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CHOICE OF NORM FOR EVALUATING TRADE–OFF SOLUTIONS IN MULTI–CRITERIA OPTIMISATION PROBLEMS IN THE CONTROL OF COMPLEX OBJECTS

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When formulating multi-criteria optimisation tasks, there are two important problems to consider. The first is the selection of a number of sub-criteria such that their importance, number, and order correspond to the modelled decision-making situation, while the second is the choice of the solution method to ensure its optimality with respect to individual sub-criteria. Therefore, the solution boils down to the search for a certain compromise that takes into account the influence of individual sub-criteria on the obtained result. For these reasons, the article presents conditions for selecting the number of sub-criteria, yielding conditions of completeness, consistency, and nonredundancy that the adopted vector criterion must satisfy, as well as conditions for selecting a compromise solution to ensure the utility of all sub-criteria and a lower limit of the maximum value of the individual loss. Using the formulated conditions, a vector quality criterion was selected and a compromise solution was chosen for the task of controlling a ship in a collision situation. The method proposed in the paper can be useful for modelling any decision situation, especially in systems where the task can be solved using the ideal-point method.

Keywords: decision making, vector criterion, polyoptimisation.

1. Introduction

In the current design and construction of control systems for complex facilities that will be operated in conflict situations with many other facilities, under conditions of high uncertainty and risk, there is a high complexity of related decision-making problems (Ceballos *et al.*, 2016). These problems involve decisions that control complex systems (Zak and Balicki, 1991; Zak, 2001) as well as design issues (Zak, 1993; 1994; Papalambros and Wilde, 2000), organisational management (Siskos *et al.*, 2014; Garcia *et al.*, 2016; Stewart, 2010; Cotana *et al.*, 2019; Marseglia *et al.*, 2019; UN, 2019), finance (Marseglia *et al.*, 2019; Doumpos and Zopounidis, 2001), and the management of political (Zopounidis *et al.*, 2015) and marketing activities (Liu *et al.*, 2019).

The decision-making problems addressed in the cited publications are characterised by their high dimension, taking into account many aspects that affect decisions (Podviezko, 2015) and the presence of sources of uncertainty and risk factors (Govindan *et al.*, 2013) or with redundant and incomplete information under a fuzzy environment (Xia *et al.*, 2022). In such a case, it is important to reconcile the conflicting goals we want to achieve, take into account multiple criteria in decision making, and seek compromise solutions (Ishizaka and Nemery, 2013). In such a case, decision makers face the complexity of decision-making situations and require decision support methods and systems (Stewart *et al.*, 2013). In response to such a demand, a number of solutions have been developed dedicated to selected areas of activity, as well as general-purpose methods (Ehrgott *et al.*, 2010).

In this context, multicriteria decision analysis (MCDA) methods are widely used. In addition to their formal basis, these methods are characterised by their ability to deal with multiple conflicting objectives, as well as various stakeholders within the decision-making process (Greco, 1997). In recent years, the rapid development of MCDA methods has been evident (Mardani *et al.*, 2015; Jafaryeganeh *et al.*, 2020; Zhu *et al.*, 2020), as evidenced by scientific publications appearing in journals on the subject. However, they differ considerably in many respects, such as complexity, the way in which preferences and evaluation criteria are represented, the type of data aggregation, the ability to account for uncertain data, and the availability of implementation in decision support systems or criteria compensation (Vansnick, 1986; Saaty and Ergu, 2015; Roy and Słowiński, 2013).

The huge number of possible MCDA methods allows one to find a solution to a decision-making situation properly formulated. Therefore, an important research issue, which has not yet been fully solved, is to determine a method for selecting the number of criteria to evaluate situations and make a choice of the most satisfactory decision that is a compromise that takes into account the impact of the adopted evaluation criteria on the decision made. That is why the formulated research problem is so important for methods of managing industrial, technological, or social systems. In particular, it is an important issue in safety and control systems for complex objects in dynamic states when interacting with other dynamic objects under conditions of uncertainty and risk arising from conflict situations. The complexity is usually due to the very large number of factors influencing the decisions made and controls determined, non-scalar criterion functions, and the difficulty of determining a direct relationship between the components of the criterion function and those influencing the decisions. In addition, in this type of system, especially in conflict situations, there are usually strong restrictions on the allowable time to solve the problem. An additional impediment to their implementation for the decision-maker is the stress resulting from high responsibility for the decisions made and usually a strong restriction on the time for making them. The conditions mentioned above imply the legitimacy of efforts to capture the problems in question in the form of optimisation tasks with multiple quality indicators. Thus, by all means, it is desirable to provide computer support for the activities performed by the decision maker in the distinguished phases of the decision-making process.

