Appl. Math. and Comp. Sci., 1993, vol.3, No.2, 329–339

CURRENT MINIMIZATION OF THE NON–IDEAL VOLTAGE SOURCE WITH PERIODICALLY TIME–VARRYING PARAMETERS BY MEANS OF AN ACTIVE COMPENSATOR

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In this paper, the problem of matching a receiver to the real voltage source is solved for the first time. The active current has been correctly determined in the real source with an inner parametric impedance and a parametric receiver. It has also been shown that a compensatory circuit should be selected from a larger class of two–terminal networks with changeable parameters. The results have been obtained by means of optimization methods and variational principles in discrete time–domain. By employing proper algorithms, active current and the compensatory circuit with periodically changeable parameters are determined by a computer, which is able to analyze the situation in the network and to give the results in diagrams currently.

1. Introduction

Problems of the theory of nonsinusoidal currents and voltages are valid for theoretical electrotechnics and electrical measurements. Many authors have tried lately to define, besides an active, power additional harmful powers, called reactive powers and powers of deformations (Czarnecki, 1987; Czarnecki and Swietlicki, 1990; Depenbrock, 1979; Kusters and Moore, 1980; Shepherd and Zakikhani, 1973; Slonim and Van Wyk, 1988). Even the authors themselves do not think these definitions to be sufficient; thus, there is still some discussion about the notions of additional powers. Current orthogonal decomposition (Czarnecki, 1987; Kusters and Moore, 1980; Shepherd and Zakikhani, 1973) into the active component and other components which are harmful, does not give quite positive results. Moreover, it is not true for nonideal sources (Siwczyński, 1990; Siwczyński and Kłosinski, 1991; Walczak, 1990).

The present formulation of the theory has three main drawbacks. First, it does not take into consideration the inner impedance of the source what causes that the

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active current is undetermined. Second, it considers the situation when current and voltage deformations are caused by contents of higher harmonics in a signal of an electromotive force (emf) of the source. In practice, an emf of the source is often sinusoidal, while deformations come from the load. Up till now a circuit impedance has been identified as linear two-terminal networks time invariable means of a spectral decomposition. Third correction circuits (compensators) are still insufficient. They have been selected from LC or ±RLC classes (Czarnecki, 1987; Pasko and Walczak, 1989; Pasko, 1991; Pasko and Grzesik 1992). These circuits are not universal and most often they cannot act successfully in a situation when the sources have sinusoidal voltages and deformations are caused by nonlinearity or time-variability of the receiver.

In the present paper the active current has been correctly determined in the real source circuit with inner parametric impedance and a parametric receiver. It has also been shown that a compensatory circuit should be selected from the larger class of two-terminal networks with changeable parameters. Such a circuit is universal. The results have been obtained by means of optimization methods and variational principles in a discrete time-domain. Due to proper algorithms the active current and the compensatory circuit with periodically changeable parameters are determined by a computer, which is able to analyze the situation in network and to give the results in diagrams currently. Many experiments have been carried out during which a large class of RL two terminal networks with periodically changeable resistance and inductance has been simulated. These two-terminal networks have modeled both the inner impedance and the load as well.

2. Algorithm of an Optimum Current Determination

The scheme of the electrical system under consideration of the real voltage source has been presented in Figure 1. The voltage source is characterized by an optional T-periodical signal $e(t)$ and a linear operator of a Z^e inner impedance of the two-terminal network with periodically changeable parameters characterized by a Y^0 admittance operator. We assume that both operators are integral operators in a continuous time-domain. In a steady state they take a form (Siwczyński, 1990; Siwczyński and Kłosinski, 1992).

