

1. Introduction

Control of multivariable dynamic plants is still the subject of studies and is the source of many unresolved issues, especially those concerning nonlinear systems. Nonlinear control systems are commonly encountered in many different areas of science and technology. In particular, problems difficult to solve arise in motion and/or position control of various vessels, like drilling platforms and ships, sea ferries, special purpose ships as well as submarines. Complex motions and/or complex-shaped bodies moving in the water, and in the case of ships also at the boundary between water and air, give rise to resistance forces dependent in a nonlinear way on velocities and positions, thus causing the floating bodies to become strongly nonlinear dynamic plants.

In general, there are two basic approaches to solve the control problem for nonlinear plants. The first one, called "nonlinear", consists in synthesizing a nonlinear controller that would meet certain requirements over the entire range of control signals variability. The second approach, called "linear", consists in designing an adaptive linear controller with varying parameters to be systematically tuned up in keeping with changing plant operating conditions determined by system nominal "operating points". Nominal "operating points" are usually defined in steady states of the plant; however, these also can be determined in its transient regimes.

The "nonlinear" approach may include techniques based on the second Lyapunov method, for example, by employing the sequential backstepping procedure (Fossen and Strand, 1999; Witkowska *et al.*, 2007) or methods that consist in system linearizing through a plant output (or state) related nonlinear feedback, supported by feedforward compensators with characteristics being inverse to nonlinear functions contained in the plant description (Fabri and Kadrikamanathan, 2001; Zwierzewicz, 2008). In the case when nonlinear descriptions of the plant are not known accurately, advantage can be taken of methods employing artificial intelligence techniques, for example, those using neural approximators (Tzirkel-Hancock and Fallside, 1992; Fabri and Kadrikamanathan, 2001; Pedro and Dahunsi, 2011). Substantial difficulties encountered in employing this "nonlinear" approach are due to the fact that control plants are multivariable.

However, in practice, the second approach, called "linear", is more convenient to use, since advantage can be taken of already proven procedures and commonly known mathematical methods employed in the design (synthesis) of linear controllers. Here, linearization of nonlinear MIMO plants is a prerequisite for the methods to be employed. The most frequently used way of linearization consists in taking a Taylor series expansion of a nonlinear function and then taking only the first order term of the expansion. After linearization, local linear models are obtained, valid for small deviations from "operating points" of the plant.

The obtained linear models with known parameters or those to be identified are the starting point for applying many known methods for linear control system design. These can be both traditional ones to design classic PID control systems, although being difficult to implement in the case of MIMO plants, and relatively simple ones to synthesize systems with multivariable modal controllers (or possibly LQR/LQG) based on the Luenberger observer or the Kalman filter (Antoniou and Vardulakis, 2005; Bańka, 2007; Kaczorek, 1992; Wolovich, 1974). Since properties exhibited by linear models at different (distant) "operating points" of the plant may substantially vary, the controllers used should be either robust (usually of a very high order, as has been observed by Gierusz (2005)) or adaptive, switched (Zhai and Xu, 2010; Tomera, 2010; Bańka *et al.* 2010b; 2011a) or with parameters being tuned in the process of operation (Aström and Wittenmark, 1995).

If the description of the nonlinear plant is known, then it is possible to make use of systems with linear controllers prepared earlier for possibly all "operating points" of the plant. Such controllers can create either a set of controllers with switchable outputs from among which one controller designed for the given system "operating point" (Bańka *et al.*, 2010a) is chosen, or multi-controller structures whose control signal components are formed as weighted means of outputs of a selected controller group (fuzzy cluster) according to Takagi–Sugeno–Kang rules. The weights could be proportional to the degree of their membership of appropriately fuzzified areas of plant outputs or other auxiliary measured signals (Tatjewski, 2007).

What all the above-mentioned multi-controller structures, where not all controllers at the moment are utilized in a closed-loop system, have in common is that all controllers employed in these structures must be stable by themselves, as opposed to a single adaptive controller with varying (tuned) parameters. This means that system strong stability conditions should be fulfilled (Vidyasagar, 1985).

In the paper an adaptive modal MIMO controller with (stepwise) varying parameters in the process of operation is studied. As already mentioned, the controller can also be physically realized as a multi-controller structure of (stable) modal controllers with switchable outputs. In such a case, the number of controllers should be limited to a cluster of controllers with fewer number of controllers. This cluster should be designed for the near surroundings of the current operating point of the system. The remaining controllers, in such a case, could be stored on the disk or redesigned in on-line mode adequately to the needs. The modal controllers making up the adaptive (multi-controller) control system considered will be designed for all possible "operating points" of the nonlinear MIMO plant. The appropriate controller (appropriate set of parameter values of the tuned controller) will be selected during system operation on the basis of two auxiliary measured signals, on which the "operating points" of the nonlinear plant are dependent.

The organization of this paper is as follows. In Section 2 a mathematical description of the adopted nonlinear control plant is presented. In Section 3 we discuss the structure of the proposed control system based on a set of linear modal controllers that may create an adaptive controller with (stepwise) varying parameters conditioned by two additional auxiliary signals, namely, the ship transitional velocity measured with respect to water and the calculated difference between the sea current angle and the actual ship's yaw angle. In Section 4 we carry out a survey of synthesis methods for multivariable modal controllers in both time and frequency domains using the polynomial approach with and without solving Diophantine polynomial matrix equations. Section 5 contains results of controller synthesis obtained by means of the methods presented in Section 4. The operation of the found controller sets is tested in Section 6 by simulation of the designed tuned controller system with the nonlinear plant model. We end the paper in Section 7 with conclusions.

2. Description of the control plant

The MIMO nonlinear dynamic control plant is exemplified here by the drillship *Wimpey Sealab* having $L_{pp} = 94.49$ [m] in length, $B = 15.24$ [m] in beam, with an average draught of $H = 5.49$ [m] and with a displacement of $m = 5670\,DWT$. When operated, the ship was equipped with a simple (clinometric) Dynamic Positioning System (DPS) with classical autonomous PID controllers. The system enabled the vessel to keep on course and position over the sea bed drilling point with the help of a 2013 [kW] main engine and four azimuth Schottel propellers of 746 [kW] each.

The adaptive (multi-controller) control system structure considered is studied by means of a 3DOF nonlinear mathematical model of the ship's low-frequency motions, which has been developed on the basis of tests carried out on a physical model on a scale of 1:20 in an American ship model basin (Wise and English, 1975). The yaw angle and the ship's position in DSP are defined in an Earth-based fixed reference system whose axes are directed northwards (N) and eastwards (E), and whose origin is located over the drilling point on the seabed. By contrast, force and speed components with respect to water are determined in a moving system related with the ship's body and the axes directed to the front and the starboard of the ship with the origin placed at its gravity center. These are shown in Fig. 1.

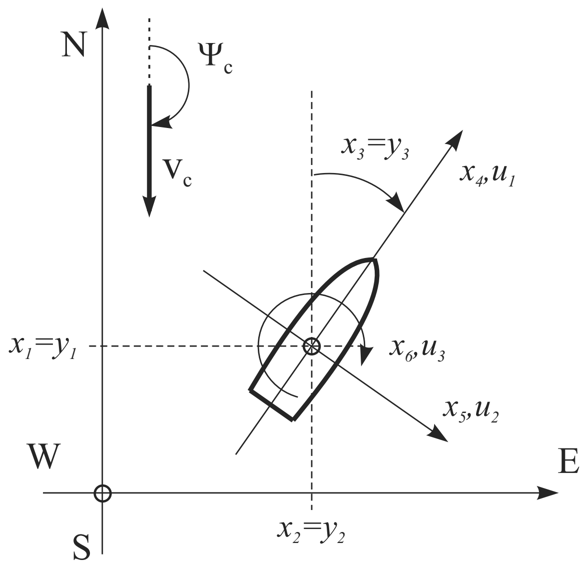

Fig. 1. Ship's co-ordinate systems.

The mathematical description of the plant is given in the form of nonlinear state space and linear output equations:

$$\begin{aligned}
\dot{x}_1 &= x_4 \cos x_3 - x_5 \sin x_3 + V_c \cos \Psi_c,\\
\dot{x}_2 &= x_4 \sin x_3 + x_5 \cos x_3 + V_c \sin \Psi_c,\\
\dot{x}_3 &= x_6,\\
\dot{x}_4 &= 0.088x_5^2 - 0.132x_4 V_s + 0.958x_5x_6 + 0.958u_1, \qquad (1)\\
\dot{x}_5 &= -1.4x_5V_s - 0.978x_5^3/V_s - 0.543x_4x_6\\
&\quad + 0.037x_6\,|x_6| + 0.544u_2,\\
\dot{x}_6 &= (0.258x_5V_s - 0.764x_4x_5 - 0.162x_6\,|x_6| + u_3),\\
y_1 &= x_1, \quad y_2 = x_2, \quad y_3 = x_3,
\end{aligned}$$

where $V_s = \sqrt{x_4^2(t) + x_5^2(t)}$ is the translational velocity of the ship measured with respect to water. The coefficient $a = k_{zz}^2 + 0.0431$ describes the ship's inertia moment together with water associated with the angle motion of the ship around its vertical axis, where k_{zz} is the relative inertia radius referenced to the ship's length L_{pp}. V_c and Ψ_c are, respectively, the velocity and direction of the sea current as indicated in Fig. 1.

All the signals appearing in (1) are dimensionless, i.e., referenced to the ship's dimensions and displacement as follows:

$$\begin{aligned}
&u_1(t) = \frac{F_x(t)}{mg}, \quad u_2(t) = \frac{F_y(t)}{mg},\\
&u_3(t) = \frac{M_z(t)}{mgL_{pp}},\\
&x_1(t) = \frac{y_1(t)}{L_{pp}}, \quad x_2(t) = \frac{y_2(t)}{L_{pp}},\\
&x_3(t)\,[\text{rad}], \qquad (2)\\
&x_4(t) = \frac{v_x(t)}{\sqrt{gL_{pp}}}, \quad x_5(t) = \frac{v_y(t)}{\sqrt{gL_{pp}}},\\
&x_6 = \frac{\omega_z(t)}{\sqrt{g/L_{pp}}},
\end{aligned}$$

together with the dimensionless time $t = t_r/\sqrt{L_{pp}/g} \approx 0.32\,t_r$.