There are practically no publications in the literature on methods of selecting sub-criteria including the selection of the number and aspects of evaluation. Few publications such as (Wang *et al.*, 2019; Xia and Wu, 2007; Kannan and Haq, 2007) deal mainly with the selection of evaluation criteria in the process of selecting suppliers in the supply chain. Most often, this problem is solved by experts who arbitrarily determine individual sub-criteria. Such an approach can lead to an unnecessary increase in the dimension of the decision evaluation vector or a failure to take into account relevant criteria, and thus this leads to the lack of best solutions. A prerequisite for optimising decisions is to have a measure for evaluating their quality (Lachowicz, 2015), formulated so that all aspects of the problem are taken into account (Zak, 2020). For this reason, this article will present a method for selecting the number of criteria for assessing the quality of control of complex objects, and, in particular, a method for determining a compromise solution in control problems of complex objects in conflict situations, taking into account both safety and economic aspects.

2. Problem formulation

A key feature of decision optimisation tasks is the existence of a non-empty set of possible solutions, denoted by X. These solutions may represent, for example, the outcomes resulting from a particular decision. When making a decision, it is essential to evaluate the possible outcomes. In complex scenarios, relying on a single criterion is often insufficient to make the best choice. Consequently, decisions are usually evaluated on the basis of multiple scalar criteria. This process can be expressed as follows:

$$F : X \longrightarrow \mathbb{R}^N, \tag{1}$$

where $\mathbb{R}$ is the real number set.

The function F assigns the evaluation to the solution $x \in X$, as follows:

$$F(x) = (F_1(x), F_2(x), \dots, F_n(x), \dots, F_N(x)) \in \mathbb{R}^N, \tag{2}$$

where $N = (1, 2, \dots, n, \dots, N)$ is the number of coordinates of the vector criterion.

We are considering a set of possible solutions $\langle x_1, x_2, \cdots, x_i, \dots, x_m \rangle \in X$. We assume that solutions x_i, are evaluated using the following criteria:

$$F(x_i) = (F_1(x_i), F_2(x_i), \dots, F_N(x_i)) \in \mathbb{R}^N. \tag{3}$$

The challenge in formulating vector optimisation tasks is to select N scalar criteria in such a way that their meaning, number and order are consistent with the decision context being modelled. As a result, the task of defining the vector criterion (1) can be expressed as follows (Zak and Balicki, 1991; Zak, 2001; Zak, 2020):

$$Q(F^*) = \sup\{Q(F)\}, \quad F \in \mathbf{F}, \tag{4}$$

where $\mathbf{F}$ is the set of acceptable vector criteria and $Q(F)$ is the quality indicator.

In the system design process, both the user and the designer determine acceptable scalar criteria by evaluating the quality of the solutions to the decision-making task. Therefore, the user's preference for the desired characteristics of the solutions should be taken into account when selecting the number of sub-criteria and determining the appropriate dominance relationship. This allows the user to express his or her preferences when formulating a multi-criteria optimisation task. The correct choice of the number of sub-criteria is essential when modelling a specific decision scenario. From the user's point of view, the inclusion of an irrelevant sub-criterion may result in recommending solutions with undesirable characteristics and omitting solutions with valuable characteristics. Multi-criteria optimisation provides solutions that are optimal under multiple criteria, where it is not always possible to maximise each criterion individually. If there is no solution that satisfies each of the sub-criteria, a Pareto-optimal or non-dominated solution is sought.

Finding a polyoptimal solution for engineering problems can be done using one of the following algorithmic approaches.

- construction of the so-called meta-criterion in the form of a weighted sum of sub-criteria or a weighted sum of the degree of fulfilment of sub-criteria,
- application of the criterion substitution principle, i.e., reduction to a one-dimensional problem (minimisation of only one criterion),
- minimising the distance to the 'ideal point.'