$$Z^e i(t) = \int_0^T z^e(t,t')i(t')\mathrm{d}t; \qquad Y^0 u(t) = \int_0^T y^0(t,t')u(t')\mathrm{d}t \tag{1}$$

where $z^e(t,t')$, $y^0(t,t')$ are T-cyclic parametric impulse functions of respective two-terminal networks. These operators can be presented in the time-discrete form. We also assume that the two-terminal networks Z^e, Y^0 are non-negative for any periodical signals $i(t)$, $u(t)$ and they equal zero only for zero signals. The scalar product in a voltage-current space is defined in the following way

$$(u|i) = \frac{1}{T}\int_0^T u(t)i(t)\mathrm{d}t \tag{2}$$

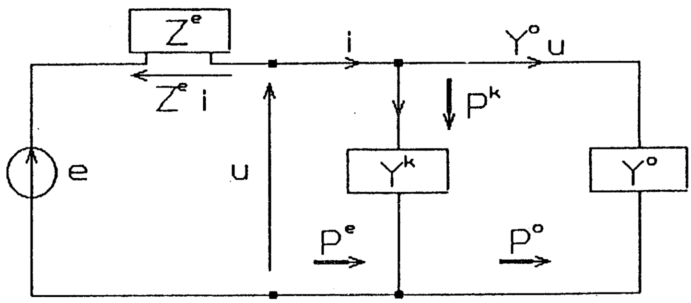


Fig. 1. A model of a real source voltage.

In such a circuit electrical signals may be deformed not only by higher harmonics in $e(t)$ signal, but by two-terminal networks of the source load and inner impedance as well. It has not been possible in the theoretical models that have been presented so far. A practical task is to minimize a positively definite functional of the source current at a given flux of active power P^e leaving the source. Because of practical reasons the functional is most often a RMS current

$$(i|i) = \frac{1}{T}\int_0^T [i(t)]^2 \mathrm{d}t \to \min \tag{3}$$

but it can also take a general form containing a current derivative (Siwczyński and Kłosinski, 1992; Walczak, 1990)

$$\int_0^T \alpha_0[i(t)]^2\mathrm{d}t + \int_0^T \alpha_1[i^{(1)}(t)]^2\mathrm{d}t + ... + \int_0^T \alpha_l[i^{(l)}(t)]^2\mathrm{d}t \to \min \tag{4}$$

where α_l–arbitrarily except ed non-negative weights. Functional (4) after integration by parts can be given in a general form

$$(Li|i) = \frac{1}{T}\int_0^T [\alpha_0 i(t) + (-1)\alpha_1 i^{(2)}(t) + ... + (-1)^l \alpha_l i^{(2l)}(t)]i(t)\mathrm{d}t \tag{5}$$

where

$$L(D) = \sum_{k=0}^{l}(-1)^k\alpha_k D^{2k}, \qquad D = \frac{\mathrm{d}}{\mathrm{d}t} \tag{6}$$

is a self-conjugate positively definite linear operator.

We can minimize also another, more general functional, for example active power losses in a transmission system which is in the form of a four-terminal network connected between the source and the receiver. Such a cause is described in the work (Siwczyński and Pasko, 1992). Here we shall deal with minimization of the functional

$$(Li|i) \to \min \tag{7}$$

with an optimal linear self-conjugate positive operator L at the condition of active power given by the source.

$$P^e - (u|i) = 0. \tag{8}$$

Optimization conditions (7)–(8) are to be given by a compensatory two-terminal network Y^k connected to the receiver terminals. In other words, this two-terminal network plays a role of an adjusting circuit in the sense of (7)–(8). As it has been proved in the works (Siwczyński, 1990; Siwczyński and Kłosinski, 1991), the problem of minimization (7)–(8) has a unique solution fulfilling the equations:

$$(L + \lambda A)i = \lambda e \tag{9}$$

$$\frac{1}{2}(Ai|i) - (e|i) + P^e = 0 \tag{10}$$

where the operator A is defined by the expression:

$$A = Z^e + Z^{e*} \tag{11}$$

(asterisk denotes a conjugate operator), λ is a Lagrange's multiplier. A current optimum signal $i^{\mathrm{opt}}(t)$ minimizing the functional (7) at power condition (8) is the solution of equations (9)–(10). In general, equations (9)–(10) can be solved only numerically in an iterative way. As it has been proved in several works eg. (Siwczyński, 1990; Siwczyński and Pasko, 1992; Siwczyński and Kłosinski, 1991), Newton's method yields the best results. The method is in this case totally convergent. It gives the following iterative process with two operational equations:

$$(L + \lambda_k A)x_k = \lambda_k e \to x_k \to (L + \lambda_k A)y_k = \frac{1}{\lambda_k} L x_k$$

$$\lambda_{k+1} = \lambda_k + \frac{P^e - \frac{1}{2}(e|x_k) - \frac{1}{2\lambda_k}(Lx_k|x_k)}{\frac{1}{\lambda_k}(Lx_k|y_k)}, \tag{12}$$

$$k = 0, 1, 2, \ldots, \qquad \lambda_0 = 0$$

The sequence of signals (x_k) obtained is uniformly convergent to the signal of optimum current $i^{\mathrm{opt}}(t)$. As it has been shown by computer simulations the optimum currents received from conditions (7), (8) require active compensators in the sense that their active powers (compare Figure 1) P^k are most often negative. To improve the working conditions of the compensatory circuit another condition of a power balance can be taken

$$P^e = P^0 \tag{13}$$

and that means that the compensator neither consumes nor gives out an active power. In the work (Siwczyński, 1990) it has been shown that the optimization equations of problem (7)–(13) can be obtained from equations (9)–(10) by putting

$$A \to A + B, \quad e \to e + Ce, \quad P^e \to \frac{1}{2}(Ge|e) \tag{14}$$

where auxiliary operators G, C, B are defined in the following way

$$G = Y^0 + Y^{0*}, \quad B = Z^{e*} G Z^e, \quad C = Z^{e*} G. \tag{15}$$

A properly designed original computer program at a given periodical signal $e(t)$ and operator date Z^e, Y^0 determines a pair of signals

$$i^{\text{opt}}, \quad u^{\text{opt}} = e - Z^e i^{\text{opt}}$$

in a few seconds on the computer of IBM PC AT type. The program works immediately in discrete time, without any unnecessary FFT processes, which makes it undoubtedly advantageous. It is simpler, quicker and more exact than in a frequency domain. On the program output a pair of signals

$$u^{\text{opt}}, \quad i^k = i^{\text{opt}} - Y^0 u^{\text{opt}}$$

appears, being an input for a proper procedure of a compensatory circuit selection.

3. Procedure of Compensatory Circuit Synthesis

A compensatory circuit should realize a given pair of periodical signals

$$u^{\text{opt}}(t) = e(t) - Z^e i^{\text{opt}}, \quad i^k(t) = i^{\text{opt}}(t) - Y^0 u^{\text{opt}}(t).$$

Because the two-terminal networks being outside of the compensator are linear circuits with periodically changeable parameters, the compensator should be looked for also in this class. It should consist of the possibly smallest number of periodically changeable elements. These conditions are fulfilled by a parallel circuit $r(t)$, $c(t)$ presented in Figure 2.

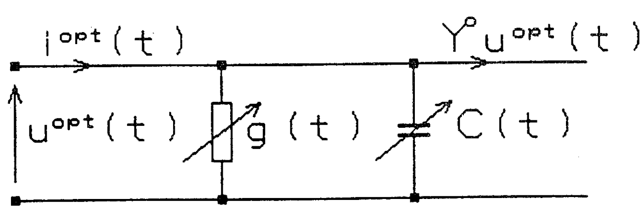


Fig. 2. Scheme of a parametric parallel compensator.

The equations of the compensatory circuit takes the form

$$g(t) u^{\text{opt}}(t) + c(t) \frac{du^{\text{opt}}}{dt} = i^k(t) \tag{16}$$

In this equation we look for periodical conductance functions $g(t)$ and functions of parametric condensator capacitance $c(t)$, at given periodical signals $u^{\text{opt}}(t)$, $i^{\text{opt}}(t)$. It can be proved that it is a minimum system that acts correctly. In the case of a resistor its conductance would have some points of *escape* in all places where voltage passes through zero, whereas in the case of a parametric condensator its capacitance has points of *escape* in the moments when voltage derivative disappears. A unique selection of functions of circuit conductance and capacitance changes is possible after passing to the domain of discrete time. Let us introduce the following notions

$u_n^{\text{opt}} = u^{\text{opt}}(t_n)$; $i_n^k = i^k(t_n)$; $g_n = g(t_n)$; $c_n = \frac{1}{\tau}c(t_n)$ – sampled values of suitable signals

t_n – moments of sampling in equal time intervals τ, $n = 0, 1, 2, ..N-1, N$,

N – number of samples in a period.