It should be noted that dividing by a signal representing the ship's translational velocity $V_s(t)$ with respect to water takes place in the above nonlinear ship motion model. This accounts for undefined behavior of the nonlinear model at zero-valued ship velocity, i.e., when dividing by $V_s(t) = 0$ occurs. This has some consequences not only during system simulation, but also for control system synthesis, since linear models become undefined at $V_s = 0$. Hence, controllers with a structure like that determined for normal operation conditions at $V_s(t) \neq 0$ cannot be found in this case. This is attributable to the fact that hydrodynamic resistance disappears at $V_s(t) = 0$, which substantially affects the character of the described phenomena and brings about, among others, the zeroing of respective terms in Eqn. (1). Such a situation takes place when the ship is carried along by currents or when the ship stands still over the drilling point in calm water at $V_c = 0$.

According to the linear approach adopted in the paper, the linearization of the model (1) is performed for ship typical locations within the area of admissible positions over the drilling point in steady state when $V_s(t) = -V_c$. The nominal values of the state vector $\mathbf{x}_o$ and forces, as well as the moment $\mathbf{u}_o$ enabling to overcome hydrodynamic resistances of the ship's hull, given the known values of $V_c \neq 0$ and Ψ_c, can be calculated from the system of nonlinear algebraic equations

$$\begin{aligned}
\mathbf{0} &= \mathbf{f}(\mathbf{x}_o, \mathbf{u}_o, V_c, \Psi_c), \qquad (3)\\
\mathbf{y}_o &= \mathbf{C}\mathbf{x}_o.
\end{aligned}$$

As a result of the linearization performed in the whole range of the yaw angle $x_{30} \in [-\pi, \pi]$ [rad], under various sea current velocities $V_c \in [0.05 \div 3.5]$ [knot] and at $\Psi_c = \pi$ [rad], the linear state-space models are obtained

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}[\mathbf{x}(t) - \mathbf{x}_o] + \mathbf{B}[\mathbf{u}(t) - \mathbf{u}_o], \\ \mathbf{y}(t) - \mathbf{y}_o &= \mathbf{C}[\mathbf{x}(t) - \mathbf{x}_o],\end{aligned} \tag{4}$$

where

$$\mathbf{A} = \left[\frac{\partial}{\partial \mathbf{x}} \mathbf{f}^{\mathrm{T}}(\mathbf{x}, \mathbf{u}, \cdot)\right]^{\mathrm{T}}_{\substack{\mathbf{x}=\mathbf{x}_o \\ \mathbf{u}=\mathbf{u}_o}} = \begin{bmatrix} 0 & 0 & 0 & a_{14} & a_{15} & 0 \\ 0 & 0 & a_{23} & a_{24} & a_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & 0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.958 & 0 & 0 \\ 0 & 0.544 & 0 \\ 0 & 0 & 1/a \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

with the entries a_{ij} depending on the difference between the sea current angle Ψ_c and values of the ship's yaw angle $y_{30} = x_{30}$ adopted for the purpose of linearization, and on the current velocity V_c. All the obtained models of the ship are unstable with three invariable poles $s_1 = s_2 = s_3 = 0.0$ and three variable poles s_4, s_5, s_6. Their matrix transfer functions in the complex domain can be presented in the form of a relatively right prime (r.r.p.) polynomial Matrix Fraction Description (MFD),

$$\mathbf{T}(s) = \mathbf{B}_1(s)\mathbf{A}_1^{-1}(s), \tag{5}$$

where

$$\mathbf{B}_1(s) = \begin{bmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ 0 & 0 & 1/a \end{bmatrix},$$

$$\mathbf{A}_1(s) = \begin{bmatrix} s^2 + a_1 s & a_2 s & a_3 s + a_4 \\ a_5 s & s^2 + a_6 s & a_7 s + a_8 \\ a_9 s & a_{10} s & s^2 + a_{11} \end{bmatrix},$$

with variable parameters b_i, $i = 1, 2, 3, 4$ and a_j, $j = 1, 2, \ldots, 11$. The gain matrix of the plant is defined as

$$\begin{aligned}\mathbf{K}_p &= [\mathbf{B}_1(0)]\left[\mathbf{A}_1^{-1}(0)\right] \\ &= \begin{bmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ 0 & 0 & 1/a \end{bmatrix} \begin{bmatrix} 0 & 0 & a_4 \\ 0 & 0 & a_8 \\ 0 & 0 & a_{11} \end{bmatrix}^{-1} \to \infty,\end{aligned} \tag{6}$$

which demonstrates in general the integration property of the control plant. However, this observation is not applicable to all control paths, among others, to those acting on the ship's course angle. The coefficient a appearing in Eqns. (1) and (6) depends on the extent to which the ship is loaded and on the mass distribution on board the ship. Since the numerator matrix $\mathbf{B}_1(s)$ in the transfer function (5) is a real matrix, all ship linear models are minimum phase, i.e., non-minimum phase transmission zeroes do not occur there.

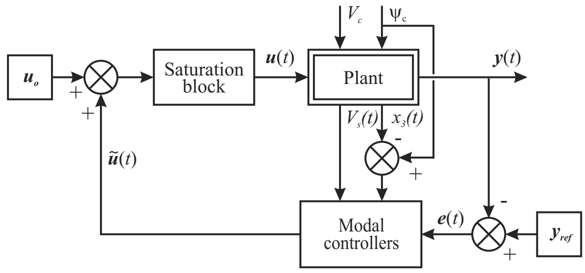

Fig. 2. Block diagram of the proposed control system structure.

3. Description of the proposed control system structure

The block diagram of the control system for ship course and position over the drilling point is depicted in Fig. 2. The above control system for the nonlinear MIMO plant with specified set points $\mathbf{y}_{\mathrm{ref}}$ consists of a set of multivariable modal controllers realized either as a single adaptive controller with stepwise switchable parameter values or as a set of controllers with a common input $\mathbf{e}(t)$ and switchable outputs $\tilde{\mathbf{u}}(t)$. All modal controllers making up the above structure are designed for different ship linear models obtained for adopted operating points of the plant at different sea current velocities V_c and yaw angles $y_{3o} = x_{3o}$ of the ship standing still over the drilling point. The points are determined by nominal values of the plant state vector $\mathbf{x}_o$ and nominal values of the control signals $\mathbf{u}_o$ in steady states. These are found from the system of algebraic equations (3). For such a kind of plants, $\mathbf{x}_o$ and $\mathbf{u}_o$ depend exclusively on the yaw angle set point y_{30}, as well as on the velocity V_c and the sea current angle Ψ_c.

In the proposed multi-controller structure, the controller parameter values are changed (or controller outputs are changed over, respectively) on the basis of auxiliary variables measured. These are in the case under study: the ship's current transitional velocity $V_s(t)$ measured with respect to water (it is negative if the ship sails astern, i.e., at $x_4(t) < 0$) and the systematically calculated difference between the sea current angle and the ship's yaw angle $\Psi_c - x_3(t)$. During the system operation the incremental values $\tilde{\mathbf{u}}(t)$ generated by the adaptive controller are added to the nominal values $\mathbf{u}_o$.

Modal controllers used in the proposed control system structure are multivariable dynamic systems with parameters defined in time domain by

$$\begin{aligned}\dot{\mathbf{x}}_r(t) &= \mathbf{A}_r\mathbf{x}_r(t) + \mathbf{B}_r\mathbf{e}(t),\\ \tilde{\mathbf{u}}(t) &= \mathbf{C}_r\mathbf{x}_r(t) + \mathbf{D}_r\mathbf{e}(t).\end{aligned} \tag{7}$$

These can be presented in their natural form, which is called "standard", with the following matrices:

$$\begin{aligned}\mathbf{A}_r &= \mathbf{A} - \mathbf{BF} - \mathbf{LC}, & \mathbf{B}_r &= \mathbf{L},\\ \mathbf{C}_r &= -\mathbf{F}, & \mathbf{D}_r &= \mathbf{0},\end{aligned} \tag{8}$$

where $\mathbf{F}$ is the matrix of proportional feedbacks that are related to state vector components (reconstructed by the observer) of the plant linear models, and $\mathbf{L}$ is the gain matrix of full order Luenberger observers that reconstruct the state vector of the plant linear models (4). Another possibility (although this is a necessity if the polynomial approach with solving polynomial matrix equations is employed) is to present Eqn. (7) in an appropriate canonical form (most common an observable one) with the matrices

$$\mathbf{A}_{ro},\ \mathbf{B}_{ro},\ \mathbf{C}_{ro} \text{ and } \mathbf{D}_r = \mathbf{0}. \tag{9}$$

Unlike the matrices in the "standard" form, these are characterized by a minimal number of parameters different from "0" or "1". The above controllers represent strictly causal dynamic systems with $\mathbf{D}_r = \mathbf{0}$. In the s-domain they are described by strictly proper matrices of rational transfer functions in the form of relatively left prime (l.r.p.) polynomial matrix fractions

$$\begin{aligned}\mathbf{T}_c(s) &= \mathbf{C}_r(s\mathbf{I}_n - \mathbf{A}_r)^{-1}\mathbf{B}_r\\ &= \mathbf{C}_{ro}(s\mathbf{I}_n - \mathbf{A}_{ro})^{-1}\mathbf{B}_{ro} = \mathbf{M}_2^{-1}(s)\mathbf{N}_2(s),\end{aligned} \tag{10}$$

with the polynomial matrices: $\mathbf{M}_2(s) \in \mathbb{R}[s]^{m\times m}$ is a nonsingular row-reduced denominator matrix and $\mathbf{N}_2(s) \in \mathbb{R}[s]^{m\times l}$ is a numerator matrix that fulfills the strict inequalities

$$\deg_{rj}\mathbf{N}_2(s) < \deg_{rj}\mathbf{M}_2(s), \quad j = 1, 2, \ldots, m, \tag{11}$$

where $\deg_{rj}[\cdot]$ denote row degrees of $[\cdot]$.

Static properties of MIMO modal controllers under discussion depend directly on their gain matrices

$$\mathbf{K}_c = \mathbf{C}_r(-\mathbf{A}_r)^{-1}\mathbf{B}_r = \mathbf{M}_2^{-1}(0)\mathbf{N}_2(0), \tag{12}$$

and the dynamic properties are determined by poles *pole_reg*, defined by the eigenvalues of the matrix $\mathbf{A}_r$ of each of the controllers, which represent zeroes of the determinants

$$\det\mathbf{M}_2(s) = \det[s\mathbf{I}_n - \mathbf{A}_r] = 0. \tag{13}$$

In general, the controllers considered can be stable or unstable. By definition, they cannot exhibit integration properties and should be stable in the proposed structure. In the case under discussion these will be multivariable (MIMO) controllers whose behavior is close to that PD ones with time lag.