Out of the algorithmic approaches presented above, the 'ideal point' method allows the determination of a compromise solution to a decision task, taking into account equally the decisions to be made for all partial criteria occurring in the defined vector criterion function. The compromise solutions are determined on the basis of the value of a certain norm containing a parameter p, which assesses the distance of individual solutions in the space of the criterion from the ideal point. The parameter p has important influence on the selection of a compromise solution, so its value should be sought to ensure the minimum value of the maximum loss and the highest group utility of the sub-criteria. This task can be formulated as a multi-criteria optimisation task of the form

$$(p, F, \leq), \tag{5}$$

where F is a criterion function with two sub-criteria, i.e., the highest group utility and the lowest individual loss.

3. Method description

3.1. Conditions for selecting sub-criteria. By selecting a set of N scalar sub-criteria, $F_1, F_2, \ldots, F_n, \ldots, F_N$ representing the coordinates of the vector criterion, this set should satisfy three basic conditions: the completeness condition, the consistency condition and the non-redundancy condition (Zak, 1994).

Satisfaction of the first condition excludes a situation where the user chooses one of two different solutions that have the same value. However, this condition does not hold if the user chooses one of two non-dominant solutions according to the definition of the dominance relation, so that in this situation the user does not prefer x to z. Satisfying the second condition, the consistency condition, prevents the following scenario: two different solutions $x, z \in X$ with identical images $F(x) = F(z)$ are modified. In the first case, the solution is modified by 'improving' at least one of its subgrades, so that $F(x^+), F(x) \in R$, where R represents the dominance relation in the polyoptimisation task. By contrast, the second solution is modified by selecting the solution that has at least one 'worse' sub-rating according to the dominance relation, resulting in $F(z), F(z^-) \in \mathbb{R}$. In this situation, it would not be rational for the user to prefer the z^- solution to the x^+ solution, as this suggests a faulty formulation of the criterion function.

The non-redundancy condition for sub-criteria in a vector criterion is met if removing one of the scalar criteria the remaining sub-criteria makes fail the completeness and consistency conditions. Therefore, sub-criteria should not be removed if they reflect user preferences. However, it is also important not to include too many sub-criteria, as this would prevent the non-redundancy condition from being met. Through initial modelling, it is usually possible to identify a set of basic scalar sub-criteria that form a vector base criterion F. However, in multi-criteria optimisation problems, there are often constraints imposed on the scalar sub-criteria that effectively limit their values.

The vector base criterion F can be modified by deleting or adding a sub-criterion according to the principles of vector criterion modification and related theorems with proofs presented by Zak (2020). Based on the methodology presented in this thesis, it is possible to formulate a vector criterion formulation scheme when we have a set of scalar criteria that affect the quality of the solutions obtained and thus the impact of the decisions made. When a vector criterion is modified, the effectiveness of such a decision is assessed by the relationship between a function that determines the overall confidence level for the base criterion and after modification.

The modification can be an extension of the vector criterion, understood as the addition of a scalar sub-criterion to the vector base criterion, or a reduction of the vector criterion by removing the scalar sub-criterion. If the reduction or addition of a sub-criterion does not change the subjective confidence levels determined for the individual sub-criteria, and the overall confidence level of the resulting vector criterion after modification is greater than before, then the modification is considered effective (Zak, 2020) and the modified criterion should be used for evaluation and selection of control decisions.

3.2. Conditions for the selection of a compromise solution. The search for the best solution will be mediated by comparing the solutions determined in the criterion space. In order to make such an evaluation, we need to determine a certain relation $R \in Y \times Y$, called the dominance relation, which will allow us to determine whether or not the evaluation of the state of the system $F(x_1) = y$ is better than $F(x_2) = w$.

Definition 1. The relation $R \in Y \times Y$, being the set of pairs (y, w) such that $\{y, w\} \in Y$, and that the decision maker prefers y to w, will be called the *dominance relation*.

Definition 2. The *multicriteria optimisation task* is defined as the ordered triple (X, F, R).

Definition 3. The *set of results dominant in the task* (X, F, R) is understood as

$$Y_D^R = \{y \in Y | (y, z) \text{ at } R \text{ for every } z \in Y - \{y\}\}. \quad (6)$$

The *set of dominant states* X_D^R of the task (X, F, R) is defined as the preimage of the set Y_D^R:

$$X_D^R = F^{-1}\{Y_D^R\} \qquad \text{for every } x \in X | F(x) \in Y_D^R. \quad (7)$$

However, this set is often empty. Therefore, another definition of the solution of the task (X, F, R) is used.