Digitized differential equation (16) takes the form

$$u_n^{\text{opt}} g_n + \Delta u_n^{\text{opt}} c_n = i_n^k \tag{17}$$

where $\Delta u_n^{\text{opt}} = (u_n^{\text{opt}} - u_{n-1}^{\text{opt}})\text{Mod}N$. Each of N equations (17) has two unknowns g_n, c_n. To determine them, there must be an additional condition which guarantees the best smoothness of changes for the samples g_n, c_n:

$$(g_n - g_{n-1})^2 + (c_n - c_{n-1})^2 \to \min \tag{18}$$

In this way we receive a minimization problem of functional (18) at the equality condition (17). Considering the samples g_{n-1}, c_{n-1} as temporarily given, and g_n, c_n as parameters searched for the solution of optimization problem (18)–(17) is received in the form of linear equations system

$$\begin{bmatrix} u_n^{\text{opt}} & \Delta u_n^{\text{opt}} & \\ 2 & & u_n^{\text{opt}} \\ & 2 & \Delta u_n^{\text{opt}} \end{bmatrix} \begin{bmatrix} g_n \\ c_n \\ \lambda_k \end{bmatrix} = \begin{bmatrix} i_n^k \\ 2g_{n-1} \\ 2c_{n-1} \end{bmatrix} \tag{19}$$

where λ is a Lagrange's multiplier.

It is good to put system solution (19) in the form of a representation $(g_{n-1}, c_{n-1}) \to (g_n, c_n)$, which takes the form

$$\begin{bmatrix} g_n \\ c_n \end{bmatrix} = \frac{1}{(u_n^{\text{opt}})^2 + (\Delta u_n^{\text{opt}})^2} \left(\begin{bmatrix} (\Delta u_n^{\text{opt}})^2 & -u_n^{\text{opt}} \Delta u_n^{\text{opt}} \\ -u_n^{\text{opt}} \Delta u_n^{\text{opt}} & (u_n^{\text{opt}})^2 \end{bmatrix} \begin{bmatrix} g_{n-1} \\ c_{n-1} \end{bmatrix} + \begin{bmatrix} u_n^{\text{opt}} i_n^k \\ \Delta u_n^{\text{opt}} i_n^k \end{bmatrix} \right)$$

Denoting the vector (g_n, c_n) with x_n, we can shorten it to

$$x_n = A_n x_{n-1} + a_n \tag{20}$$

here $n = 1, 2, ...N-1, N$; $x_n = x_0$; $A_n = A_0$; $a_n = a_0$.
The solution to cyclic equation (20) can be looked for in the form

$$x_n = B_n x_0 + b_n. \tag{21}$$

Thus

$$x_n = A_n(B_{n-1}x_0 + b_{n-1}) + a_n = A_n B_{n-1} x_0 + A_n b_{n-1} + a_n \tag{22}$$

It results from the comparison of expressions (21) and (22) that matrices B_n and vectors b_n fulfil the recurrent equations

$$B_n = A_n B_{n-1} \tag{23}$$

$$b_n = A_n b_{n-1} + a_n \tag{24}$$

for $n = 1, 2, ..., N$; at the initial conditions: $B_0 = I$, $b_0 = \Theta$ (I – unit matrix, Θ – zero vector). A vector $x_0 = (g_n, c_n)$ unknown till now is determined as solution of a linear equations system:

$$(I - B_N)x_0 = b_N \tag{25}$$

Next samples $x_n = (g_n, c_n)$ are calculated recursively from formula (20) or (21). This algorithm is easy to be programmed in a computer, so it has been formed as a procedure. Curves (g_n, c_n) are in diagrams. Matrix $I - B_N$ of equation system (25) must be nonsingular and it is enough for the compensatory circuit. Tests that have been carried out with the help of a computer show that the cases of singular matrices practically do not occur.