In order to limit the effect of excessive forces and moments produced by the adaptive set of modal controllers, we introduce constraints imposed on the maximal values of control signals $\mathbf{u}(t) = \tilde{\mathbf{u}}(t) + \mathbf{u}_o$. In a real ship control system, a block of propulsion distribution among individual propellers and the main engine has been used instead of a block for constraining control signals $\mathbf{u}(t)$.

If the values of $\mathbf{u}_o$ are known and the modal controllers are properly designed (for the given operating points), there exists a theoretical possibility that the residual steady-state error will tend to zero $\mathbf{e}_{st}(t) \rightarrow \mathbf{0}$ as $\tilde{\mathbf{u}}(t) \rightarrow \mathbf{0}$. In real situations, the values $\mathbf{u}_o$ can be corrected manually in the block for compensation of steady-state errors in such a way as to eliminate (or reduce) possible deviations of the ship's course and/or position in steady state for the reason that the effect of some environmental disturbances (wind, motion of the sea) has been neglected here and/or the actual ship operation conditions differ from the "nominal" adopted for linearization. Another reason is the lack of knowledge about "real" nominal values of control signals required to maintain the ship's position in steady state.

It should be noted that steady state errors may be brought about not only by the effect of additional long-lasting forces and moments turning the ship produced by, among others, the (averaged) action of wind and sea waves, but also for the reason that not all paths of the ship's multivariable model exhibit integration properties. This is the case with modal PD controllers with time lag (Bańka and Latawiec, 2009).

4. Methods of modal controller synthesis

The synthesis of modal controllers is based on using the technique of pole placement in stable regions of the s-plane. In the case of SISO, pole placement determines the system dynamics in one control path only, so the task is easy to accomplish and results of calculations are unambiguous, i.e., they are independent of the structure of source data.

The synthesis of modal controllers with MIMO plants is much more complex, since the dynamics of many control paths are to be shaped. The system poles in each path may take different values in accordance with to the dynamics required for each of the paths. This raises the question of how to provide the location of a specific pole for an appropriate path of the control system to be designed. The task is not easy to perform and, as it turns out, the final result depends not only on selecting an appropriate design method, but also on setting a concrete data structure used for the design. Additionally, the results may depend on whether the poles are real or complex and on the order in which the poles occur in the set of data taken for design. In a polynomial approach it is required to divide the set of pre-determined poles into appropriate pole blocks with a specific number of poles in each block. If the poles are conjugate complex, each pair of them must be an element of the same block. This is particularly essential for plants of odd order *n*, and also if an odd number of poles is required for individual blocks of adopted pole values. As a result, completely different modal controllers may be obtained for the same input data depending on the adopted design method and the adopted data structure used for design.

The synthesis of modal controllers can be performed directly in time domain with the plant linear models (4) as a starting point and in s-domain using the polynomial approach with the transfer function matrices (5) as a starting point. Using the polynomial approach with solving polynomial matrix equations usually yields causal controllers described in s-domain by matrices of proper rational transfer functions obtained directly from solutions of Diophantine polynomial matrix equations. If we decide on (strictly causal) modal controllers based on full-order Luenberger observers, the design performed directly in time domain (and also in s-domain without solving polynomial matrix equations) boils down to separate determination of the feedback matrix $\mathbf{F}$, which forces the closed-loop eigenvalues to the pole locations specified by the adopted (stable) pole values *pole_sys*, and the weight matrix $\mathbf{L}$ of the full-order Luenberger observer for appropriately chosen observer poles *pole_obs*. The real parts of the latter should be more negative than those selected for the *pole_sys* set.

Assuming the modal control plant is given by the linear MIMO system described by differential state-space equations (4), the first step on the road to synthesizing a modal control system in time domain is to determine the state feedback gain matrix $\mathbf{F} \in \mathbb{R}^{m\times n}$ in

$$\mathbf{u}(t) = -\mathbf{F}\mathbf{x}(t), \tag{14}$$

which shifts the poles of a linear plant model to desired locations specified by the preassigned *a priori* values of *pole_sys*. These correspond to respective eigenvalues λ_i, $i = 1, 2, \dots, n$, of the matrix $\mathbf{A}$ and s_i, $i = 1, 2, \dots, n$, for the matrix $\mathbf{A} - \mathbf{BF}$. The latter are the roots of the characteristic equation

$$\begin{aligned}\det[s\mathbf{I}_n - \mathbf{A} + \mathbf{BF}]& \\ &= s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0 \end{aligned}\tag{15}$$

with coefficients $a_0, a_1, \dots, a_{n-1}$ calculated from preassigned eigenvalues s_i (poles *pole_sys*) of the matrix $\mathbf{A} - \mathbf{BF}$.

The eigenvalues λ_i of the plant matrix $\mathbf{A}$ correspond to the eigenvectors $\mathbf{m}_i$, $i = 1, 2, \dots, n$, which represent the solution of the equation system

$$[\mathbf{A} - \lambda_i \mathbf{I}_n]\,\mathbf{m}_i = 0 \quad \text{for } i = 1, 2, \dots, n. \tag{16}$$

Usually they are found by taking $\mathbf{m}_i$ as a nonzero (arbitrary) column of the adjugate matrix $[\mathbf{A} - \mathbf{I}_n\lambda_i]_{ad}$. From them a matrix of eigenvectors

$$\mathbf{M} = \begin{bmatrix}\mathbf{m}_1 & \mathbf{m}_2 & \dots & \mathbf{m}_n\end{bmatrix} \tag{17}$$

can be created, which will be nonsingular, provided $\mathbf{m}_i$ are chosen as linear independent columns from consecutive matrices $[\mathbf{A} - \mathbf{I}_n\lambda_i]_{ad}$ for $i = 1, 2, \dots, n$.

Hence, the sought-for matrix $\mathbf{F}$ can be determined in time domain through the eigenvalues of the matrices $\mathbf{A}$ and $\mathbf{A} - \mathbf{BF}$ or on the basis of eigenvectors corresponding to eigenvalues of the matrices (Kaczorek, 1992). Determining the weight matrices $\mathbf{L}$ for full order observers is carried out in a dual way by using eigenvalues or eigenvectors of the matrices $\mathbf{A}$ and $\mathbf{A} - \mathbf{LC}$, respectively.

4.1. Eigenvalues method. Making an exclusive use of eigenvalues requires that the plant description (4) be converted into the controllable second canonical form with matrices $\hat{\mathbf{A}} = \hat{\mathbf{P}}\mathbf{A}\hat{\mathbf{P}}^{-1}$ and $\hat{\mathbf{B}} = \hat{\mathbf{P}}\mathbf{B}$, where characteristic nonzero rows occur having numbers $n_i = \sum d_i$, $i = 1, 2, \dots, m$, and d_i are Kronecker controllability indices of the plant. The form may be obtained by a homothetic transformation with matrix $\hat{\mathbf{P}}$ created appropriately from the controllability matrix for the pair ($\mathbf{A}$,$\mathbf{B}$) of the plant model (4).

Taking into consideration the nonzero rows of the matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, denoted respectively by $\hat{\mathbf{A}}_{n_i}$ and $\hat{\mathbf{B}}_{n_i}$, $n_i = \sum d_i$, $i = 1, 2, \dots, m$, the following matrix is created:

$$\bar{\mathbf{F}} = \begin{bmatrix} \hat{\mathbf{A}}_{n_1} - \mathbf{e}^T_{n_1+1} \\ \hat{\mathbf{A}}_{n_2} - \mathbf{e}^T_{n_2+1} \\ \cdots \\ \hat{\mathbf{A}}_{n_{m-1}} - \mathbf{e}^T_{n_{m-1}+1} \\ \hat{\mathbf{A}}_{n_m} - \mathbf{a}^T \end{bmatrix}, \tag{18}$$

where $\mathbf{e}^T_i$ is the i-th row of the identity matrix $\mathbf{I}_n$, and $\mathbf{a}^T := [a_0, a_1, \dots, a_{n-1}]$ is the row made up of coefficients of the characteristic polynomial (15), and the matrix

$$\hat{\mathbf{B}}_m = \begin{bmatrix} \hat{\mathbf{B}}_{n_1} \\ \hat{\mathbf{B}}_{n_2} \\ \cdots \\ \hat{\mathbf{B}}_{n_m} \end{bmatrix} = \begin{bmatrix} 1 & * & \cdots & * \\ 0 & 1 & \cdots & * \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{19}$$

is formed from nonzero rows of the matrix $\hat{\mathbf{B}}$. Then the sought-for feedback matrix $\mathbf{F}$, which shifts the poles of the closed-loop system to desired locations on the left half plane $s \in \mathbb{C}$, can be determined from

$$\mathbf{F} = \hat{\mathbf{B}}_m^{-1} \bar{\mathbf{F}} \hat{\mathbf{P}}. \tag{20}$$

This can be done by calling the function $[\mathbf{F}] = modal(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, pole\_sys)$, which represents an implementation of the above described procedure in the Polynomial Toolbox for MATLAB. Determining weight matrices $\mathbf{L} \in \mathbb{R}^{n \times l}$ for the full-order Luenberger observer in time domain can be carried out by utilizing the already mentioned function *modal.m* in a dual way, namely, by calling $[\mathbf{L}] = modal(\mathbf{A}', \mathbf{C}', \mathbf{B}', \mathbf{D}, pole\_obs)'$.

4.2. Eigenvectors method. In the event that matrix $\mathbf{A}$ has different eigenvalues λ_i, $i = 1, 2, \dots, n$, the *eigenvectors method* comes down to determining the matrix of eigenvectors (17) and creating a diagonal matrix

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 - s_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 - s_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & \lambda_n - s_n \end{bmatrix}, \tag{21}$$

whose elements are differences between eigenvalues λ_i of the matrix $\mathbf{A}$ and roots s_i of the characteristic equation (15). Then the feedback matrix $\mathbf{F}$ can be calculated directly from

$$\mathbf{F} = \mathbf{M} \mathbf{\Lambda} \mathbf{M}^{-1}. \tag{22}$$

However, this way of calculating the feedback matrix becomes complicated if eigenvalues of the plant matrix $\mathbf{A}$ are complex or multiple real, and if, for some reason or other, such eigenvalues are preassigned for the control system to be designed. The standard function *place.m* of the Control Toolbox for MATLAB/Simulink represents an implementation of the above-mentioned procedure with restrictions imposed on the maximal multiplicity of preassigned poles *pole_sys*, which may not excess the number of plant inputs m.