Definition 4. The *set of non-dominated solutions* X_N^R of the task (X, F, R) is defined as the preimage of the set of non-dominated results X_N^R,

$$X_N^R = F^{-1}\{Y_N^R\} \qquad \text{for every } x \in X | F(x) \in Y_N^R, \quad (8)$$

where

$$X_N^R = \{y \in Y | \text{ there is no } z \in Y \text{ such that } (z, y) \in R\}. \quad (9)$$

Dominated and non-dominated solutions of multi-criteria optimisation tasks have many disadvantages from the point of view of their practical application. Here are some of them:

- the set of dominant solutions is very often empty;
- the set of non-dominated solutions is usually very large or empty;
- the task of determining all non-dominant solutions is generally very difficult;
- in the case of an extensive set of non-dominant solutions, a practical dilemma arises as to which of the many non-dominant solutions to use.

The practical need of decision-making models for definitions of the solution x of a multi-criteria optimization task is expressed by the following postulates:

- a solution x should exist for as the widest possible class of tasks,
- a solution x should be the unique,
- a solution x should be a non-dominated solution as long as the set of non-dominated solutions is not empty.

The above-mentioned postulates are generally realised by the so-called compromise solution. The concept of compromise in choosing the final solution of the task $(X, F, >)$ can be reduced to two aspects:

- selection of an ideal point $\hat{y} \in \mathcal{F}(X)$, which will be considered the most desirable result lying in the extended solution space $\mathcal{F}(X)$; the coordinates of this point can be determined by solving the optimization task sequentially for the individual coordinates of the vector criterion;
- choosing the form of the norm of the vector $(\hat{y} - y)$, determining the distance of the point $y \in Y$ from the ideal point $y \in \mathcal{F}(X)$.

The choice of the ideal point $\hat{y}$ can be dictated by the considerations and specifics of the decision situation being modelled and mainly by the dominance relation adopted. It should be a point of the extended criterion space $\mathcal{F}(X)$ with the most desirable properties, not necessarily belonging to the set $Y \in F(X)$. If $\hat{y} \in Y$, then the set of solutions is obviously $F^{-1}(\hat{y}) = 0$. However, most of the time this is not the case. For the task $(X, F, >)$, the ideal point can be defined as follows:

$$\hat{y}_n = \sup_{x \in X} F_n(x), \quad n \in \mathbb{N}. \quad (10)$$

The distance of the result $y \in Y$ from the result $\hat{y}$ can be determined by introducing some metric or simply a vector norm $(\hat{y} - y), y \in Y$.

Definition 5. The *distance of the result y from the ideal result* $\hat{y}$ is defined as the value of the norm $R^{\hat{y}}(y)$ of the vector $(\hat{y} - y), y \in Y$. The distance of the result $(y \in Y)$ from the ideal result $\hat{y}$ can be measured by any function $R^{\hat{y}}(\cdot)$, which has the norm property. The specific form of the function $R^{\hat{y}}(\cdot)$ should result from the analysis of the decision-making situation.

Definition 6. The set of compromise solutions with parameter $p \geq 1$, which has an impact on how all the sub-criteria are taken into account in the evaluation, is understood as the set

$$X_p^{\hat{y}} = F^{-1}(Y_p^{\hat{y}}), \tag{11}$$

where

$$Y_p^{\hat{y}} = \{y \in Y | R_p^{\hat{y}}(y) = \inf_{y \in Y} R_p^{\hat{y}}(y)\}$$

while

$$R_p^{\hat{y}} = \|\hat{y} - y\|_p = \Big(\sum_{n \in N} (|\hat{y} - y|)^p \Big)^{\frac{1}{p}}, \quad y \in Y. \tag{12}$$

From the above, we conclude the following:

- In the case of $p = 1$, we get a weighted sum of sub-criteria. This is the case that is often used, although not very interesting, because it is necessary to determine the weights of each sub-criteria, which is affected by the subjectivity of the designer, and thus the result of the solution.