4. Examples and Conclusions

The electrical circuit under consideration is presented in Figure 3.

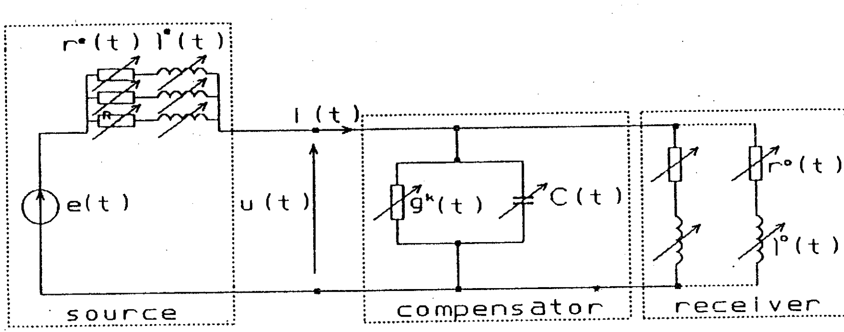
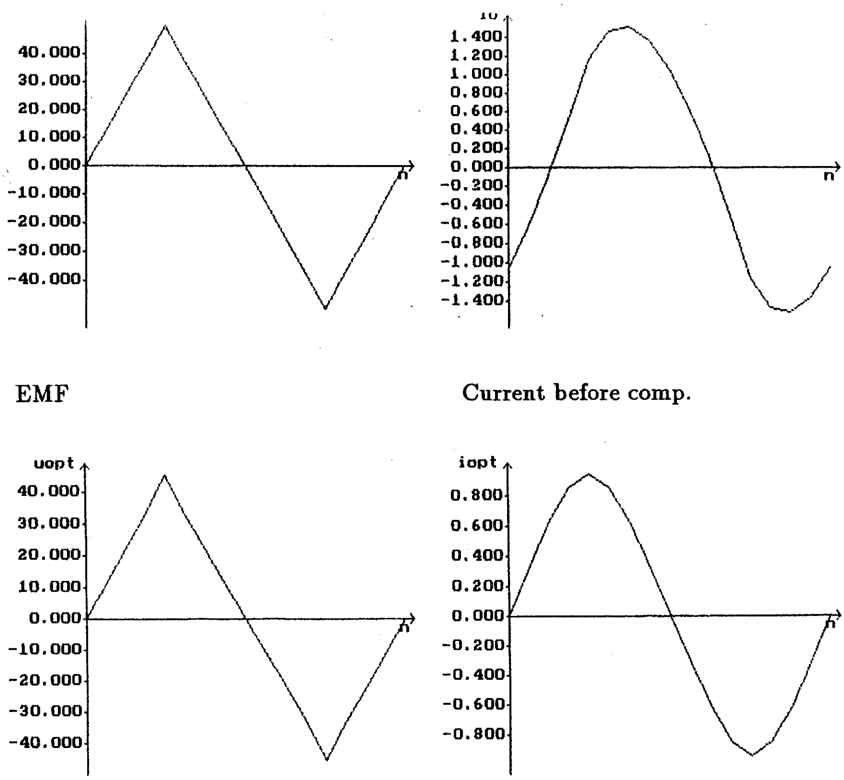


Fig. 3. Actual scheme of the system: source, compensator, receiver.

In series, parallel two–terminal network r, l with periodically changeable resistance and periodically changeable inductance was applied as a load. The number of parallel branches could be optional. A set of signals to control the resistance and inductance was called in a computer program out of a suitable file. The program also made it possible to set amplitudes and phases of controlling signals. A similar structure was in the two terminal network of the source inner impedance. In such a system the deformations of the source current may be introduced not only from the electromotive force $e(t)$ signal side, but also, what is very important, from the side of load and inner impedance two terminal networks. Such two–terminal networks may also serve as a certain approximation of loads and nonlinear sources. In optimum currents and voltages, and compensatory circuit conductance and capacitance (Fig. 4) are shown in the computer diagrams.