Determining weight matrices $\mathbf{L} \in \mathbb{R}^{n \times l}$ for the full-order Luenberger observer in time domain can be carried out utilizing the already mentioned function *place.m* in a dual way, namely, by respective calling $[\mathbf{L}] = place(\mathbf{A}', \mathbf{C}', pole\_obs)'$, where *pole_obs* is the set of eigenvalues (poles) s_i, $i = 1, 2, \dots, n$, specified for the matrix $\mathbf{A} - \mathbf{LC}$.

It is easy to note that calculations performed with the use of the above functions do not ensure that eigenvalues of $\mathbf{A} - \mathbf{BF}$ and $\mathbf{A} - \mathbf{LC}$, i.e., the poles *pole_sys* and *pole_obs*, will be located in *a priori* specified control system paths since in MIMO systems many different matrices $\mathbf{A} - \mathbf{BF}$ and $\mathbf{A} - \mathbf{LC}$ of identical determinants $\det(s\mathbf{I}_n - \mathbf{A} + \mathbf{BF})$ and $\det(s\mathbf{I}_n - \mathbf{A} + \mathbf{LC})$ may exist. The actual pole location can be verified only through simulations of the designed control system, preferably with a modal controller in its standard form (8). Then the pole location can be assessed whether or not it is proper from observations of time responses of state variables for the plant model and the Luenberger observer, where $\mathbf{x}_r(t) = \hat{\mathbf{x}}(t) \to \mathbf{x}(t)$ for $t \to \infty$.

If we use the *eigenvector method*, the final result may additionally depend on the order the elements in sets *pole_sys* and *pole_obs* are listed due to the freedom of choice of the sequence of eigenvectors $\mathbf{m}_i, i = 1, 2, \dots, n$, in (16). This means that, depending on the method used, many different matrices $\mathbf{F}$ and $\mathbf{L}$ may exist for the same poles *pole_sys* and *pole_obs* yielding, as a result, entirely different modal controllers (7).

Furthermore, when employing synthesis methods in time domain, there is no impediment to make use of different functions, e.g., *place.m* of the *eigenvector method* while determining the matrix $\mathbf{F}$ and *modal.m* of the *eigenvalue method* when finding the matrix $\mathbf{L}$ or vice versa, which further extends the range of solutions possible to obtain. However, this makes the results of design ambiguous in the sense that many different modal controllers are obtained for the same input data.

4.3. Polynomial method. The matrices $\mathbf{F}$ and $\mathbf{L}$ can be found using the polynomial approach without solving polynomial matrix equations utilizing the well-known Wolovich structure theorem (Wolovich, 1974). According to this theorem and the procedure described by Bańka (2007) as well as Bańka and Dworak (2011a), these matrices can be determined directly from the following relationships:

$$\mathbf{C}_1(s) - \mathbf{A}_1(s) = -\mathbf{F}(s) = -\mathbf{F}\hat{\mathbf{P}}\hat{\mathbf{S}}(s) \tag{23}$$

and

$$\mathbf{C}_2(s) - \mathbf{A}_2(s) = \tilde{\mathbf{S}}(s)\tilde{\mathbf{P}}\mathbf{L}, \tag{24}$$

where $\mathbf{C}_1(s) \in \mathbb{R}[s]^{m \times m}$ and $\mathbf{C}_2(s) \in \mathbb{R}[s]^{l \times l}$ are generated on the basis of specified (stable) pole values of *pole_sys* and *pole_obs*, respectively.

The structure of the polynomial matrices $\mathbf{C}_1(s)$ and $\mathbf{C}_2(s)$ should comply with those of the denominator matrices $\mathbf{A}_1(s)$ and $\mathbf{A}_2(s)$ of the plant transfer function matrix (5), that is, the matrix $\mathbf{C}_1(s)$ generated on the basis of poles *pole_sys* should have a column matrix of leading coefficients

$$\mathbf{\Gamma}_c(\mathbf{C}_1(s)) = \mathbf{\Gamma}_c(\mathbf{A}_1(s)), \tag{25}$$

with $\deg_{ci}\mathbf{C}_1(s) = \deg_{ci}\mathbf{A}_1(s) = d_i, i = 1, 2, \dots, m$, and the row structure of the matrix $\mathbf{C}_2(s)$ generated from the poles *pole_obs* should comply with that of the polynomial matrix $\mathbf{A}_2(s)$, namely,

$$\mathbf{\Gamma}_r(\mathbf{C}_2(s)) = \mathbf{\Gamma}_r(\mathbf{A}_2(s)) \tag{26}$$

at $\deg_{rj}\mathbf{C}_2(s) = \deg_{rj}\mathbf{A}_2(s) = \bar{d}_j, j = 1, 2, \dots, l$.

The matrix $\mathbf{A}_2(s)$ is the denominator matrix of the plant transfer function matrix (5) converted to the dual (r.l.p.) form of transfer matrix $\mathbf{T}(s) = \mathbf{A}_2^{-1}(s)\mathbf{B}_2(s)$.

The polynomial matrices $\hat{\mathbf{S}}(s)$ and $\tilde{\mathbf{S}}(s)$ occurring in (23) and (24) have the following form:

$$\hat{\mathbf{S}}^T(s) = \begin{bmatrix} 1 & s & \cdots & s^{d_1-1} & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & s^{d_m-1} \end{bmatrix}, \tag{27}$$

$$\tilde{\mathbf{S}}(s) = \begin{bmatrix} 1 & s & \cdots & s^{\bar{d}_1-1} & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & s^{\bar{d}_l-1} \end{bmatrix}. \tag{28}$$

The structure of $\hat{\mathbf{S}}(s)$ depends on controllability indices d_i, $i = 1, 2, \ldots, m$, $\sum d_i = n$, and that of $\tilde{\mathbf{S}}(s)$ on plant observability indices $\bar{d}_j, j = 1, 2, \ldots, l$, $\sum \bar{d}_j = n$. The matrices $\hat{\mathbf{P}}$ and $\tilde{\mathbf{P}}$ are transformation matrices obtained in the process of transforming the original plant state-space equations (4) into the second Luenberger-Brunovsky canonical forms, controllable and observable, respectively.

Unlike the *eigenvalue* and the *eigenvector methods*, where no possibility exists to locate intentionally the poles in specified paths of the MIMO system to be designed, the method considered here permits the poles *pole_sys* to be assigned to plant inputs, and the poles *pole_obs* to plant outputs. This can be done in the process of generating the matrices $\mathbf{C}_1(s)$ and $\mathbf{C}_2(s)$ first in diagonal structures with polynomials of orders equal to controllability indices d_i, $i = 1, 2, \ldots, m$, for the first matrix and to observability indices $\bar{d}_j, j = 1, 2, \ldots, l$, for the second matrix, and then bringing these diagonal matrices to forms that satisfy the conditions (25) and (26), respectively. To this end the set of preassigned poles *pole_sys* should be divided into subsets with $d_i, i = 1, 2, \ldots, m$, elements, and the set of *pole_obs* into l subsets with $\bar{d}_j$ $j = 1, 2, \ldots, l$, elements. The sequence in which the individual pole subsets are used in the process of system synthesis does matter and has a pronounced effect on static and dynamic properties of the obtained controllers. Although the above procedure does not provide full possibility to locate the system and observer poles in specified paths of the control system, the design of modal control systems with MIMO plants is made thereby easier.

4.4. Polynomial matrix equations method. In the above presented methods the synthesis of MIMO modal controllers has been based on separately finding the matrices $\mathbf{F}$ and $\mathbf{L}$ for which, according to (8), their "standard" state-space equations have been formulated. These equations can be converted to appropriate state-space canonical forms with the matrices (9), and then, if desired, the matrices of controller transfer functions (10) can be determined on their basis.

However, instead of separately calculating the matrices $\mathbf{F}$ and $\mathbf{L}$ when designing modal controllers in s-domain, a more typical polynomial procedure may be employed, where the controller transfer function matrix $\mathbf{T}_c(s) = \mathbf{M}_2^{-1}(s)\mathbf{N}_2(s)$ is directly obtained at one go by solving the Diophantine left polynomial matrix equation

$$\mathbf{M}_2(s)\mathbf{A}_1(s) + \mathbf{N}_2(s)\mathbf{B}_1(s) = \Delta(s) = \mathbf{Q}(s)\mathbf{C}_1(s), \tag{29}$$

where $\mathbf{A}_1(s)$ and $\mathbf{B}_1(s)$ are known polynomial matrices describing the control plant (5), and $\mathbf{M}_2(s)$ and $\mathbf{N}_2(s)$ are a pair of unknown polynomial matrices that Eqn. (29) is to be solved for. In the case of MIMO systems obtaining minimal solutions of Eqn. (29) (of minimal degree with respect to the matrix $\mathbf{N}_2(s)$), which should satisfy the conditions $\deg_{rj}\mathbf{N}_2(s) \leq \deg_{rj}\mathbf{M}_2(s), j = 1, 2, \ldots, m$, is much more complex than in the case of SISO systems. In SISO systems, $\mathbf{Q}(s)$ and $\mathbf{C}_1(s)$ are generated in a simple way as stable Hurwitz polynomials on the basis of preassigned respective pole values *pole_obs* and *pole_sys*. For solutions of the polynomial equation (29) to exist, only the necessary condition is to be met that roots of these polynomials be separable with those of polynomials $\mathbf{A}_1(s)$ and possibly $\mathbf{B}_1(s)$.

However, in MIMO systems the polynomial matrix $\Delta(s) = \mathbf{Q}(s)\mathbf{C}_1(s)$ in addition to that it should be relative right prime (r.r.p.) with matrices $\mathbf{A}_1(s)$ and $\mathbf{B}_1(s)$, it should also have a row-column-reduced structure with a nonsingular matrix of the highest (diagonal) coefficients (Callier and Kraffer, 2005; Bańka, 2007)

$$\begin{aligned} \mathbf{\Gamma}_h(\Delta(s)) &= \mathbf{\Gamma}_r(\mathbf{Q}(s))\mathbf{\Gamma}_c(\mathbf{C}_1(s)) \\ &= \mathbf{\Gamma}_r(\mathbf{M}_2(s))\mathbf{\Gamma}_c(\mathbf{A}_1(s)). \end{aligned} \tag{30}$$

The matrices $\mathbf{Q}(s)$ and $\mathbf{C}_1(s)$ should have determinants $\det \mathbf{Q}(s)$ and $\det \mathbf{C}_1(s)$ with zeroes equal to the preassigned poles, i.e., *pole_obs* for the observer and *pole_sys* for the system.