- For $p = 2$, we obtain the Euclidean norm. The result obtained $y \in Y$ such that $R_p^{\hat{y}}(y) = \inf_{y \in Y} R_p^{\hat{y}}(y)$, is the result closest to the result $\hat{y} \in Y$ in the sense of the geometric distance in the extended criterion space. In the case of $p = \infty$, we obtain the following:

$$R_\infty^{\hat{y}}(y) = \sup_{n \in \mathbb{N}}(|\hat{y}_n - y_n|). \tag{13}$$

  and

$$\begin{aligned} Y_\infty^{\hat{y}} = \{y \in Y | &\sup_{n \in \mathbb{N}}(|\hat{y}_n - y_n|) \\ &= \inf_{y \in Y} \sup_{n \in N}(|\hat{y}_n - y_n|)\}. \end{aligned} \tag{14}$$

Compromise solutions with parameter $p > 1$ have a number of very interesting properties and are significantly related to non-dominated solutions with the relation $>$ (optimisation in the Pareto sense). The most important properties of the results that determine the compromise solutions with the parameter $p > 1$ are as follows:

- if the set Y is a compact set, then for $p > 1$ there exists a non-empty set $Y_p^{\hat{y}}$;

- the total loss represented by the $\|\hat{y} - y^p\|_p$ is the minimum loss that can be obtained by having the results from the set Y,

$$\|\hat{y} - y^p\|_p = \inf_{y \in Y} \|\hat{y} - y\|_p; \tag{15}$$

- the result $y^p \in Y_p^{\hat{y}}$ is an intermediate result the influence of all sub-criteria $F_n(\cdot)$ on the choice of the final solution $F^{(-1)}(\hat{y}^p)$ is taken into account;

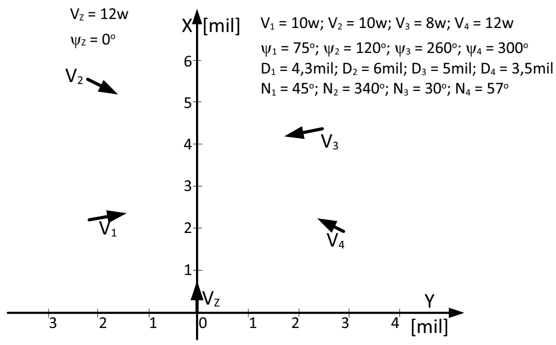

Fig. 1. Collision situation of our own ship with four ships encountered, with V, Ψ, D, N as the parameters of the encountered objects, i.e., speed, course, distance and bearing of the 1-st, 2-nd, 3-rd and 4-th encountered ship, respectively; V_z and Ψ_z are parameters of our own ship motion.

- for $1 < p < \infty$, the set $Y_p^{\hat{y}}$ is a subset of the set $Y_N^>$ of non-dominated outcomes in the Pareto sense; thus any compromise solution with $1 < p < \infty$ is optimal in the Pareto sense;

- if the set Y is convex, then for $1 < p < \infty$ the set $Y_p^{\hat{y}}$ is a singleton.

If one were to interpret $y_n^p = F_n(x)$ as an evaluation of the solution x in terms of the n-th criterion, then $\sum y_n^p$ would be the sum of the utilities of all criteria, In turn, the number $\sup_{n \in \mathbb{N}}(\hat{y}_n - y_n^p)$ determines the maximum value of the individual loss when accepting the compromise result y^p.

4. Example of the choice of parameter p

The issue of forming a multidimensional form of the criteria used for decision evaluation, as well as the method of determining a compromise solution, are important problems in many decision-making tasks. In the article, as an example of the application of the presented method of determining the compromise solution, the task of forming control signals of a complex floating object in a collision situation using a vector criterion for evaluating the quality of control will be considered.

Analysing the process of controlling a floating object in a collision situation and taking into account the requirements for collision avoidance, we conclude that the evaluation of control should be done in terms of traffic safety, and this aspect of control is the most important. In the set of safe control systems, we can look for solutions that provide optimal control in economic terms. Therefore, in forming the form of the vector criterion, we can take into account scalar quantities evaluating traffic safety and economic aspects.