Optimal voltage

$Po = 16.320$ $Pk = 0.000$

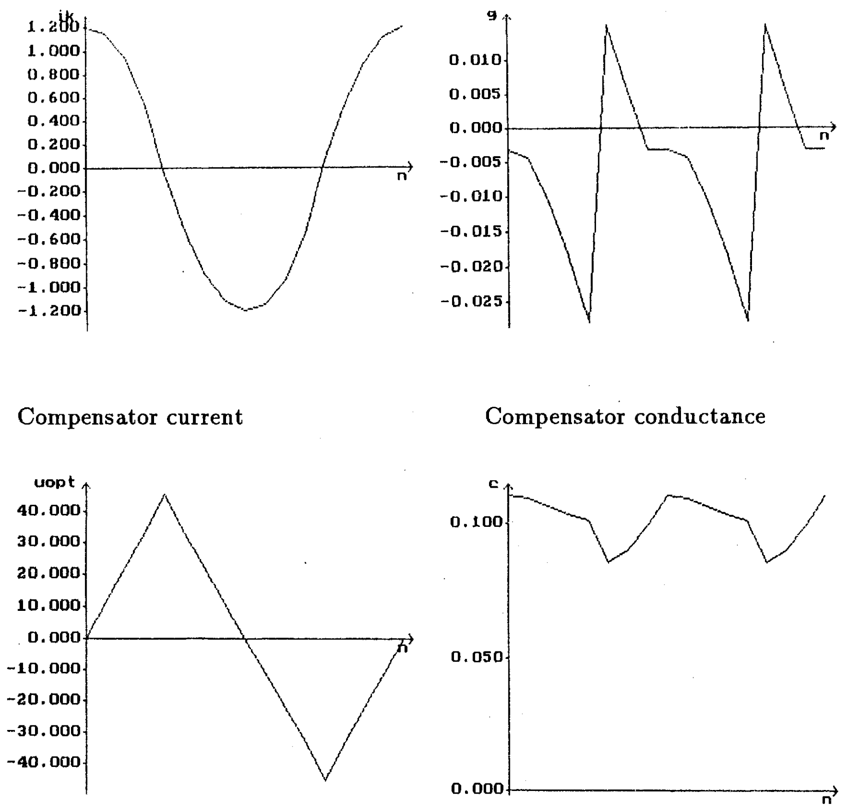
$Pol = 16.687$

Optimal current

$(Lio, io)1/2 = 1.271$ $(Liopt, iopt)1/2 = 0.748$

$\|io\| = 1.084$ $\|iopt\| = 0.644$

Fig. 4a. Illustrative plot of the functions: e, u^{opt}, i_0, i^{opt}.



Compensator current

Compensator conductance

Optimal voltage

$Po = 16.320$ $\quad Pk = 0.000$

$Pol = 16.687$

Compensator capacity

$(Lio, io)1/2 = 1.271$ $\quad (Liopt, iopt)1/2 = 0.748$

$\|io\| = 1.084$ $\quad \|iopt\| = 0.644$

Fig. 4b. Illustrative plot of the functions: i_k, u^{opt}, g, c.

Various functionals of the quality and power conditions were chosen. Two–terminal networks with periodically changeable parameters on the side of the receiver and the source were various,too. Some deformations from the side of the electromotive force were introduced. It can be observed from the signals received that the compensator conductance generally changes the sign. Such a conductance can be realized in the system R–switched with one permanent negative resistance. The keys are controlled periodically by means of a specially adjustments digital automatic machine. Information about the machine adjusting is received from the computer according to g function wave. Capacitance part of the compensator may be realized in the system C–periodically switched, usually without a simulated negative capacitance. Such systems are possible nowadays due to the development of a thyristor inverter technique.

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Received December 27, 1992
Revised May 8, 1993