In selecting the matrices $\mathbf{Q}(s)$ and $\mathbf{C}_1(s)$ we have a great freedom of choice of their structure, since, as previously, many different polynomial matrices may exist of the given dimensions with identical determinants. In the method proposed here the matrix $\mathbf{C}_1(s)$ can be generated as in the *polynomial method*, i.e., in accordance with the column structure of the denominator matrix $\mathbf{A}_1(s)$. A circumstance that may present some problems is the choice of an appropriate structure of the matrix $\mathbf{Q}(s)$ so that the structure of the matrix $\Delta(s)$ is row-column-reduced, which guarantees that the obtained solutions will have the form of proper transfer function matrices (10) for each sought modal controller.

This is not an easy task and requires great skills or additional *a priori* information acquired, for example, in the process of system synthesis in time domain. Fortunately, the matrix $\mathbf{Q}(s)$ in systems with controllers of full order *n* may frequently have a diagonal structure with polynomials of orders $\bar{r}_j = \deg_{rj}\mathbf{Q}(s), j = 1, 2, \ldots, m$, selected so that $\sum \bar{r}_j = n$. As was reported by Bańka (2007), it is also possible to obtain solutions of Eqn. (29) in the form of strict proper transfer function matrices for full-order controllers. However, this is feasible if the polynomial matrix is selected in a special way, and in general, only for plant models described by the strict proper transfer function matrices (5) (Callier and Kraffer, 2005; Bańka, 2007).

Furthermore, Eqn. (29) may also deliver modal controllers of reduced order built on the basis of functional Luenberger observers of reduced order $n_1 = m(\nu - 1)$, where $\nu = \max\{\bar{d}_j, j = 1, 2, \ldots, l\}$. Then the matrix $\mathbf{Q}(s)$ assumes a regular structure, i.e., with identical row degrees $\bar{r}_j = \deg_{rj}\mathbf{Q}(s) = \nu - 1; \sum \bar{r}_j = n_1$ (Bańka, 2007; Wolovich, 1974). In this case the controllers of reduced order $n_1 = m(\nu - 1)$ will always be obtained in the form of matrices of proper transfer functions. They can be realized in time domain exclusively in canonical observable forms of state-space equations with $\mathbf{D}_{ro} \neq \mathbf{0}$.

Additionally, it might be good to mention that there exists a possibility to design modal control systems by solving the Diophantine (dual) right polynomial matrix equation

$$\mathbf{A}_2(s)\mathbf{M}_1(s) + \mathbf{B}_2(s)\mathbf{N}_1(s) = \tilde{\Delta}(s) = \mathbf{C}_2(s)\mathbf{Z}(s) \tag{31}$$

with the use of equivalent polynomial descriptions concerning the plant and the controller in the forms $\mathbf{T}(s) = \mathbf{A}_2^{-1}(s)\mathbf{B}_2(s)$ and $\mathbf{T}_c(s) = \mathbf{N}_1(s)\mathbf{M}_1^{-1}(s)$, and with an appropriately chosen row-column-reduced matrix $\tilde{\Delta}(s) \in \mathbb{R}[s]^{l\times l}$, where zeroes of the matrix $\mathbf{C}_2(s)$ correspond to the preassigned values of *pole_obs*, and zeroes of the matrix $\mathbf{Z}(s)$ correspond to the values of *pole_sys* (Bańka, 2007). These will not be considered in this paper, as well as structures with reduced-order controllers, mainly because matrices $\mathbf{D}_{ro} \neq \mathbf{0}$ occur in time domain realizations of the matrix transfer function of such controllers, thus increasing quite significantly the number of parameters to be tuned.

5. Synthesis of ship modal controllers

In the case of linear models obtained in the form of state-space equations (4) or transfer function matrices (5) for the drillship *Wimpey Sealab* given by nonlinear state-space equations (1) with the effects of wind gusts and wave action having been neglected for clarity's sake, each of the above discussed synthesis methods leads to yielding strict causal modal full-order controllers described by the space-state equations (7) with matrices $\mathbf{D}_{ro} = \mathbf{0}$, which are defined by the strict proper transfer function matrices (10) in s-domain. In order to obtain solutions with the minimal number of parameters whose values are different from "0" or "1", the state-space equations for all controllers to be yielded will be presented exclusively in canonical forms with matrices (9). The following sets of stable pole values have been adopted for the system and the full-order Luenberger observer:

$$\begin{aligned}&pole\_sys\\&\quad = \{-0.40, -0.45, -0.14, -0.15, -0.15, -0.16\}\end{aligned}$$

and

$$\begin{aligned}&pole\_obs\\&\quad = \{-0.80, -0.90, -0.28, -0.30, -0.30, -0.32\}.\end{aligned}$$

Such a choice of the poles *pole_sys* was performed experimentally to obtain control processes without excessive overshoots on the course and the ship's coordinate position with "reasonable" times needed to achieve steady-state control conditions and possible without crossing the limits on the control signals. On the other hand, the values of *pole_obs* were chosen with negative values of its real parts, twice larger than the negative values of the corresponding values of *pole_sys* that ensure the vanishing of transitional processes in the Luenberger observer two times faster than processes occurring in particular paths of the closed-loop control system.

Employing the above synthesis methods yielded four different sets of 3650 modal controllers described by the state-space equations (7) in the second Luenberger–Brunovsky canonical observable form with the matrices

$$\mathbf{A}_{ro} = \begin{bmatrix} 0 & a_{12} & 0 & a_{14} & 0 & a_{16} \\ 1 & a_{22} & 0 & a_{24} & 0 & a_{26} \\ 0 & a_{32} & 0 & a_{34} & 0 & a_{36} \\ 0 & a_{42} & 1 & a_{44} & 0 & a_{46} \\ 0 & a_{52} & 0 & a_{54} & 0 & a_{56} \\ 0 & a_{62} & 0 & a_{64} & 1 & a_{66} \end{bmatrix}, \tag{32}$$

$$\mathbf{B}_{ro} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \\ b_{51} & b_{52} & b_{53} \\ b_{61} & b_{62} & b_{63} \end{bmatrix},$$

$$\mathbf{C}_{ro} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_{ro} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

They have 36 variable entries: a_{ij}, $i = 1, 2, \ldots, 6$, $j = 2, 4, 6$, and b_{ij}, $i = 1, 2, \ldots, 6$, $j = 1, 2, 3$, dependent on the ship's velocity $V_s = \mathrm{sign}(x_4)\sqrt{x_4^2 + x_5^2}$ and on deviations of the ship's yaw angle $y_{30} = x_{30}$ from the sea current angle Ψ_c. The controllers have been synthesized for velocities lying in the range $V_s \in [-4.9 \div 4.9]$ [knots] with the resolution of 0.2 [knot] over the entire range of round angle, that is, over the range $\Psi_c - x_{30} \in [0 \div 360°]$ with the resolution of 0.0873[rd], for the adopted ship relative "radius of gyration" $k_{zz} = 1/4$.

As might be expected, the use of different synthesis methods for modal controllers yielded different results for the same data taken for calculations. The differences in the obtained results are fundamental both in terms of constructing from them an adaptive controller with stepwise varying parameters (or a switchable multi-controller structure) and also in terms of operation quality provided by these controllers in the designed control system. Nonetheless, all the obtained modal controllers are stable exhibiting a time lag affected PD behavior. Their dynamic and static properties for different ship velocities V_s and yaw angles $\Psi_c - x_3$ within the group of controllers obtained by one of the discussed design methods experience some fluctuations, since they have variable poles *pole_reg* defined as eigenvalues of the matrices $\mathbf{A}_{ro}$ and variable gain matrices

$$\mathbf{K}_c = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \mathbf{C}_{ro}(-\mathbf{A}_{ro})^{-1}\mathbf{B}_{ro}. \quad (33)$$

It is worth noting that, despite employing always the same pole values *pole_sys* and *pole_obs* without introducing any changes in their sequence, there have been obtained entirely different sets of controllers with varying entries of $\mathbf{A}_{ro}$ and $\mathbf{B}_{ro}$, which yield different (variable) gain matrices $\mathbf{K}_c$ for these controllers at varying (over different ranges) values of always stable poles *pole_reg*. The variable entries of matrices $\mathbf{A}_{ro}$, $\mathbf{B}_{ro}$ and $\mathbf{K}_c$ may be depicted in the form of 3-D surfaces as functions of ship velocity V_s and yaw angles $\Psi_c - x_3$.

The synthesis results both the most unreliable in operation and the most difficult to realize an adaptive controller (or a switchable multi-controller structure) have been obtained by means of the *eigenvalues method*. They will not be shown in the paper. On the other hand, the best control responses, i.e., smooth and overshoot-free ones, were provided by controllers obtained by the *eigenvector method*. Unfortunately, the parameters of such obtained controllers depend in a complex way on the ship's velocity V_s and yaw angles $\Psi_c - x_3$, which makes realization of the proposed control system structure difficult. The character of parameter variability for controllers obtained by this method is illustrated by 3-D surfaces published earlier by Bańka *et al.* (2011a).

The most promising results both in terms of ease of realization of the proposed control system structure and also in terms of the quality of controller operation in the multi-controller structure are delivered by the *polynomial matrix equation method*. The controllers obtained by this method are characterized by moderately "smooth" surfaces of parameters variation and, at the same time, meet sufficiently the quality requirements placed on control of the ship's nonlinear model. The variability of entries of matrices $\mathbf{A}_{ro}$, $\mathbf{B}_{ro}$ versus the ship's velocity V_s and yaw angle $\Psi_c - x_3$ obtained by this method is illustrated by 3-D surfaces shown in Figs. 3 and 4, respectively.

For comparison, 3-D surfaces for all entries of gain matrices $\mathbf{K}_c$ of modal controllers obtained by means of all the above-mentioned synthesis methods (except for results yielded by the *eigenvalue method*, which were unacceptable from every point of view) are depicted in Figs. 5–7.