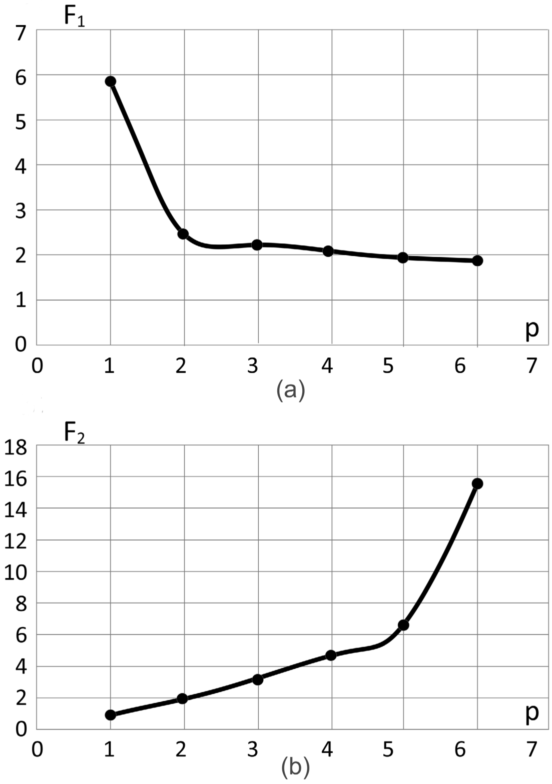

Fig. 2. The coordinates of the vector criterion as a function of the parameter p evaluating the choice of a compromise solution y^p : the value of group utility (a), the maximum value of individual loss (b).

In the first group of criteria, we can distinguish the following quantities:

- the smallest proximity distance between the floating object and the objects encountered,
- the time remaining to reach the shortest proximity distance,
- the collision risk index,
- the collision risk angle,
- the aspect of the j-th object.

On the other hand, the following criteria can be distinguished among those evaluating steering from the point of view of economics:

- the time lost per anti-collision maneuver,
- the fuel consumption per anti-collision maneuver,
- the road loss per anti-collision maneuver,
- the deviation from the set trajectory,

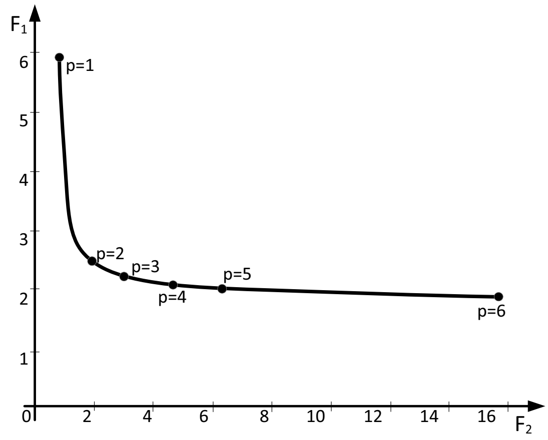

Fig. 3. The waveform, in the criterion space, of the function evaluating the choice of parameter p .

- the deviation from the set course.

The forms of the first group of criteria depend on the motion parameters of the floating objects in a collision situation and their mutual position, while the second group depends solely on the quantities that characterise its own floating object. The mathematical relationships of these criteria are presented in (Zak, 2020). Based on the specified sub-criteria, according to the methodology presented in Sections 3.1 and 3.2, a vector criterion was formulated to assess the quality of control, taking into account the aspect of traffic safety and the economic aspect.

An example of selecting the form of the vector criterion for evaluating control quality for a floating object in a collision situation is presented in the paper by Zak (2020). The vector criterion takes the following form:

$$F(x) = [F_1(x), F_2(x), F_3(x), F_4(x)], \qquad (16)$$

where $F_1(x)$ is the smallest distance from the ideal point, $F_2(x)$ is the time remaining to reach the smallest proximity distance, $F_3(x)$ is the time lost for the anticollision manoeuvre, and $F_4(x)$ is the fuel consumption for the anticollision manoeuvre.

The following constraints are imposed on the individual coordinates of the vector criterion:

$$F_1(x) \geq D_b, \quad F_2(x) \geq T_b, \quad F_3(x) \geq T_p, \quad F_4(x) \geq Z_p, \qquad (17)$$

where D_b is the safe proximity distance, T_b is the time remaining to reach the safe proximity distance, T_p is the planned time for four own ship to pass along a given trajectory without an anti-collision maneuver, Z_p is the planned fuel consumption of our own ship along a given trajectory without an anti-collision maneuver.

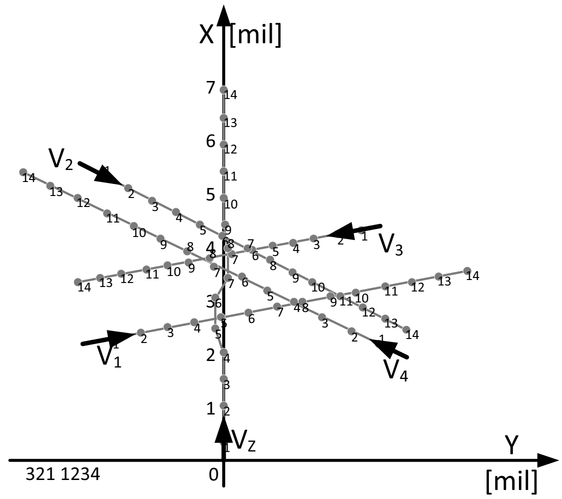

Fig. 4. The motion trajectories of our own ship and four ships encountered being in the collision situation shown in Fig. 1, using the vector criterion.