From these plots it can be seen that parameters of all obtained modal controllers change their values (both the absolute value and the sign) at different values of yaw angle $y_3 = x_3$ and the ship's velocity V_s. Particularly violent changes, especially for controllers obtained by the *eigenvector method*, take place in the vicinity of values that correspond to yaw angles $0°$, $90°$, $180°$ and $270°$ and at the ship's velocities close to $V_s = 0$. This particularly concerns parameters which constitute the last columns of matrices $\mathbf{A}_{ro}$ and $\mathbf{B}_{ro}$ and the last column of gain matrix $\mathbf{K}_c$ (not presented here), i.e., the entries having a direct influence on signals associated with the ship's course control.

For parameters of controllers obtained by the remaining methods, i.e., the *polynomial* and the *polynomial matrix equation* methods, the yielded surfaces are already more smooth except for controllers obtained by the *eigenvalue method*. The latter exhibit sharp spikes (not presented here) in canonical forms for yaw angles equal to $90°$ and $270°$ occurring at high ship velocities V_s.

All of this makes a quite complex picture of problems connected with implementation of the proposed multi-controller structures of linear modal controllers designed for steady states, but actually utilized for control transients. This is possible as evidenced below by results of simulations carried out with the ship's nonlinear model (1) for all obtained sets of modal controllers realized here as a single adaptive controller with tuned parameters.

6. Results of simulation tests

All simulation tests have been carried out without regard for the effect produced by the wind and wave action in the presence of sea current of $V_c = 2$ [knots] at $\Psi_c = 180°$ with the use of the ship's nonlinear model (1) that describes low frequency varying ship motions in

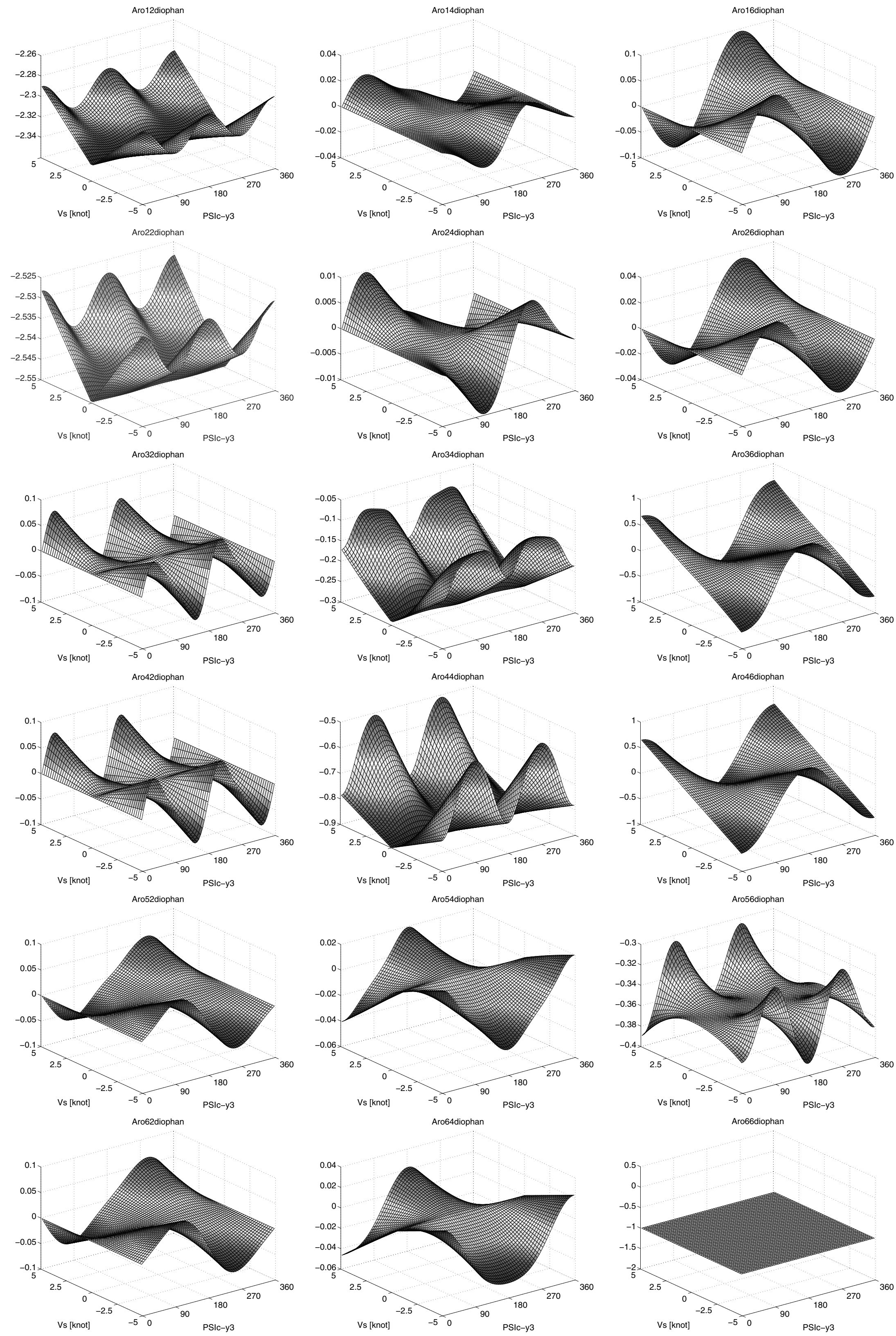

Fig. 3. Entries of the matrix $\mathbf{A}_{ro}$ vs. ship velocity and yaw angle obtained by the *polynomial matrix equation method*.

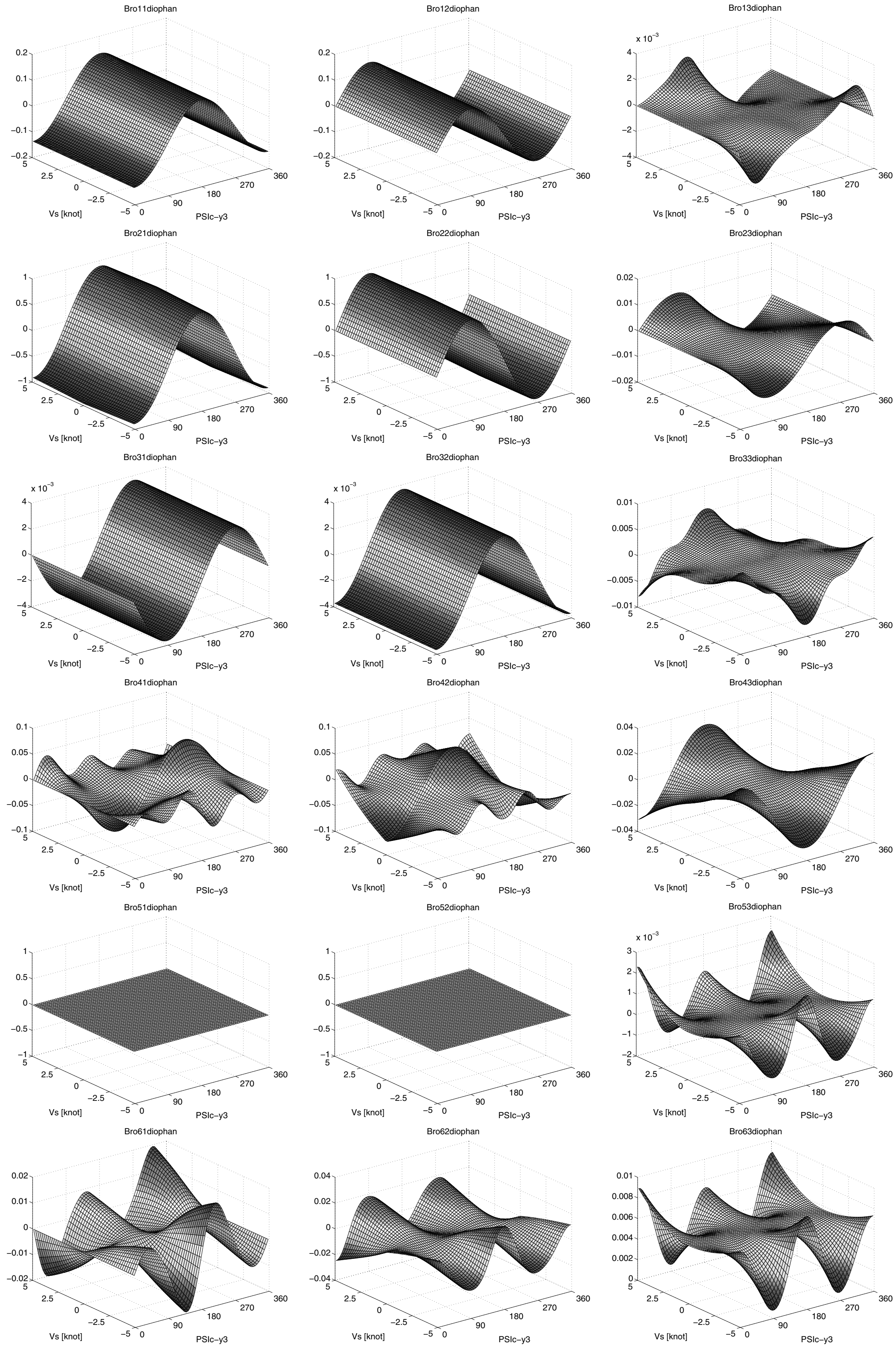

Fig. 4. Entries of the matrix $\mathbf{B}_{ro}$ vs. ship velocity and yaw angle obtained by the *polynomial matrix equation method*.

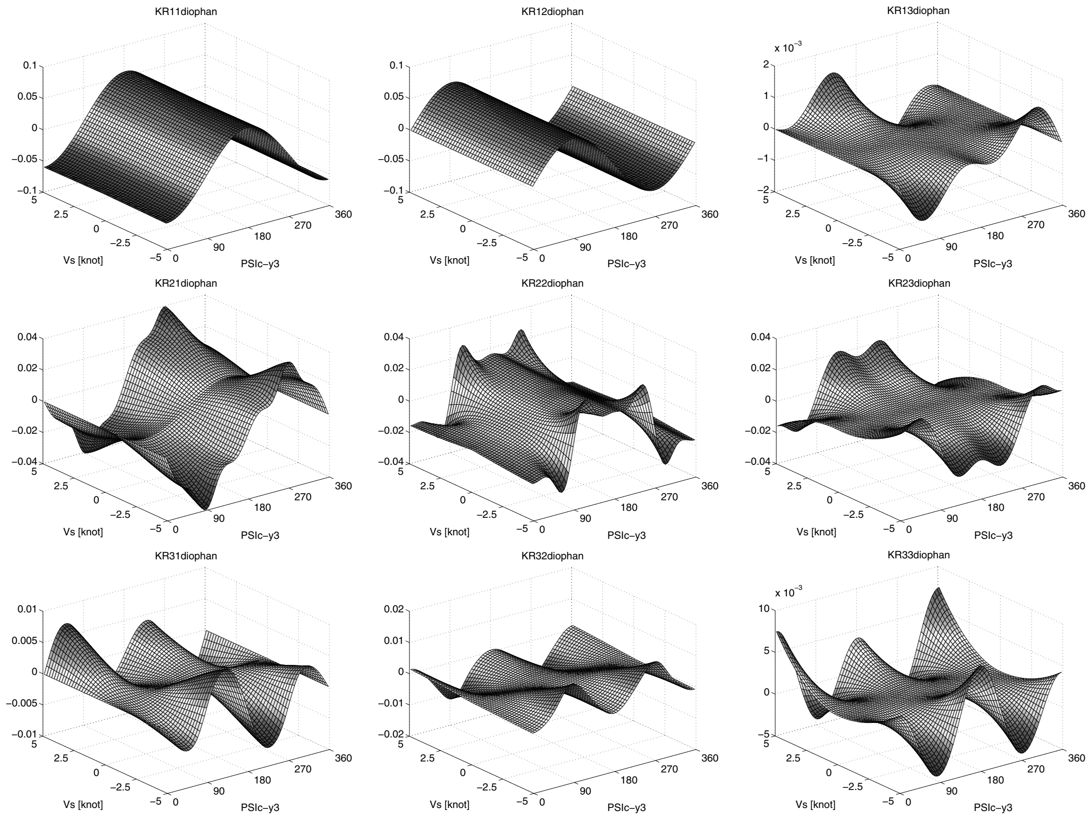

Fig. 5. Entries of the matrix $\mathbf{K}_c$ vs. ship velocity and yaw angle obtained by the *polynomial matrix equation method*.

3DOF. The tests have been conducted for many different initial states defined by appropriate ship positions, yaw angle and ship initial velocities. In typical situations, that is, when the ship sailed bow on against the current at $V_s \neq 0$, all controllers behaved quite correctly yielding time responses without excessive oscillations experienced by the control signals $\mathbf{u}(t)$. In that case the ship could always be brought to the drilling point and assumed the preset yaw angle, and then she could be moved to any specified position. This is demonstrated by time responses depicted in Fig. 8 obtained with controllers found by the *eigenvector* and the *polynomial matrix equation* methods.

During these simulations the ship was brought to the drilling point from a position at a distance of about 100 [m] ($r = 1$) distant situated on the left below the drilling point with the adopted initial yaw angle $x_3(0) = 35°$ and velocity components $x_4(0)$ and $x_5(0)$, which corresponds to the ship's sailing bow on against the current $V_s(0) = V_c = 2$ [knots]. After reaching the drilling point with the specified yaw angle equaling $y_{3\mathrm{ref}} = x_{30} = 0°$ after 150 dimensionless time units (corresponding to about 7.5 min. of real time), the yaw angle was changed to $y_{3\mathrm{ref}} = x_{30} = 60°$. Then at $t = 200$ of time units the reference values have been changed stepwise for both ship position coordinates so that the ship moved through a distance of about 100 [m] from the right over the drilling point and come to a standstill at a distance of 100 [m] with a velocity of $V_s = V_c = 2$ [knots] relative to water and bringing the ship's yaw angle $y_3(t) = x_3(t)$ to the preset value $y_{3\mathrm{ref}} = x_{30} = 60°$.

However, more interesting and instructive are responses obtained for a typical situations when the ship sails stern-first against the current, especially at changes in the sign of the linear ship velocity V_s in the vicinity of $V_s = 0$. This may happen when the ship is brought to the drilling point with the current (conditioned, for example, by an unfavorable direction of the wind or sea waves being in opposition to the sea current direction Ψ_c) or when changing the ship's yaw angle over the drilling point caused by a change in the wind or waves direction.

To investigate the matter, the remaining simulations have been performed for the ship situated initially about 100 [m] on the left over the drilling point with initial yaw angle $x_3(0) = 125°$ and velocity components $x_4(0)$ and $x_5(0)$, which corresponds at the beginning of simulations to moving astern at $V_s(0) = -V_c = -2$ [knots]. In these simulations the preset ship yaw angles $y_{3\mathrm{ref}} = x_{30}$ over the drilling point, as well as making later changes in the yaw angle and ship final positions, have been performed just in the same way as earlier, keeping as far as possible the same time conditions for manoeuvring.

Simulation results obtained for sets of controllers

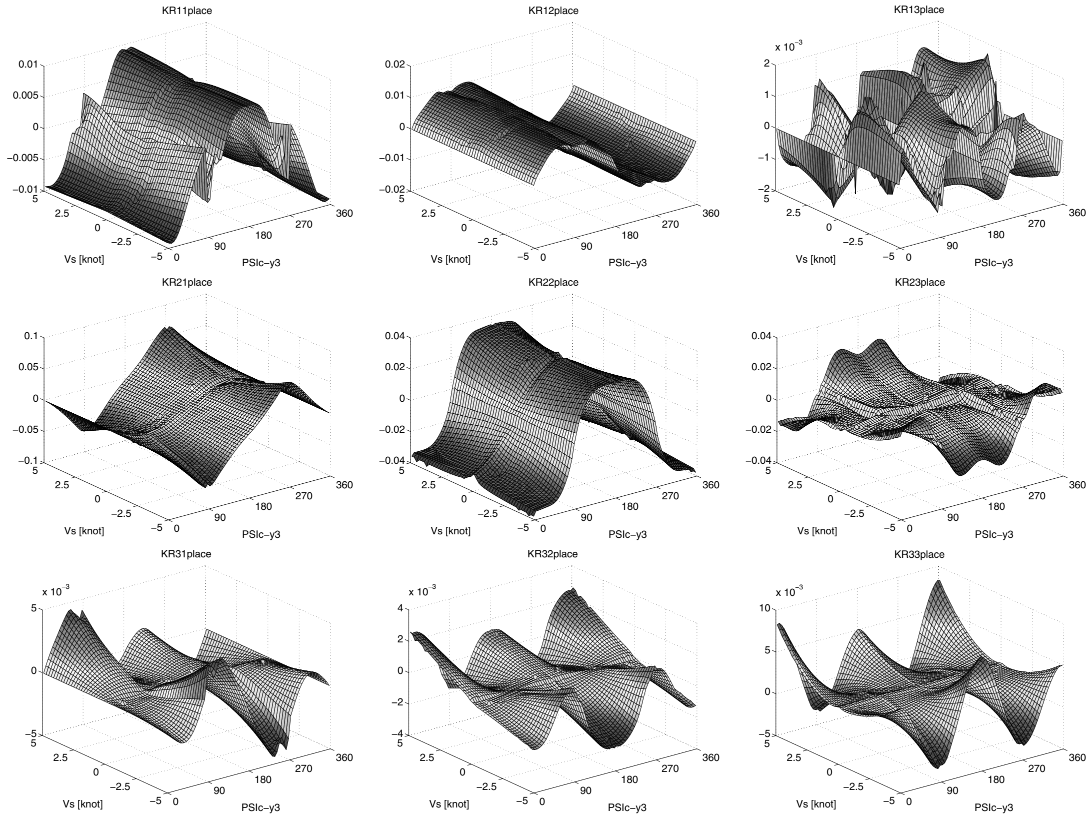

Fig. 6. Entries of the matrix $\mathbf{K}_c$ vs. ship velocity and yaw angle obtained by the *eigenvector method*.

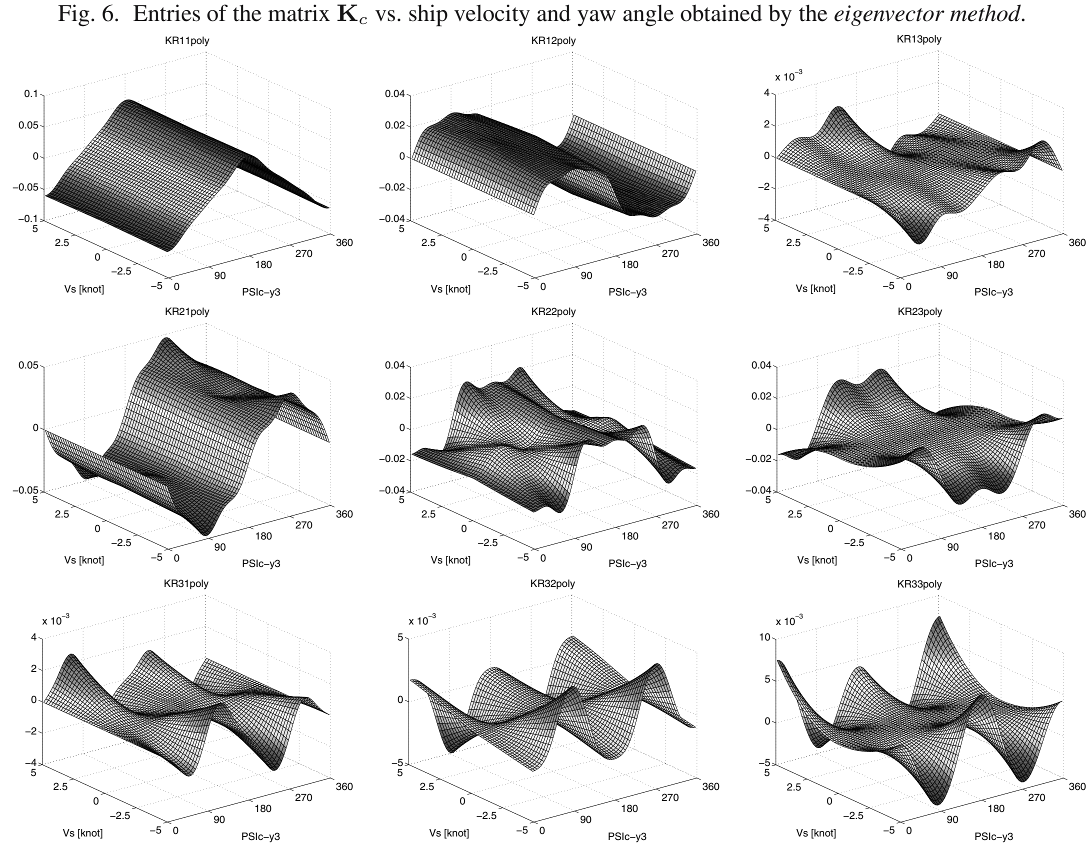

Fig. 7. Entries of the matrix $\mathbf{K}_c$ vs. ship velocity and yaw angle obtained by the *polynomial method*.