Using the imposed constraints, it is possible to normalize the vector criterion, which we will write in the form

$$F(x) = [\overline{F_1(x)}, \overline{F_2(x)}, \overline{F_3(x)}, \overline{F_4(x)}], \tag{18}$$

where

$$\overline{F_1(x)} = \frac{F_1(x)}{D_b} \geq 1,$$
$$\overline{F_2(x)} = \frac{F_2(x)}{T_b} \geq 1,$$
$$\overline{F_3(x)} = \frac{F_3(x)}{T_p} \geq 1,$$
$$\overline{F_4(x)} = \frac{F_4(x)}{Z_p} \geq 1.$$

The formulated form of the criterion was used to determine the trajectory of motion of a floating object in the collision situation shown in Fig. 1 with four floating objects encountered.

For the collision situation presented, assuming that the encountered ships do not manoeuvre, we are looking for a compromise solution of the multi-criteria optimisation task formulated in the form $(X, \overline{F}, \leq)$. For such a formulated task, the problem can be reduced to two aspects:

- selection of an ideal point that is the most desirable result lying in the extended criterion space $\hat{y} \in \mathcal{F}(X)$,
- the choice of the form of the norm of the vector $(\hat{y} - y)$ determining the distance of the point $y \in F(x)$ from the ideal point $\hat{y} \in \mathcal{F}(X)$.

The choice of the ideal point $\hat{y}$ is dictated by the considerations and specifics of the decision situation to be modelled and mainly by the adopted dominance relation. For tasks $(X, F, \leq)$ the coordinates of the ideal point can be defined as follows:

$$\hat{y}_N = \inf_{x \in X} F_N(x), \quad n \in \mathbb{N}. \tag{19}$$

so in this case it is an ideal point with coordinates $\hat{y} = [1, 1, 1, 1]$. In order to determine the distance of the result $y \in Y$ from the ideal result, we take a vector norm with parameter p in the form

$$\begin{aligned} R_p^{\hat{y}} &= \|\hat{y} - y\|_p \\ &= \left(\sum_{n \in \mathbb{N}} (|\hat{y} - y|)^p \right)^{\frac{1}{p}}, \quad y \in Y. \end{aligned} \tag{20}$$

For the collision situation shown in Fig. 1, for the possible controls, the values of the norm that evaluate each control were determined and the compromise controls were selected for different values of the parameter p, where $1 < p < \infty$.

The task of selecting the value of the parameter p was reduced to a multi-criteria optimization task formulated in the form $(p, F, \leq)$, with the criterion function taking the form

$$F(F_1, F_2). \tag{21}$$

where F_1 is the utility value of the group expressed by the relation, and F_2 means the maximum value of the individual loss.

Interpreting the value of $y_n^p = F_n(x)$ as an evaluation of the solution x in the sense of the n-th criterion, the value $F_1 = \sum y_n^p$ would be the sum of the utility of all criteria, while the value $F_2 = \sup_{n \in \mathbb{N}} (\hat{y}_n - y_n^p)$ determines the maximum value of the individual loss when accepting the compromise result y^p. For the collision situation presented in Fig. 1, the changes in the group utility and the smallest individual loss were determined for different parameters p. The results obtained are shown in Figs. 2 and 3.

In the set of non-dominated solutions, for different values of the parameter p, we are looking for a compromise solution with a dominance relation of the form $\leq$. With such a relation, the ideal point will be the origin of the coordinate system. Therefore, to select a compromise solution, we will use the norm of the form

$$R_q = \|F\|_q = \left(\sum_{n}^{2} (F_n)^q \right)^{\frac{1}{q}}. \tag{22}$$

The norm values determined for different parameters p are shown in Table 1. From these results, it can be seen that the norm value in the criterion space of $R_q = 3.07$ is the closest result to the ideal result for the adopted

Table 1. Values of the norm evaluating compromise solutions for different values of p.