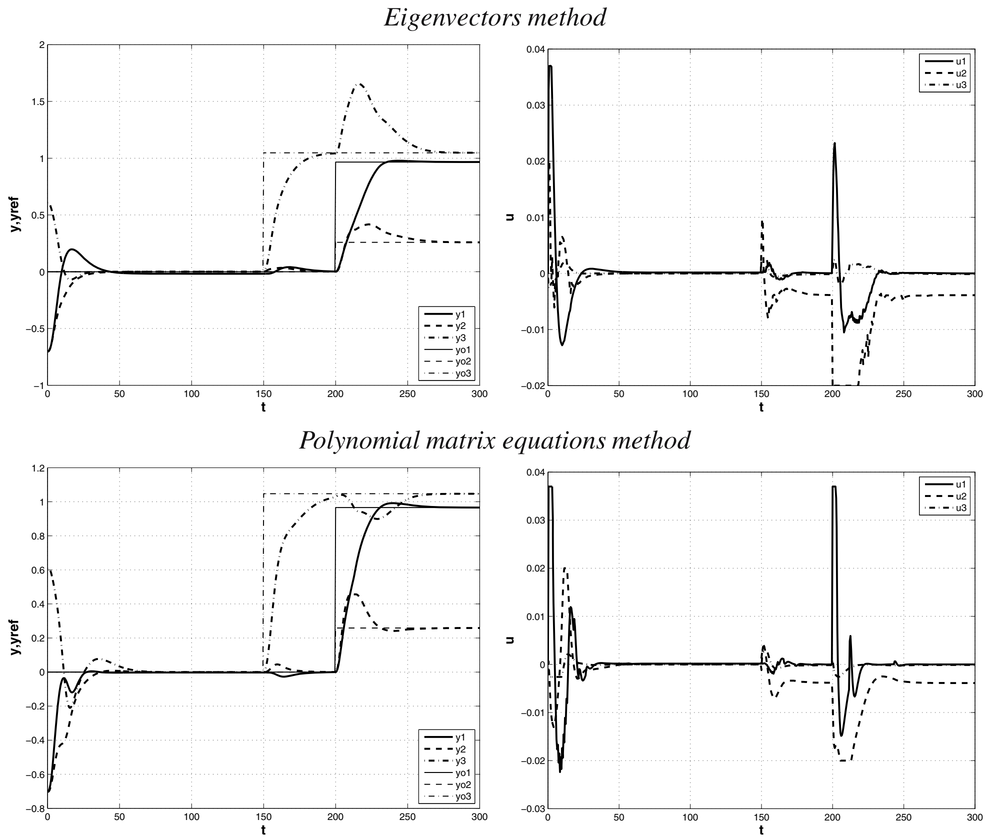

Fig. 8. Plots of controlled variables and control signals $\mathbf{u}(t)$ in the process of bringing the ship to the drilling point bow on against the current.

designed by the *eigenvector* and the *polynomial matrix equation* methods are shown in Fig. 9.

Steady-state errors in each of the tested systems have been eliminated by applying nominal values $\mathbf{u}_o$ calculated from the system of equations (3) for preset final ship positions at the yaw angle $y_{3\mathrm{ref}} = x_{30} = 60°$.

The time responses presented above feature smaller overshoots and shorter settling times than those delivered by other (switchable) structures described in earlier papers (e.g., Bańka *et al.*, 2010a; 2010b). A drawback to these responses is that the responses of control signals $\mathbf{u}(t)$ are (oscillating) nonsmooth, caused mainly by stepwise changes in parameter values of the controller in the vicinity of “operating points” that cause trouble.

7. Concluding remarks

It follows from the simulation tests carried out that the proposed concept of control of a nonlinear model of a MIMO drillship by the use of an adaptive structure of a linear MIMO controller with tuned parameters on the basis of two auxiliary measured signals is feasible. It can also be realized by a set of linear MIMO modal controllers in a multi-controller structure with switchable outputs. Modal controllers, which must be stable in these systems, though predesigned for steady states, operate properly despite the fact that they actually operate in transient states (in a quasi-steady-state mode). Nevertheless, stepwise changes of parameter values in an adaptive single controller, as well as the switching over of controller outputs in a multi-controller structure, are accompanied by “nonsmooth” responses of control signals $\mathbf{u}(t)$. This especially concerns control transients taking place at the ship’s velocities close to $V_s = 0$ and/or at yaw angles corresponding to values close to $0°$, $90°$, $180°$ and $270°$. The propellers and main engine of a real ship will not be able to realize such control signals. Hence, such a problem is a main aim of researching the solutions that could give a chance to obtain smoother surfaces of parameters change shown in Figs. 3 and 4.

Apart from using different design method as well as employing different description forms of designed modal controllers, the possibility of replacing the obtained (original) surfaces of parameter change with the surface generated by means of artificial neural networks exists. As training data for neural networks the obtained surfaces of parameter changes can be used. Such a controller with “neural” parameters should provide a “smoother” system

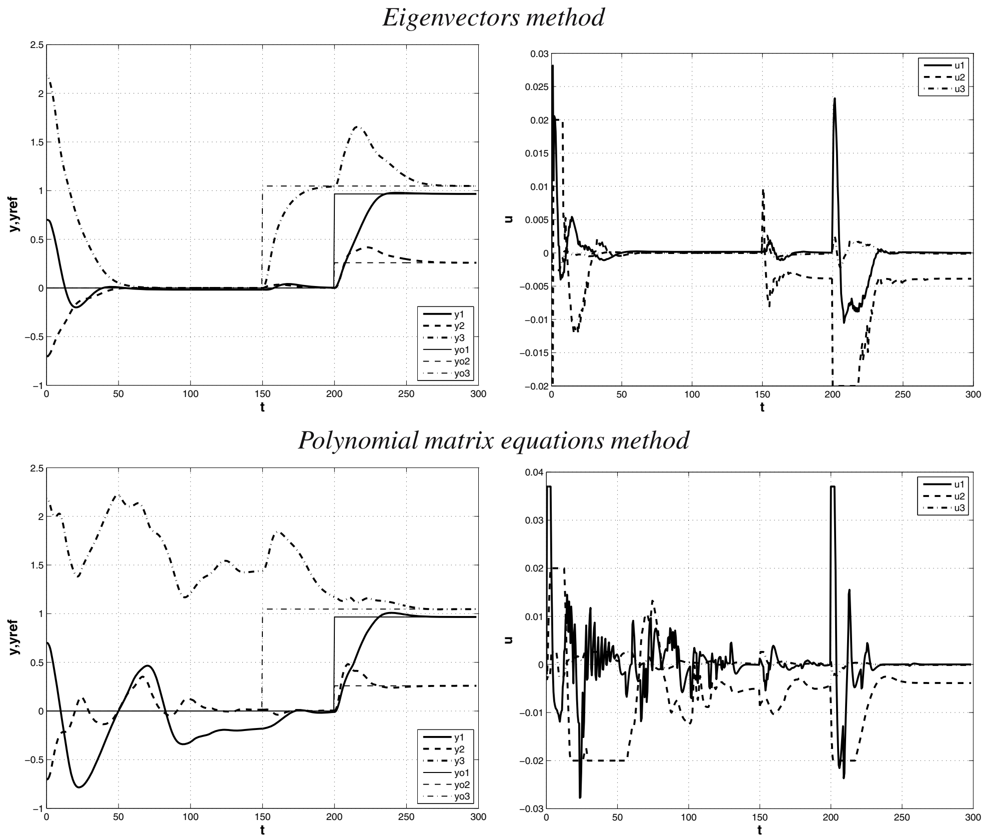

Fig. 9. Plots of controlled variables and control signals $\mathbf{u}(t)$ obtained for the ship's approaching the drilling point with the current.

operation (without any switching over) and also

- generalize the parameter values of matrices $\mathbf{A}_{ro}$ and $\mathbf{B}_{ro}$ to untrained (unknown) values following from quantization of signals V_s and $\Psi_c - x_{30}$ with acceptable resolution,

- smooth out "ridges" and "precipices" seen on the surfaces that describe variable controller parameters,

- eliminate the problem of ambiguity in operation of controllers, which assume different parameter values in the process of switching over depending on whether the auxiliary signals $V_s(t)$ and $\Psi_c - x_3(t)$ increase or decrease.

A first attempt of replacing a discussed linear structure of controller with tuned parameters (or a switchable structure of modal controllers) with an adaptive "neural" controller trained on the basis of known parameter values contained in matrices $\mathbf{A}_{ro}$ and $\mathbf{B}_{ro}$, designed by the *eigenvector method*, has been presented by Bańka *et al.* (2011b). Such an approach will be taken into consideration during further research.

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Stanisław Bańka received the M.Sc., Ph.D, and D.Sc. degrees in control engineering in 1969, 1976 and 1992, respectively. In the years 1996–2002 he was the dean of the Electrical Engineering Faculty at the Technical University of Szczecin and in 2003–2005 the director of the Institute of Control Engineering of that university. Since 2009 a professor, currently the head of the Chair of Control Engineering and Robotics at the West Pomeranian University of Technology in Szczecin. Member of the Committee on Automatic Control and Robotics of the Polish Academy of Sciences (since 2003). Author of over 80 journal papers and three books. Main scientific interests: multivariable control systems, adaptive and optimal control, estimation and parameter identification, and applications of informatics in automation.

Paweł Dworak received the M.Sc. and Ph.D degrees in control engineering in 1999 and 2005, respectively. Currently an assistant professor in the Department of Control Engineering and Robotics, West Pomeranian University of Technology in Szczecin. Main scientific interests: multivariable control systems, adaptive control, process data acquisition and visualization and industrial applications of modern control algorithms.

Krzysztof Jaroszewski received the M.Sc. (2001) and Ph.D. (2007) degrees, both in control engineering. Currently an assistant professor in the Department of Control Engineering and Robotics, West Pomeranian University of Technology in Szczecin. Main scientific interests: artificial intelligence, especially neural networks, control systems and visualization and industrial diagnostic.

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