Parameter p	1	2	3	4	5	6
F_1	5.91	2.405	2.195	2.006	2.001	1.85
F_2	0.982	1.908	3.096	4.648	6.338	15.328
R_q	5.991	3.07	3.795	5.062	6.646	15.433

method for evaluating the compromise solution of the parameter p. Thus, the value of the parameter p of the norm that evaluates the compromise solution with the vector indicator of the quality of the ship control in a collision situation is $p = 2$; this value corresponds to the Euclidean norm of the compromise solution from the ideal solution. With an increase in the parameter $p > 2$ and for $p = 1$, the value of the evaluation norm increases, i.e., the evaluation includes the increase in the maximum value of the individual loss and the decrease in the value of the utility sums of all criteria.

For the collision situation of Fig. 1, with the vector control quality index (30), non-dominated solutions in the criterion space for different controls were determined. To determine a compromise solution of this collision situation, the value of the parameter $p = 2$ was assumed in relation (32) and the values of the norm evaluating the various controls in the criterion space of the control were determined and the compromise control of our own ship was selected closest to the ideal solution in terms of geometric distance.

Assuming this solution, the trajectory of our own ship (Fig. 4) and those of the ships encountered was determined, assuming that our own ship performs the manoeuvre by changing its course while maintaining a constant speed, while the other ships do not perform course and speed manoeuvres; therefore, their trajectories are rectilinear. The successive states of our own ship and the ships encountered (described by consecutive numbers) are switched every 2.5 min.

5. Conclusions

Decision making is an indispensable part of our private and professional lives. Such decisions may concern, for example, the acceptance of a technical solution, the selection of the best solution from among the available options, or the selection of the most appropriate controls under conditions of uncertainty and risk. The above issues are related to engineering decisions that involve the search for optimal solutions, so the task must be a well-defined problem. Thus, the designer is faced with the task of formulating an optimisation problem involving the formulation of criteria for evaluating the decisions, actions, or effects of control of any system. For this reason, the article presents a method for selecting the number of criteria for evaluating the quality of decisions and searching for compromise solutions, which can find application in modelling decision-making situations, especially in engineering problems related to the control of complex systems under conflict conditions. In such cases, depending on the time horizon and external conditions, making an optimal decision can be difficult and often involves a risk of error. Therefore, when designing a control device, the designer must formulate a vector criterion function to analyse multiple alternatives in a limited time horizon and a method for determining a compromise solution that takes into account the influence of various sub-criteria on the final control result.

This work presents the conditions that must be met by the vector criterion adopted for the evaluation of the decision situation, ensuring the selection of the number of scalar criteria in the vector criterion adequate to the modelled decision-making situation and the method of selecting the parameter p in the norm ensuring the determination of a compromise solution in the modelled decision situation (Sections 3.1 and 3.2). The parameter p affects the value of the group utility of all criteria and the value of the maximum individual loss. An increase in the value of p causes, on the one hand, a decrease in group utility and, on the other hand, a decrease in the maximum value of individual loss. In this sense, the parameter p is a tool for mediating the impact on the choice of the final solution of the need for an increase in group utility, as well as the need to reduce the loss of an individual criterion. The choice of a large p to determine the appropriate compromise solution favours the interests of the individual criterion, while the utility of the group is taken into account to a lesser extent. Thus, the final choice of the parameter p depends on what should be preferred more in a given decision-making situation (Section 3.3).

The basis for the selection of the parameter p is the simulation studies of the designed system, which are carried out for various variants of the decision. Upon simulation of these variants, compromise controls are determined for different values of the parameter p (Section 4). The choice of the parameter is made based on the value of the maximum individual loss and the value of the group utility of the criterion function for different values of p. In these sets, a compromise solution is sought that provides the minimum value of the norm, evaluating the effect of the parameter p on the maximum individual loss value and the utility value of the group.

The presented method was used to formulate a decision-making task for a marine anti-collision system.

The determined vector criterion and the method of selecting the parameter p, and thus the determined compromise control based on the formulated method, ensure the selection of safe control in a collision situation of the ship's movement and is optimal in economic terms (Fig. 4).

It should be concluded that the formulated method of selecting the number of scalar criteria in the vector criterion and selecting the parameter p in the method of determining the compromise solution can be used to formulate optimal control issues, as well as to solve other decision-making tasks formulated as multi-criteria optimisation tasks. However, the applicability of this approach is limited to decision problems for which we can define scalar criteria and specify their confidence levels. However, the presented approach does not limit the number of scalar criteria that can be used to evaluate the decision situation under analysis, nor their complexity.

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